SOME COMMENTS TO ANALOG WAVE FILTERS DESIGN

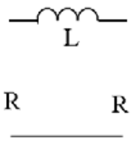
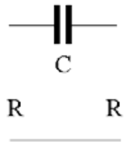
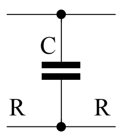
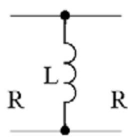
Daša Tichá [1], Pravoslav Martinek [2]

[1] University of Žilina, EF, Dept. of telecommunications, Veľký diel, 010 26 Žilina, SR
e-mail: ticha@fel.utc.sk
[2] ČVUT FEL, Dept. of circuit theory - K331, Technická 2, 166 27 Praha 6
e-mail: martinek@fel.cvut.cz

Abstract: *presented paper deals with some aspects of ARC filter design based on wave description of the LC-prototype. Realization of the basic branch model, basic loop behavior and sensitivity properties are discussed.*

Introduction

In [1] a prospective current-mode ARC filter implementation based on a wave description principle has been introduced. This idea can be easily modified to obtain sampled-data SI filter structure, as shown in [2]. Similarly to the "conventional" digital wave filters, the analog implementation is based on the description of the passive LC prototype ladder structure branches using scattering matrix. The corresponding equations, described the four basic filter branches in Tab.1, are simulated by suitable four-port model – see Fig.1.

	$[S]=\frac{1}{1+s\tau}\begin{bmatrix} s\tau & 1 \\ 1 & s\tau \end{bmatrix}$ $\tau = L/2R$		$[S]=\frac{1}{1+s\tau}\begin{bmatrix} 1 & s\tau \\ s\tau & 1 \end{bmatrix}$ $\tau = 2RC$
	$[S]=\frac{1}{1+s\tau}\begin{bmatrix} -s\tau & 1 \\ 1 & -s\tau \end{bmatrix}$ $\tau = RC/2$		$[S]=\frac{1}{1+s\tau}\begin{bmatrix} -1 & s\tau \\ s\tau & -1 \end{bmatrix}$ $\tau = 2L/R$

Tab.1: Scattering matrixes of the LC ladder structure basic branches.

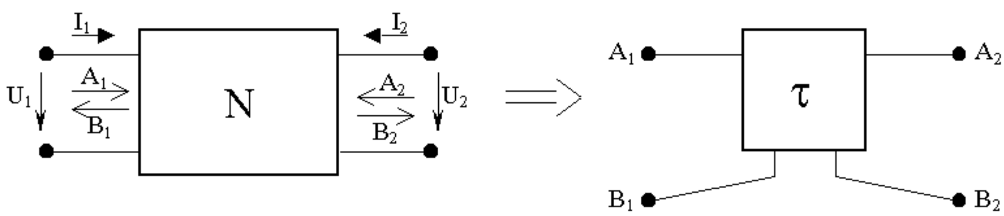
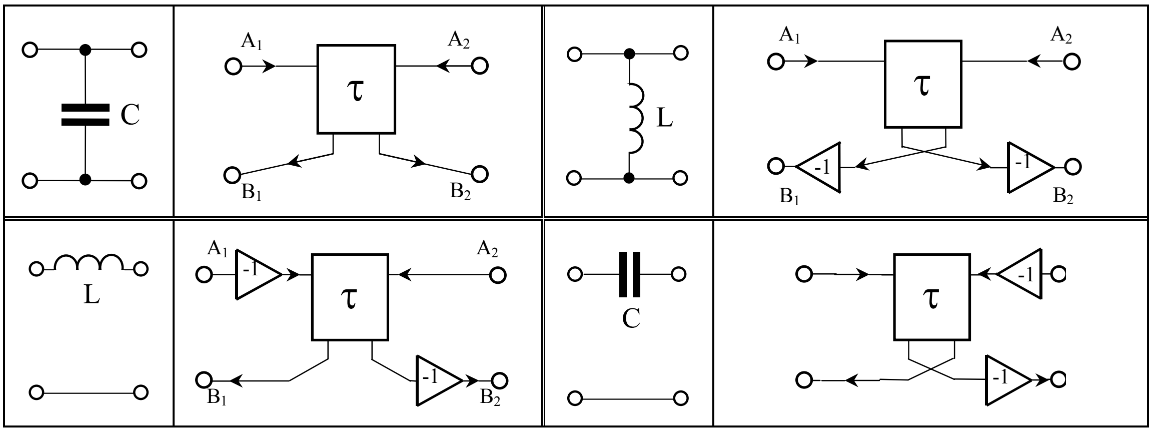


Fig.1: Four-port model corresponding to the two-port wave description

The similarity of the basic branch description, which is clear from Tab.1, makes possible to use an universal model (so called "two-pair") for the simulation of all the presented cases. In fact, there are two slightly different groups of branch models: the first includes inductive and capacitive series branch, the second includes inductive and capacitive short branch of starting LC ladder structure. Nevertheless, the branch models differ only in arrangement of input and output ports of the "basic" two-pair. Two-pair simulation of basic branches is presented in Tab.2. The shunt capacitor is considered to be the "reference branch". Note that the resulting filter structure is obtained by the correct interconnection of branch models.
In the following, some problems of two-pair implementation and sensitivity properties, which are important with respect to the practical realization, will be discussed.



Tab.2: Basic LC prototype branches simulation

Basic building block

As mentioned, the filter design method is based on suitable application of the universal 1st order building block – "two-pair". In fact, there are two possibilities of two-pair structure derivation, dependent on choice of the "reference equations" in the first or second row of Tab.1. In contrast to [1], we will prefer the shunt capacitor reference branch, because of simpler current-mode implementation. The voltage-mode direct circuit implementation of branch equations (1) is shown in Fig.2.

$$\begin{aligned} \boldsymbol{B}_1 &= -\frac{\boldsymbol{s}\tau}{1+\boldsymbol{s}\tau}\boldsymbol{A}_1 + \frac{1}{1+\boldsymbol{s}\tau}\boldsymbol{A}_2 \\ \boldsymbol{B}_2 &= \frac{1}{1+\boldsymbol{s}\tau}\boldsymbol{A}_1 - \frac{\boldsymbol{s}\tau}{1+\boldsymbol{s}\tau}\boldsymbol{A}_2 \end{aligned} \tag{1}$$

$$\boldsymbol{C}_1\boldsymbol{R}_1 = \boldsymbol{C}_2\boldsymbol{R}_2 = \boldsymbol{C}_3\boldsymbol{R}_3 = \boldsymbol{C}_4\boldsymbol{R}_4 = \tau$$

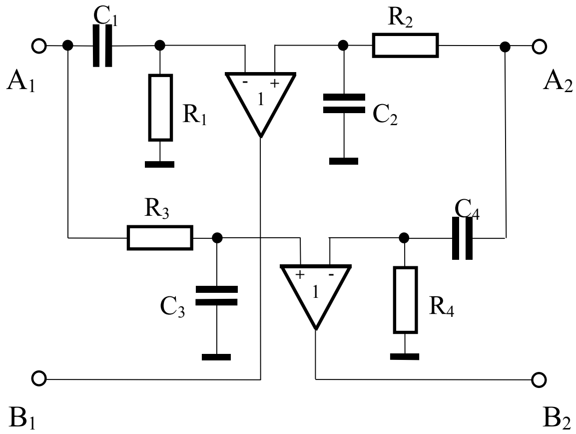


Fig.2: A direct realization of the Eqs. (1)

As evident, the two-pair circuit requires only the passive RC cells and difference unity-gain amplifiers. The circuit simplification is possible by a suitable re-arrangement of equations (1). The modified equations (2), (3) differs each other by choice of time-constant presentation only, and leads to the lossy integrator or lossy differentiator based two-pair circuits. The corresponding block diagrams are shown in Fig.3a, 3b.

$$\begin{aligned} \boldsymbol{B}_1 &= -\boldsymbol{A}_1 + \frac{1}{1+\boldsymbol{s}\tau}\left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) \\ \boldsymbol{B}_2 &= -\boldsymbol{A}_2 + \frac{1}{1+\boldsymbol{s}\tau}\left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) \end{aligned} \tag{2}$$

$$\begin{aligned} \boldsymbol{B}_1 &= \boldsymbol{A}_2 - \frac{\boldsymbol{s}\tau}{1+\boldsymbol{s}\tau}\left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) \\ \boldsymbol{B}_2 &= \boldsymbol{A}_1 - \frac{\boldsymbol{s}\tau}{1+\boldsymbol{s}\tau}\left(\boldsymbol{A}_1 + \boldsymbol{A}_2\right) \end{aligned} \tag{3}$$

Practical realization of the presented block diagrams is flexible and suitable both for current- and voltage-mode concept. Simplified example of the integrator based two-pair design is shown in Fig.4, the lossy differentiator version is in Fig.5. As can be seen, the main building blocks are current followers or inverters and transconductance amplifiers (OTA). The last version is especially interesting with respect to the simple realization of current inverters by current mirrors and it can be easily applied in sampled-data SI filter implementation.

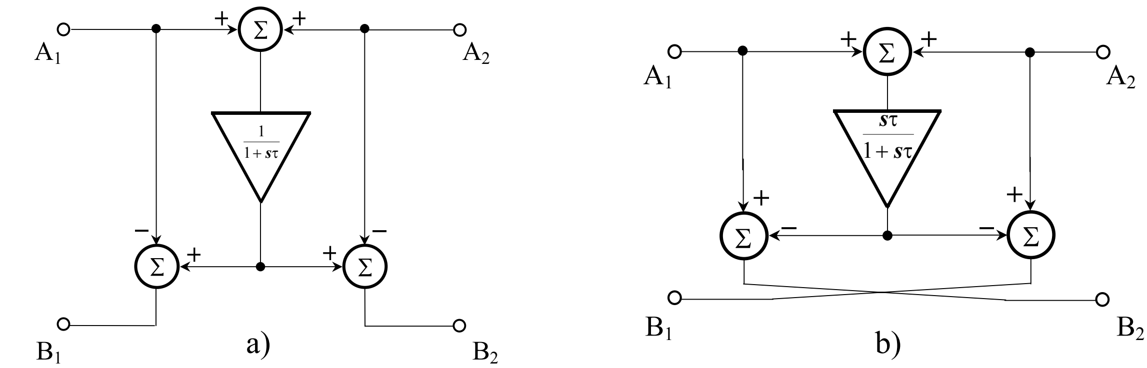


Fig.3: The block diagrams corresponding to the Eqs. (2), (3).

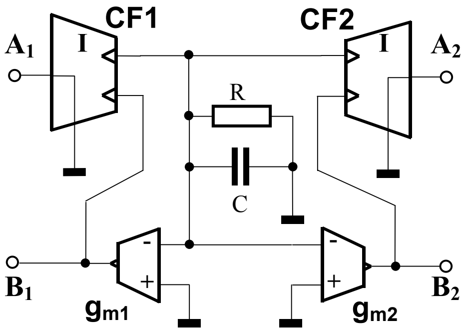


Fig.4: Integrator based CM two-pair

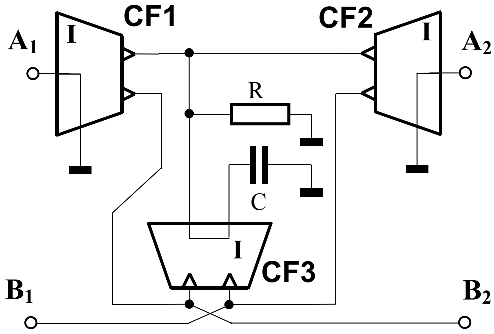


Fig.5: Differentiator based CM two-pair

Circuit equations for the presented examples can be written in the form (4) for the integrator based circuit, and (5) for the differentiator based realization. The correct form of Eqs.(4) requires to accomplish the condition $g_{m1} = g_{m2} = 1/R$.

$$\begin{aligned} B_1 &= -\frac{sCR + 1 - g_{m1}R}{1 + sCR} A_1 + \frac{g_{m1}R}{1 + sCR} A_2 \\ B_2 &= \frac{g_{m2}R}{1 + sCR} A_1 - \frac{sCR + 1 - g_{m2}R}{1 + sCR} A_2 \end{aligned} \tag{4}$$

$$\begin{aligned} B_1 &= -\frac{sCR}{1 + sCR} A_1 + \frac{1}{1 + sCR} A_2 \\ B_2 &= \frac{1}{1 + sCR} A_1 - \frac{sCR}{1 + sCR} A_2 \end{aligned} \tag{5}$$

Function principle and sensitivity properties

To demonstrate the basic behavior and sensitivity properties of the analog wave filters, we will simulate series resonant circuit using branch wave equivalents from Tab.2, as shown in Fig.6. For better sight the voltage-mode (VM) representation is used.

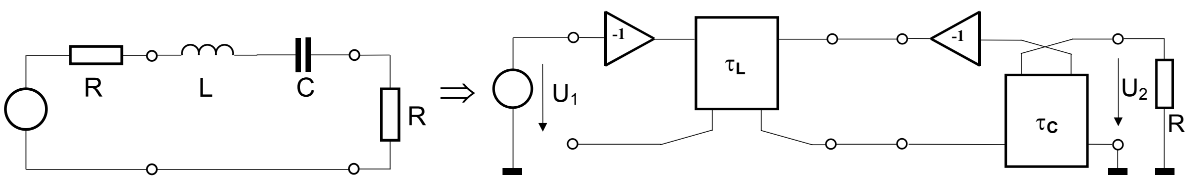


Fig.6: Series resonant circuit (SRC) simulation

Re-drawing the block presentation into detail circuit diagram as shown in Fig.7, it is possible to identify the main loop forming the transfer function complex conjugate poles. The loop consists of the both RC cells, unity gain voltage amplifiers E_{21}, E_{22} and OTA amplifiers G_{12}, G_{22}. Note that the voltage-mode two-pair model used was derived from CM model in Fig.4 using adjoint transformation (See Fig.8).

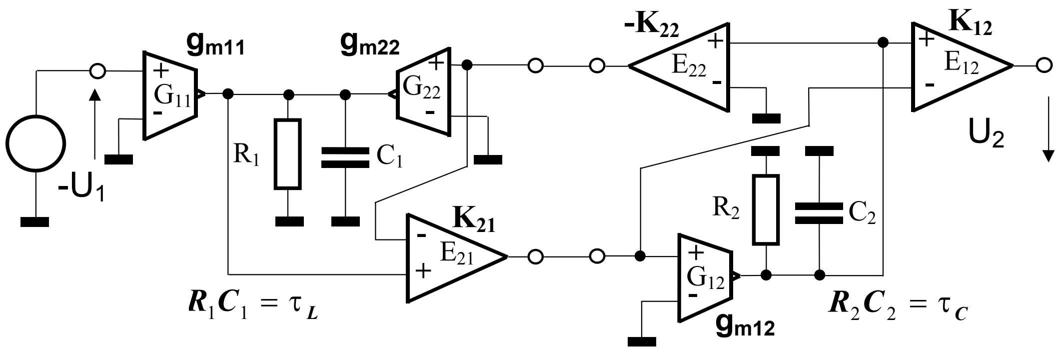


Fig.7: The detail circuit diagram of SRC wave model using VM two-port

As evident, loop configuration corresponds to the unity-gain positive feedback biquad structure [3] and it is in congruity with their sensitivity properties – i.e. low sensitivity to the passive circuit elements, relative high sensitivity to the active devices gain. This fact can be proved by expressing the transfer function parameters ω_0, Q in symbolic form (6), (7)

$$\omega_0 = \sqrt{\frac{K_{21}K_{22}g_{m21}R_1g_{m12}R_2 + 1 - K_{21}K_{22}g_{m12}R_2}{C_1C_2R_1R_2}}, \tag{6}$$

$$Q = \frac{\sqrt{K_{21}K_{22}g_{m21}R_1g_{m12}R_2 + 1 - K_{21}K_{22}g_{m12}R_2}\ \sqrt{C_1C_2R_1R_2}}{C_2R_2 + C_1R_1\left(1 - K_{21}K_{22}g_{m12}R_2\right)}. \tag{7}$$

Using assignment $C_1R_1 = \tau_L = L/2R$, $C_2R_2 = \tau_C = 2RC$, and keeping

$$1 - K_{21}K_{22}g_{m12}R_2 = 0; \quad K_{21}K_{22}g_{m21}R_1g_{m12}R_2 = 1, \tag{8}$$

the Eqs.(6), (7) correspond to the original circuit transfer function parameters. Computation of ω_0, Q sensitivity gives the results:

$$S^{\omega_0}_{R_2,C_1,C_2} = -0.5, \qquad S^{\omega_0}_{g_{m21}} = 0.5,$$

$$\left|S^{Q}_{C_1,C_2,g_{m21}}\right| = 0.5, \qquad S^{Q}_{R_1} = 1,$$

$$S^{Q}_{K_{21},K_{22},g_{m12}} = Q^2, \qquad S^{Q}_{R_2} = Q^2.$$

The Q-sensitivity is strongly influenced by condition (8), which is critical with respect to the resultant circuit parameters.

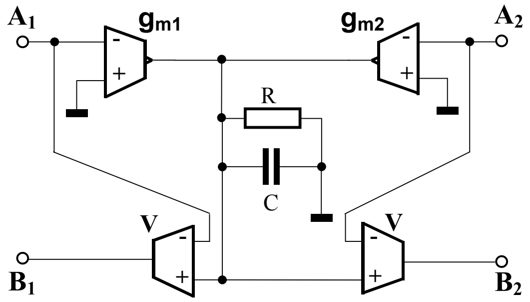


Fig.8: Integrator based two-pair

Conclusions

Presented results are valid for each type of the two-pair realization. The influence of high Q-sensitivities to the active devices gain can be minimized by a suitable design. Note that their influence to the filter parameters is suppressed by mutual relations of individual loops.

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