Modeling of a Didactic Magnetic Levitation System for Control Education

Milica B. Naumović

Abstract - **The magnetic levitation control system of a metallic sphere is an interesting and visual impressive device successful for demonstration many intricate problems for control engineering research. The dynamics of magnetic levitation system is characterized by its instability, nonlinearity and complexity. In this paper some approaches to the levitation sphere modeling are addressed, that may be validate with experimental measurements.**

Keywords - **magnetic levitation system, control engineering education, system modeling**

I. Introduction

Magnetic levitators not only present intricate problems for control engineering research, but also have many relevant applications, such as high-speed transportation systems and precision bearings. From an educational viewpoint, this process is highly motivating and suitable for laboratory experiments and classroom demonstrations, as reported in the engineering education literature [1]-[8].

The classic magnetic levitation control experiment is presented in the form of laboratory equipment given in Fig.1. The complete purchase of the Feedback Instruments Ltd. Maglev System 33-006 [9] is supported by WUS (World University Service [10]) – Austria under Grant CEP (Center of Excellence Projects) No. 115/2002. This attraction-type levitator system is a challenging plant because of its nonlinear and unstable nature. The suspended body is a hollow steel ball of 25 mm diameter and 20 g mass. This results in a visually appealing system with convenient time constants. Both analogue and digital control solutions are implemented. In addition, the system is simple and relatively small, that is portable.

This paper deals with the dynamics analysis of the considered magnetic levitation system. Although the gap between the real physical system and the obtained nominal design model has complex structure, it should be robust stabilized in spite of model uncertainties.

II. System Description

The Magnetic Levitation System (Maglev System 33-006 given in Fig. 1) is a relatively new and effective laboratory set-up very helpful for control experiments. The basic control goal is to suspend a steel sphere by means of a magnetic field counteracting the force of gravity. The Maglev System consists of a magnetic levitation mechanical unit (an enclosed magnet system, sensors and drivers) with a computer interface card, a signal conditioning unit, connecting cables and a laboratory manual.

Milica B. Naumović is with the Faculty of Electronic Engineering, University of Niš, Beogradska 14, 18000 Niš, Yugoslavia, E-mail: nmilica@elfak.ni.ac.yu

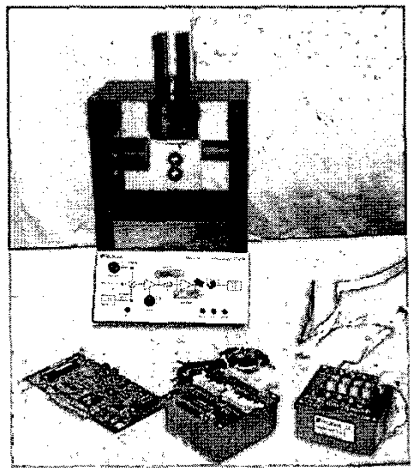

Fig. 1. Photograph of magnetic levitation system

In the analogue mode, the equipment is self-contained with inbuilt power supply. Convenient sockets on the enclosure panel allow for quick changes of analogue controller gain and structure. The bandwidth of lead compensation may be changed in order to investigate system stability and time response. Moreover, user-defined analogue controllers may be easily tested. Note, that the position of the sphere may be adjusted using the set-point control and the stability may be varied using the gain control.

In the digital mode, the Maglev System operates with MATLAB® /SIMULINK® software. Feedback Software for SIMULINK® is provided for the control models and interfacing between the PC and the Maglev system hardware.

The Maglev System, both in analogue and digital mode, allows the study of various control strategies and other issues from system theory, as follows:

- *Analogue mode*
  - Nonlinear modeling
  - System stabilization
  - Linearization about an operating point
  - Infrared sensor characteristics
  - Closed-loop identification
  - Lead-lag compensation
  - Perturbation sensitivity
  - PD control;
- *Digital mode*

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- Nonlinear modeling
- System stabilization
- Linearization about an operating point
- A/D and D/A conversion
- Closed-loop identification
- Perturbation sensitivity
- State space PD control
- Position regulation and tracking control.

III. System Modeling and Identification

A schematic diagram of the single-axis magnetic levitation system with principal components is depicted in Fig. 2. The applied control is voltage, which is converted into a current via the driver within the mechanical unit. The current passes through an electromagnet which creates the corresponding magnetic field in its vicinity. The sphere is placed along the vertical axis of the electromagnet. The measured position is determined from an array of infrared transmitters and detectors, positioned such that the infrared beam is intersected by the sphere.

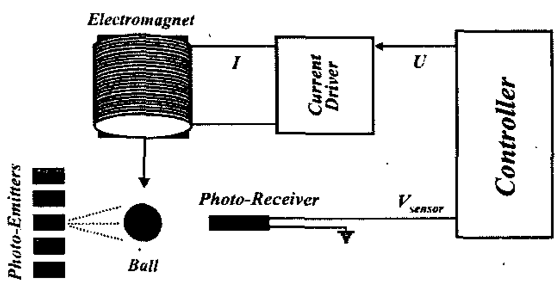

Fig. 2. Main components of the magnetic levitation system

The infrared photosensor is assumed to be linear in the required range of operation. Current I is regulated by an inner control loop and is linearly related to input voltage U.

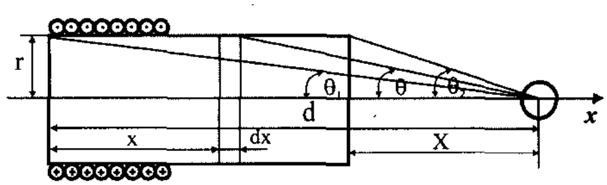

Fig. 3. Sphere and coil arrangement

Using the fundamental principle of dynamics, the behaviour of the ferromagnetic ball is given by the following electromechanical equation

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = mg + f(x,i) \quad , \tag{1}$$

where m is the mass of the levitated ball, g denotes the acceleration due to gravity, x is the distance of the ball from the electromagnet, i is the current across the electromagnet, and $f(x,i)$ is the magnetic control force.

A. Calculating the magnetic control force on the metallic sphere

Consider a solenoid with an r radius, an l length, crossed by an I current. The sphere is located on the axis of the coil as shown in Fig. 3. The effect of the magnetic field from the electromagnetic is to introduce a magnetic dipole in the sphere which itself becomes magnetized. The force acting on the sphere is then composed of gravity and the magnetic force acting on the induced dipole.

The magnetic control force between the solenoid and the sphere can be determined by considering the magnetic field as a function of the ball's distance x from the end of the coil.

The magnetic field at some given point (see Fig. 3), may be calculated according to the Biot-Savart-Laplace formula [11]. Recall, that the magnetic field produced by a small segment of wire, $\mathrm{d}l$, carrying a current I (see Fig. 4a) is given by

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{\mathrm{d}\mathbf{l}\times\mathbf{r}}{r^3} \quad , \tag{2}$$

where μ_0 is the permeability of the free space and $\mathrm{d}\mathbf{l}\times\mathbf{r}$ is the vector product of vectors $\mathrm{d}\mathbf{l}$ and $\mathbf{r}$.

Hence, the magnitude of the magnetic field becomes

$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l\sin\alpha}{r^2} \quad . \tag{3}$$

The magnetic field of a circular contour with an a radius, as shown in Fig. 4b, is given by

$$B = \frac{\mu_0 I}{2}\frac{a^2}{\left(a^2+d^2\right)^{3/2}} \quad . \tag{4}$$

Note, that from considerations of symmetry, the field component perpendicular to the coil axis $\mathrm{d}B_2$ must be zero on the axis.

In order to evaluate the field due to the many turns (N) along the axis of the coil, let n be the number of turns per metre. Also, consider the solenoid given in Fig. 3 as a series of equidistant circular contours at the mutual distances $\mathrm{d}x$, carrying the current $nI\,\mathrm{d}x$. The total axial field from all turns of the coil becomes

$$B = \int \mathrm{d}B = \frac{\mu_0}{2} nI \int \mathrm{d}x \frac{\sin^3\theta}{r} \quad . \tag{5}$$

Integrating Eq. (5) within the interval $\theta_1 \le \theta \le \theta_2$ gives

$$B=\frac{\mu_0}{2}nI\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta=\frac{\mu_0}{2}nI(\cos\theta_1-\cos\theta_2)\ , \quad (6)$$

which can be rewritten in the form

$$B=\frac{\mu_0}{2}nI\left(\frac{X+l}{\sqrt{r^2+(X+l)^2}}-\frac{X}{\sqrt{r^2+X^2}}\right) . \quad (7)$$

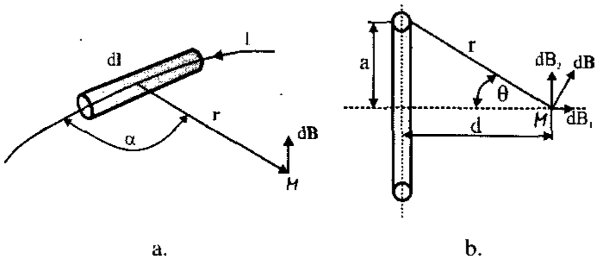

Fig. 4. Magnetic field
a. produced by a current segment $I\,dl$;
b. produced by a current circular contour.

However, the electromagnet is composed of many turns layers with variable radius $r_1 \le r \le r_2$. Using the foregoing, in conjunction with Eq. (7), the magnetic field becomes

$$dB=\frac{\mu_0}{2}nI\left(\frac{X+l}{\sqrt{r^2+(X+l)^2}}-\frac{X}{\sqrt{r^2+X^2}}\right)n\,dr . \quad (8)$$

And so, the total magnetic field is given by

$$B=\frac{\mu_0 n^2 I}{2}\left((X+l)\ln\left|\frac{r_2+\sqrt{r_2^2+(X+l)^2}}{r_1+\sqrt{r_1^2+(X+l)^2}}\right|+X\ln\left|\frac{r_1+\sqrt{r_1^2+X^2}}{r_2+\sqrt{r_2^2+X^2}}\right|\right)$$

$$=C_1\,I\,G(X) \quad (9)$$

The magnetic force on the metallic sphere can be expressed as [11]

$$f=\frac{B^2}{2\mu_0}S\ , \quad (10)$$

where S is the material surface crossed by the magnetic flux. Finally, the upwards force on the ball due to the field B is given by

$$f=-CI^2\left((X+l)\ln\left|\frac{r_2+\sqrt{r_2^2+(X+l)^2}}{r_1+\sqrt{r_1^2+(X+l)^2}}\right|+X\ln\left|\frac{r_1+\sqrt{r_1^2+X^2}}{r_2+\sqrt{r_2^2+X^2}}\right|\right)^2$$

where $$C=\frac{\mu_0 n^4}{8}S\ . \quad (11)$$

Another approach to the determination of the force on the ball due to the field is based on the fact that the force is proportional both to the induced dipole strength and field strength. Namely, the magnetic field induces a dipole in the sphere whose strength is proportional to the magnetic field and that means to the coil current. Of course, it is necessary to make the assumption, that the sphere is magnetized within the linear part of the magnetization curve and does not reach saturation.

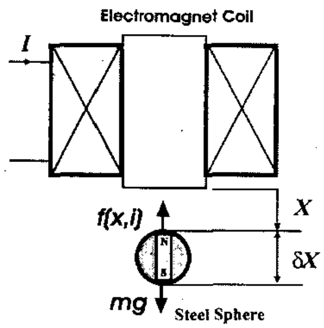

Fig. 5. Levitation of the sphere

The location of the dipole within the sphere is such that each pole is at the center of mass of its respective hemisphere (see Fig. 5).

The forces on the sphere due to the magnetic field are an attractive force on the North pole and a repulsive one on the South pole.

Hence, the upwards force on the ball due to the magnetic field B in conjunction with Eq. (9) is given by

$$f(X,I)=C_1\,C_2 I^2\left(G(X)-G(X+\delta X)\right)\ , \quad (12)$$

and hence

$$f(X,I)\approx C_1\,C_2 I^2\,\delta X\,G'(X)\ , \quad (13)$$

where $G'(X)$ denotes the derivative and δX is the dipole separation.

Using assumption that δX may be taken as a constant, because it is sufficiently small compared to the electromagnet geometry (radius and length), the magnetic force becomes

$$f(X,I)\approx K\,I^2 G'(X)\ . \quad (14)$$

B. Some practicable approximations of magnetic force between the electromagnet and the sphere

Note that the analytical expression of the magnetic force/current/displacement relationships (Eqs. (11) and (14)) is very complex for the experimental purpose. However, the magnetic force characteristics may be experimentally calibrated as a function of the applied current I and the ball position X. Namely, the complex nonlinear function $G(X)$ can be approximate by a polynomial function

$$\left(a_0+a_1X+a_2X^2+\cdots+a_nX^n\right)^{-1}\ . \quad (15)$$

In the equilibrium position we have

$$mg=f=I^2\left(a_0+a_1X+a_2X^2+\cdots+a_nX^n\right)^{-1}\ , \quad (16)$$

which can be rewritten in the form

$$\frac{I^2}{mg}=a_0+a_1X+a_2X^2+\cdots+a_nX^n\ . \quad (17)$$

The experiment consists of resting the levitation metallic sphere on an xyz- stage capable of 1 mm incremental positioning and determining the minimum current required to pick up the ball at various heights [12-13], [4]. Then the model of the force/distance relationship can be determined by means of least square fitting. Note, that the validity of so obtained curve is limited to some range $X_{\min} \leq X \leq X_{\max}$. Also, the solenoid characteristics change with temperature, and the coefficients of the curve-fit change significantly when the system has been in operation for a while. This drift, combined with other nonlinear effects, results in a dispersion such that the calibration data can deviate from the curve-fit by up to $\pm 10\%$ in extreme conditions.

However, many studies dealing with the modeling of the magnetic levitation system based on model linearization using Taylor's series [1], [5], [14-15]. Recall that this approach is really restrictive, because it is only available when the system variations are small.

IV. THE COIL INDUCTANCE

The inductance $L(x)$ is a nonlinear function of the ball's position x [1], [14]. It has the largest value when the bearing ball is next to the coil, and decreases to a constant value as the bearing ball is enough removed. Some typical approximations for $L(x)$ are given by

$$L_1(x) = L_1(\infty) + L_{10}\, e^{-x/x_{10}} \tag{18}$$

$$L_2(x) = L_2(\infty) + \frac{L_{20}}{1 + \dfrac{x}{x_{20}}} \tag{19}$$

$$L_3(x) = L_3(\infty) + L_{30}\frac{x_{30}}{x} \quad . \tag{20}$$

Note that over the range of x considered in the experiments $(x_{\min} \leq x \leq x_{\max})$, one can pick the parameters $L_i(\infty)$, L_{i0}, x_{i0}, $i = 1, 2, 3$ in Eqs. (18) to (20) for mutual approach purpose [7], [8].

V. CONCLUSION

Using the magnetic field to levitate a steel ball, the Magnetic Levitation System as a teaching aid enables the theoretical study and practical investigation of basic and advanced approaches to control of nonlinear unstable systems. The equipment may operate in stand alone mode without the PC. In the digital mode system operates with MATLAB® /SIMULINK® software. The scope of experimentation includes dynamics modeling, identification, analysis and various controller design using classical and modern methods. For the modeling of the considered didactic magnetic levitation system a set of formulas has been established.

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