Bipolar junction transistor - Basics

Introduction

Walter Brattain, John Bardeen, and William Shockley invented the bipolar junction transistor (BJT) in 1949, while working for Bell Telephone Laboratories.

This revolutionary invention changed the world.

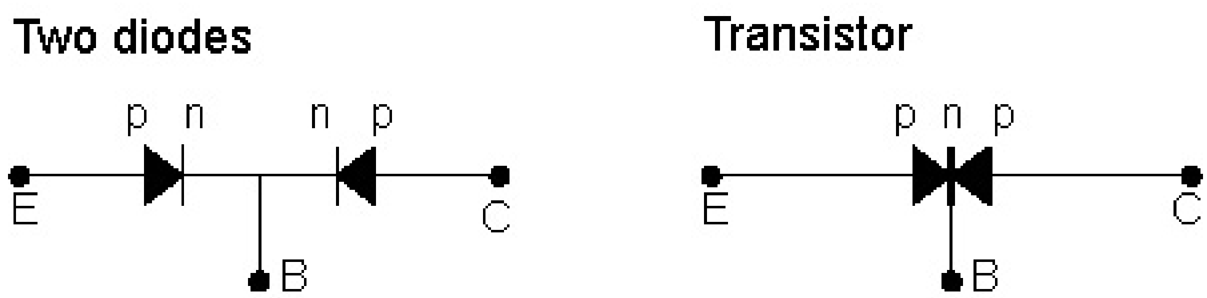
The invention of the BJT followed the invention of the point-contact transistor by Walter Brattain and John Bardeen.

The point-contact transistor has several problems that prevented it from becoming a viable device.

BJT is a three-terminal device.

BJT is used as amplifier and switch.

Circuit diagram of pnp transistor consisting of two diodes

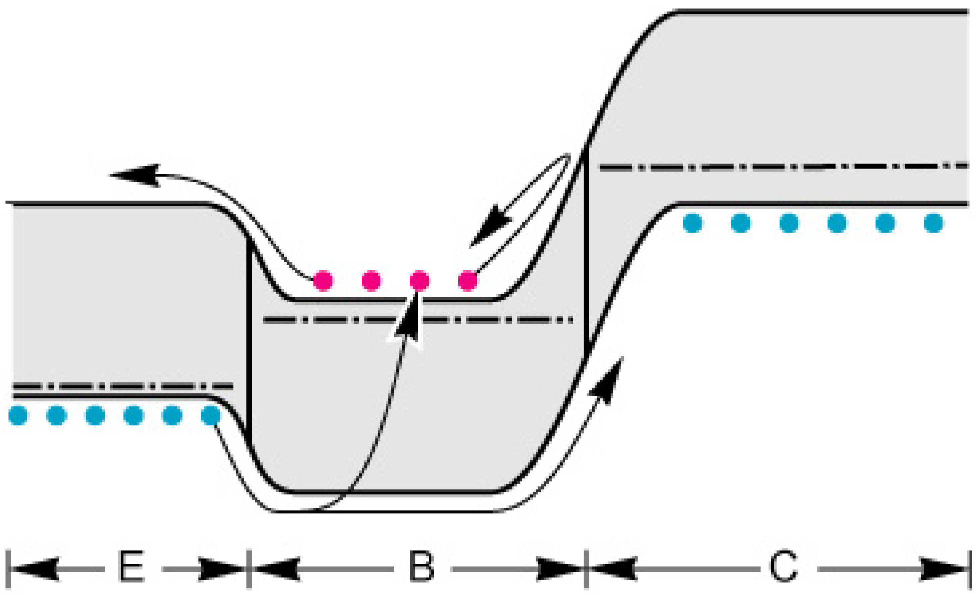


The two n-type regions merge to form a very thin base.

EB junction: Forward bias

CB junction: Reverse bias

Band diagram (PNP)

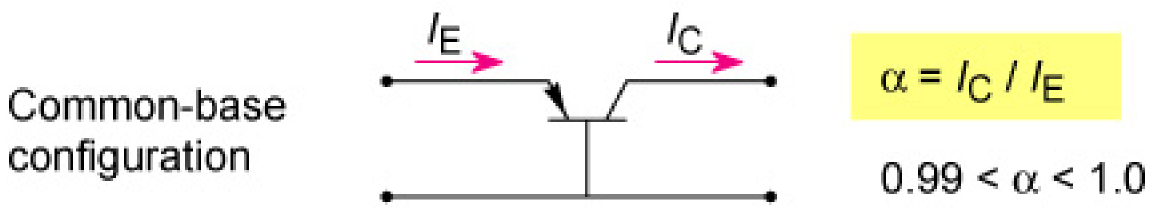


Junction bias?

Major current flows?

Basic amplifier circuits

Common-base configuration

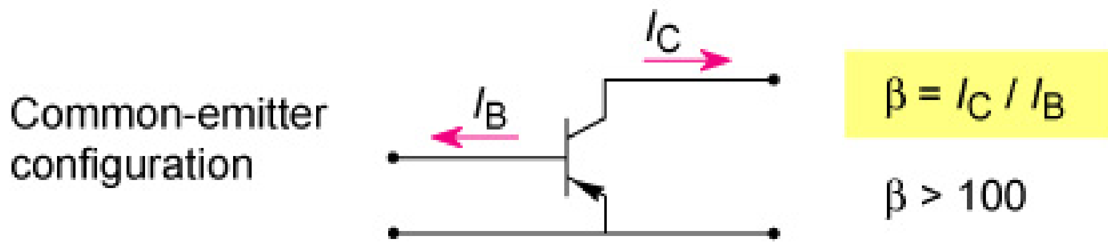


$$\alpha = \frac{I_C}{I_E} \qquad (1)$$

α = current amplification in common base circuit

Typical values: $\alpha > 0.99$ (for state-of-the-art transistor)

Common-emitter configuration

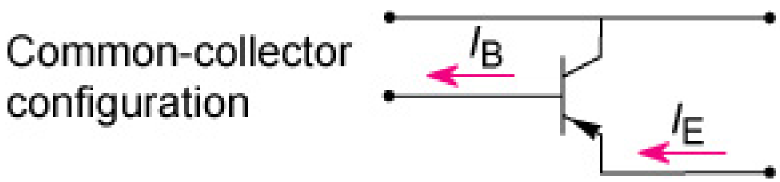


β = amplification in common-emitter circuit

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \left(\frac{1}{\alpha} - 1\right)^{-1} = \frac{\alpha}{1-\alpha} \qquad (2)$$

$\beta > 100$ for state-of-the art transistors

Common-collector configuration



$I_E / I_B = \beta / \alpha$

$\beta / \alpha > 100$

$$\frac{I_E}{I_B} = \frac{I_C / \alpha}{I_B} = \frac{\beta}{\alpha} \qquad (3)$$

Nature of bipolar transistor

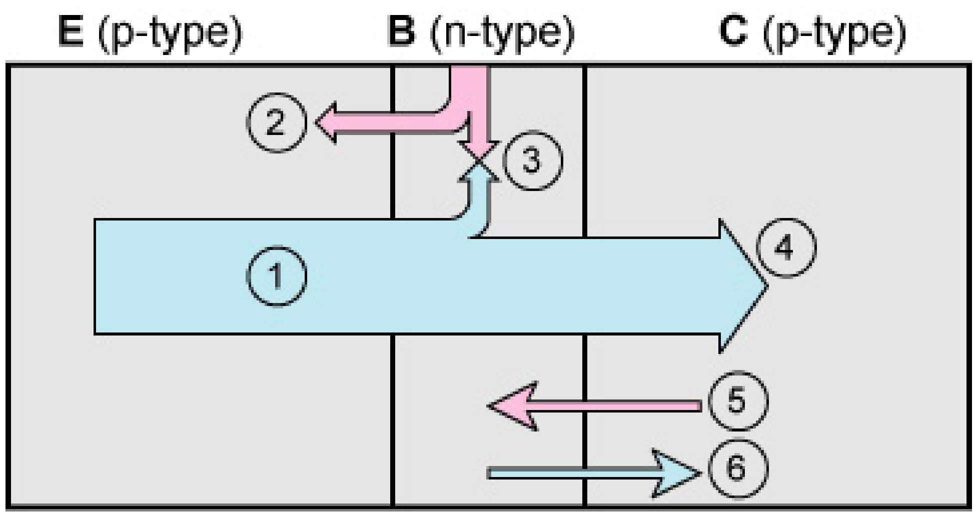
BJT is a ***current amplifier*** (not a voltage amplifier).

BJT is current-controlled current source.

BJT base current controls the emitter current and thereby the collector current.

Qualitative discussion of pnp transistor

Schematic current flows in pnp BJT



Basic ideas

EB junction is asymmetric:

$$I_{\mathrm{Ep}} \gg I_{\mathrm{En}} \qquad (4)$$

The emitter hole current is controlled by EB junction.

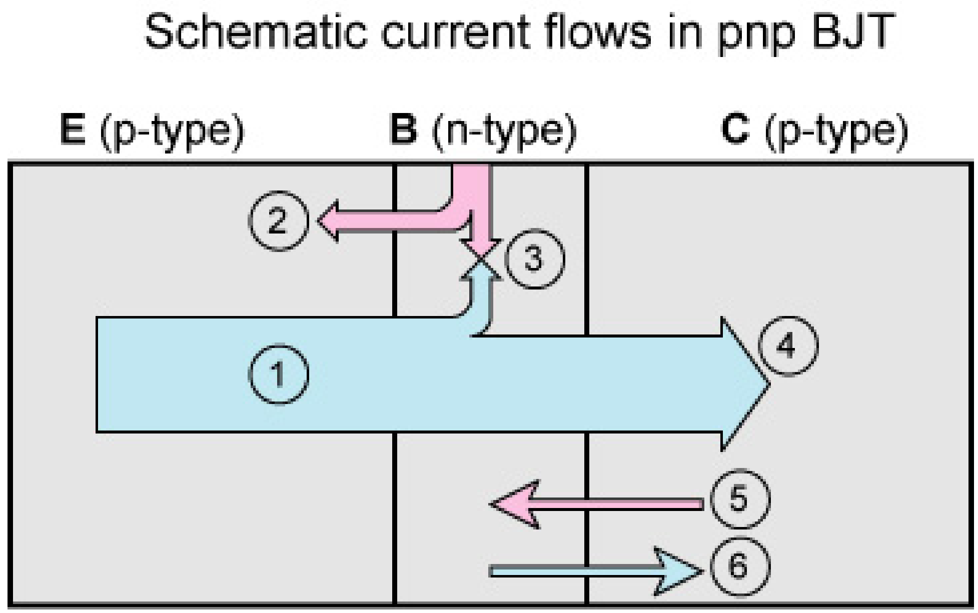
The base width is small.

$$W_{\mathrm{B}} \ll L_{\mathrm{p}} \qquad (5)$$

Most holes diffusing into the base will reach the collector if condition of Eq. (5) is met.

Thus the base current controls collector current.

Discussion of currents



EB junction currents *(EB junction is forward biased)*

(1) Holes diffusing from the E into the B
(2) Electrons diffusing from the B into the E

Base currents

(3) Recombination of holes injected into base
(4) Most holes reach C since $L_P >> W_B$

BC junction currents *(BC is reversely biased)*

(5) Electron minority carrier current from C to B
(6) Hole minority carrier current from B to C

We know that current (5) and (6) can be ***neglected for most*** practical purposes.

Basic equations

What is the fraction of the emitter hole current that reaches the collector?

$$I_{\mathrm{C}} = B\, I_{\mathrm{Ep}} \qquad (6)$$

B = Base transport factor

B = Probability that a hole injected into B reaches C

$B \leq 1$

What fraction of total emitter current is emitter hole current?

$$E_{\mathrm{Ep}} = \gamma I_{\mathrm{E}} = \gamma \left(I_{\mathrm{En}} + I_{\mathrm{Ep}} \right) \quad (7)$$

γ = Emitter Efficiency

γ = Ratio of I_{Ep} to I_{E}

$\gamma \leq 1$

$$\gamma = \frac{I_{\mathrm{Ep}}}{I_{\mathrm{En}} + I_{\mathrm{Ep}}} = \left(1 + \frac{I_{\mathrm{En}}}{I_{\mathrm{Ep}}} \right)^{-1} \approx 1 - \frac{I_{\mathrm{En}}}{I_{\mathrm{Ep}}} \quad (8)$$

Current amplification α

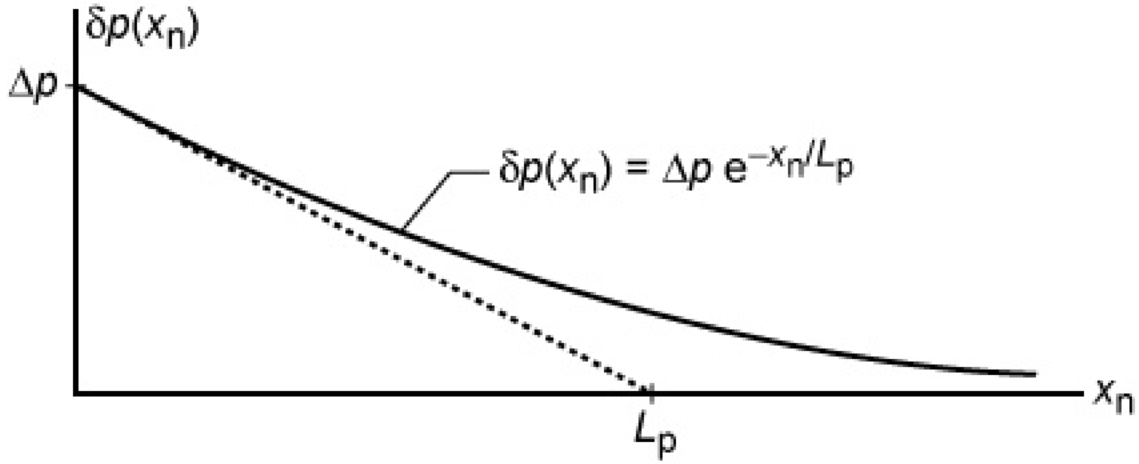
$$\alpha = \frac{I_C}{I_E} = B \frac{I_{Ep}}{I_E} = B\gamma \qquad (9)$$

We will later calculate *B* and γ in two ways:

1. Approximate calculation
2. Exact calculation

Approximate hole distribution in base (PNP)

Long base ($W_B >> L_p$)



$$\delta p(x_n) = \Delta p \, e^{-x_n / L_P} \qquad (10)$$

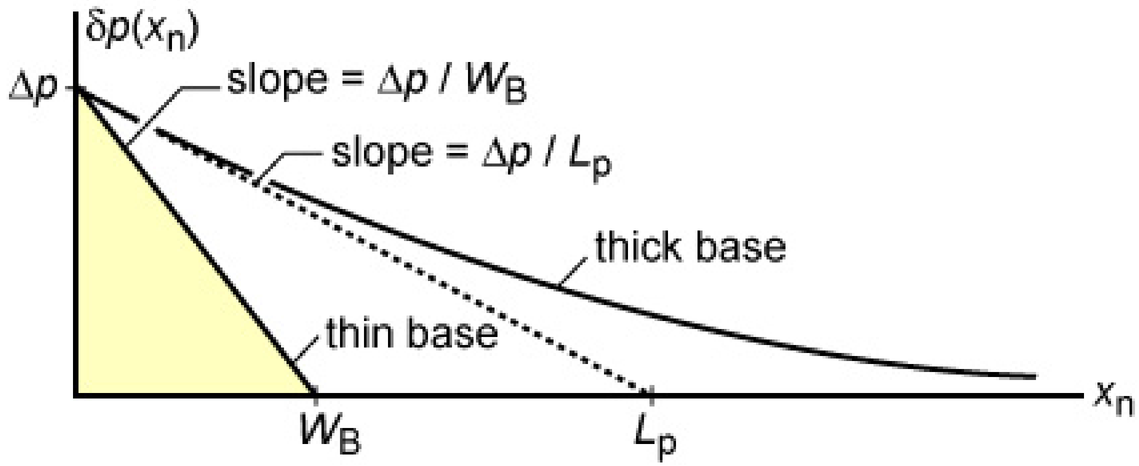
Short base ($W_B << L_p$)

Exponential function ***can be linearized***

At $x_n = 0$ it is $\Delta p = p_{n_0} \left(e^{eV_{BE}/kT} - 1 \right)$ (11)

At $x_n = W_B$ it is $\Delta p = p_{n_0} \left(e^{eV_{CB}/kT} - 1 \right) = -p_{n_0}$ (12)

That is $p(x_n = W_B) = 0$ (13)



Can you identify the ***diffusion triangle*** in the figure?

Equation for diffusion triangle:

$$p(x_n) = \Delta p \left(1 - \frac{x_n}{W_B} \right) \tag{14}$$

Note

Diffusion Current:

$$J_{\mathrm{p}} = -e\, D_{\mathrm{p}} \frac{\mathrm{d}p}{\mathrm{d}x} \qquad (15)$$

$$J_{\mathrm{p}} \propto \text{slope } (i.e.\ \mathrm{d}p / \mathrm{d}x)$$

Short base changes slope (*i. e*. d*p* / d*x*)

Approximate calculation: Emitter efficiency (PNP)

Recall the Shockley equation:

$$I = e\,A \left(\frac{D_\mathrm{P}}{L_\mathrm{P}}\, p_{\mathrm{n}_0} + \frac{D_\mathrm{n}}{L_\mathrm{n}}\, n_{\mathrm{p}_0} \right) \left(\mathrm{e}^{eV/kT} - 1 \right) \qquad (16)$$

… where first summand within first parenthesis is due to hole injection

… where second summand within first parenthesis is due to electron injection

Emitter is "long", and therefore the electron current from base into emitter is given by

$$I_\mathrm{En} = e\,A\, \frac{D_\mathrm{n}}{L_\mathrm{n}}\, n_{\mathrm{p}_0} \left(e^{eV/kT} - 1 \right) \qquad (17)$$

Base is "short", and therefore the hole current from emitter into base is given by

$$I_{\mathrm{Ep}} = e\,A\,\frac{D_{\mathrm{p}}}{L_{\mathrm{p}}}\,p_{\mathrm{n}_0}\left(e^{eV/kT}-1\right)\frac{L_{\mathrm{p}}}{W_{\mathrm{B}}} \qquad (18)$$

where last term, *i. e.* (L_{p} / W_{B}), is correction due to increase in slope

One obtains the emitter efficiency using Eqs. (8), (17), and (18)

$$\gamma = 1 - \frac{I_{En}}{I_{Ep}} = 1 - \frac{\dfrac{D_n}{L_n} \; n_{p_0}}{\dfrac{D_P}{W_B} \; p_{n_0}} \qquad (19)$$

$$\text{using} \quad n_{p_0} = n_i^2 / p = n_i^2 / N_A \qquad (20)$$

$$\text{and} \quad p_{n_0} = n_i^2 / n = n_i^2 / N_D \qquad (21)$$

one obtains:

$$\gamma = 1 - \frac{D_n \, W_B \, N_D}{D_p \, L_n \, N_A} \qquad (22)$$

How can we attain high emitter efficiency?

For a high value of γ:

1. W_B must be very short

2. $N_A >> N_D$ (23)

That is,

Emitter doping >> Base doping

Example:

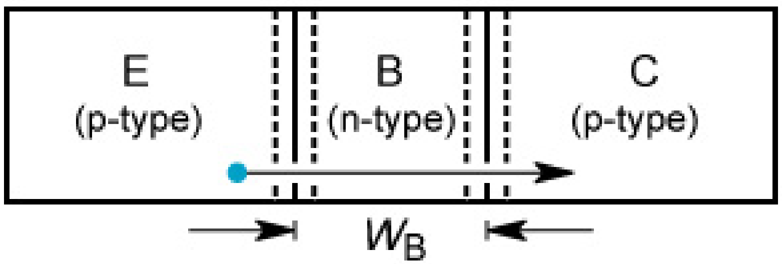
Problem: Assume a PNP transistor with the following parameters:
Emitter doping: $N_A = 1 \times 10^{18}\ cm^{-3}$
Base doping: $N_D = 1 \times 10^{17}\ cm^{-3}$
$D_p = D_n$
$W_B = 100$ nm
$L_n = 1\ \mu m$
Calculate emitter efficiency.

Solution:

$$\gamma = 1 - \frac{D_n W_B N_D}{D_p L_n N_A} = 1 - \frac{1}{100} = 0.99$$

The problem assumed reasonable parameters. For such reasonable parameters, we obtain a high current gain.

Approximate calculation: Base transport factor (PNP)

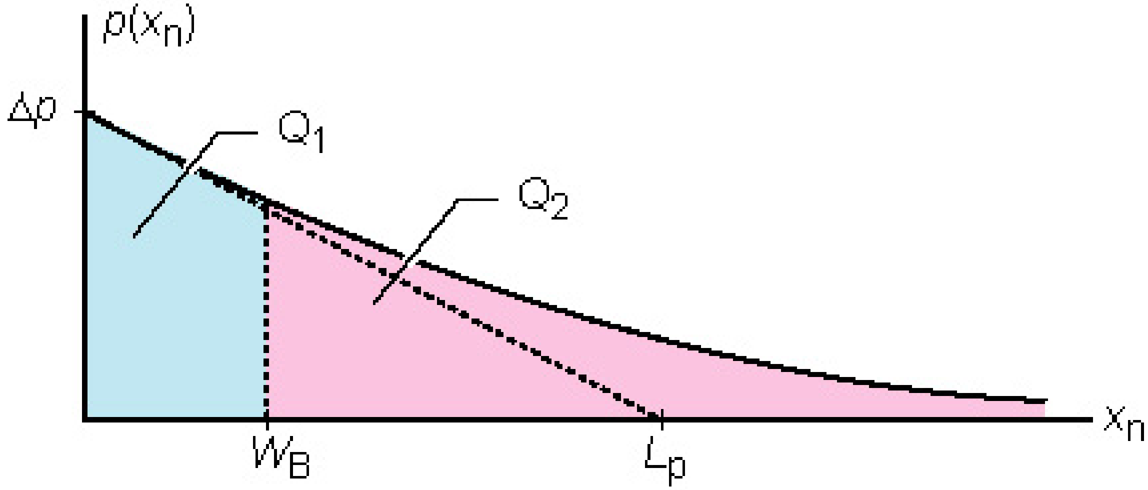


What is probability that holes reach C ?

Which meachanisms hinder holes in reaching C ?

Thought experiment: Let’s assume that the BC junction would not influence the hole distribution. Warning: Strictly speaking, this is incorrect assumption!

In this case, the following hole distribution would be obtained:



$$\text{Base recombination current} \propto Q_1/\tau \propto \Delta p\, W_B \qquad (24)$$

$$\text{Collector current} \propto Q_2/\tau \propto \Delta p\, L_p \qquad (25)$$

(Note: In Eqs. 24 and 25, we use that $W_B << L_P$)

It is

$$B = \frac{I_C}{I_{Ep}} = \frac{Q_2/\tau}{Q_1/\tau + Q_2/\tau} = \left(1 + \frac{Q_1}{Q_2}\right)^{-1} \approx 1 - Q_1/Q_2 \qquad (26)$$

Using Eqs. (24), (25), and (26) one obtains:

$$B = 1 - \frac{W_B}{L_p} \qquad (27)$$

End of thought experiment.

Warning: This thought experiment is an oversimplification and the result (Eq. 27) must not be used.

Exact hole distribution in the base (PNP)

Hole concentration at the emitter side of base

$$\Delta p_{\mathrm{E}} = \Delta p(x_{\mathrm{n}} = 0) = p_{\mathrm{n}_0}\left(\mathrm{e}^{eV_{\mathrm{BE}}/kT} - 1\right) \approx p_{\mathrm{n}_0}\mathrm{e}^{eV_{\mathrm{BE}}/kT} \tag{28}$$

Hole concentration at the collector side of base

$$\Delta p_{C} = \Delta p(x_{\mathrm{n}} = W_{\mathrm{B}}) = p_{\mathrm{n}_0}\left(\mathrm{e}^{eV_{\mathrm{BC}}/kT} - 1\right) \approx -p_{\mathrm{n}_0} \tag{29}$$

… note that V_{BC} is negative

Eqs. (28) and (29) are the ***boundary conditions*** for the hole concentration in the base

There is no electric field in the neutral region of the base. Therefore, transport can be described by the ***diffusion equation***

$$\frac{\mathrm{d}^2}{\mathrm{d}{x_\mathrm{n}}^2}\,\delta p(x_\mathrm{n}) = \frac{\delta p(x_\mathrm{n})}{{L_\mathrm{p}}^2} \qquad (30)$$

General solution of this equation is given by

$$\delta p(x_\mathrm{n}) = C_1\,\mathrm{e}^{x_\mathrm{n}/L_\mathrm{p}} + C_2\,\mathrm{e}^{-x_\mathrm{n}/L_\mathrm{p}} \qquad (31)$$

The constants C_1 and C_2 will be determined by using the boundary conditions

$$\delta p(x_\mathrm{n} = 0) \;=\; C_1 + C_2 \;=\; \Delta p_\mathrm{E} \tag{32}$$

$$\delta p(x_\mathrm{n} = W_\mathrm{B}) \;=\; C_1\,\mathrm{e}^{W_\mathrm{B}/L_\mathrm{p}} + C_2\,\mathrm{e}^{-W_\mathrm{B}/L_\mathrm{p}} \;=\; \Delta p_\mathrm{C} \tag{33}$$

Solving Eqs. (32) and (33) for C_1 and C_2 yields

$$C_1 \;=\; \frac{\Delta p_\mathrm{C} - \Delta p_\mathrm{E}\,\mathrm{e}^{-W_\mathrm{B}/L_\mathrm{P}}}{\mathrm{e}^{W_\mathrm{B}/L_\mathrm{P}} - \mathrm{e}^{-W_\mathrm{B}/L_\mathrm{P}}} \tag{34}$$

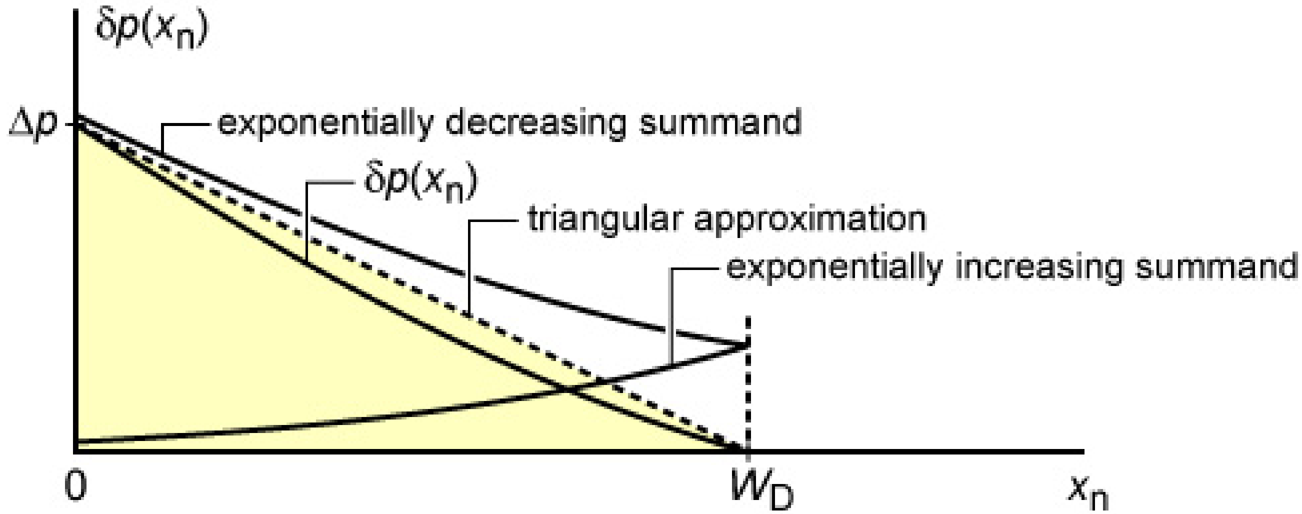
$$C_2 \;=\; \frac{\Delta p_\mathrm{E}\,\mathrm{e}^{W_\mathrm{B}/L_\mathrm{P}} - \Delta p_\mathrm{C}}{\mathrm{e}^{W_\mathrm{B}/L_\mathrm{P}} - \mathrm{e}^{-W_\mathrm{B}/L_\mathrm{P}}} \tag{35}$$

Insert the constants C_1 and C_2 into Eq. (31)

For $\Delta p_\text{C} \approx 0$, the hole concentration in the base is given by

$$\delta p(x_\text{n}) \approx \Delta p_\text{E} \frac{e^{W_\text{B}/L_\text{P}}\, e^{-x_\text{n}/L_\text{P}} - e^{-W_\text{B}/L_\text{P}}\, e^{x_\text{n}/L_\text{P}}}{e^{W_\text{B}/L_\text{P}} - e^{-W_\text{B}/L_\text{P}}} \qquad (36)$$

This function has an ***exponentially decreasing*** part and an ***exponentially increasing*** part.


δp(xn)
Δp
exponentially decreasing summand
δp(xn)
triangular approximation
exponentially increasing summand
0
WD
xn

Discussion of slopes

Recall that the ***slope*** $[\ \mathrm{d}\delta p(x_\mathrm{n}) / \mathrm{d}x_\mathrm{n}\ ]$ determines the diffusion current.

Slope is larger at $x_\mathrm{n} = 0$ as compared to $x_\mathrm{n} = W_\mathrm{B}$.

The difference in slope is due to recombination in base.

Approximation for exponential function:

For $W_\mathrm{B} \ll L_\mathrm{P}$, we can expand the exponential function into a power series:

$$\mathrm{e}^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$$

Inserting this approximation into Eq. (36) and neglecting all quadratic and higher terms in Eq. (36) yields

$$\delta p(x_n) = \Delta p_E \left(1 - \frac{x_n}{W_B}\right) \qquad (37)$$

This equation represents the ***diffusion triangle*** in the base.

The ***strictly triangular shape*** is valid for negligible recombination in the base.

Mathematics of exponential functions

Exponential function = Natural decay function

For the next section, we need some mathematical relations for exponential functions and they are summarized below:

$$\mathrm{e} = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n = 2.718\ldots$$

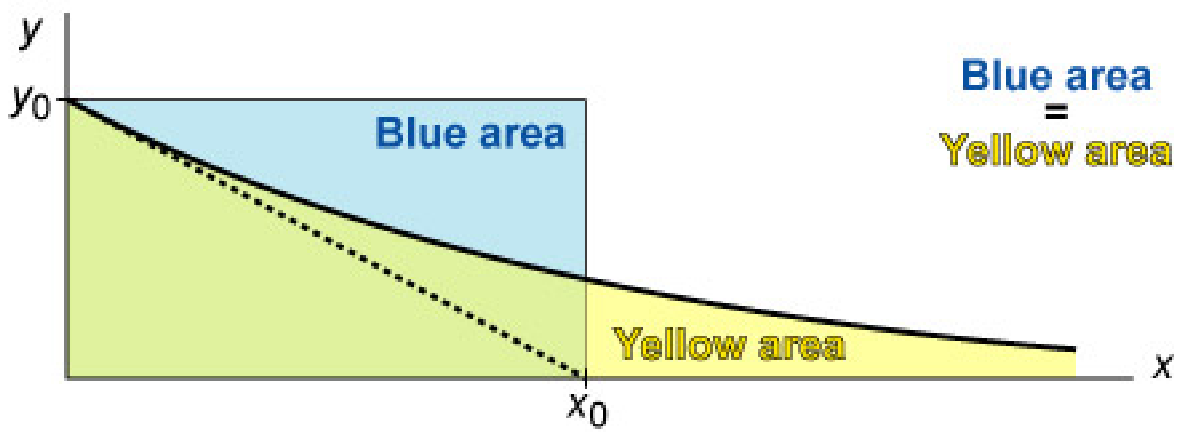
$$\mathrm{e}^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Give some examples of natural (*i. e*. exponential) decays!

Function : $y = y_0 \, e^{-x/x_0}$

Slope : $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0} = -\frac{y_0}{x_0}$

Integral : $\int_0^\infty y_0 \, e^{-x/x_0} \, \mathrm{d}x = y_0 \, x_0$



Mathematics of hyperbolic exponential functions

For the next section, we need some mathematical relations for hyperbolic exponential functions and they are summarized below:

$$\text{Hyperbolic sin function:} \qquad \sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\text{Hyperbolic cos function:} \qquad \cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

(*Note*: Hyperbolic cos function is also called “chain function”. Why?)

$$\text{Hyperbolic tan function:} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

$$\text{Hyperbolic cot function:} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

Hyperbolic secan function:

$$\operatorname{sech} x = \frac{2}{e^{x} + e^{-x}} = \frac{1}{\cosh x}$$

Hyperbolic cosecan function:

$$\operatorname{cosech} x = \frac{2}{e^{x} - e^{-x}} = \frac{1}{\sinh x}$$

Exact E, B, and C currents

We have calculated the hole distribution in the base and can now calculate the currents of the three terminals E, B, and C by using the equation:

$$I = - e A D_p \frac{d}{dx_n} \delta p(x_n) \qquad (38)$$

Emitter current

Emitter current is obtained by using Eqs. (31), (34), (35), (38)

$$I_{Ep} = I_p(x_n = 0) = e A \frac{D_p}{L_p} (C_2 - C_1) \qquad (39)$$

$$I_{\mathrm{Ep}} = e\,A\,\frac{D_{\mathrm{p}}}{L_{\mathrm{p}}}\left(\Delta p_{\mathrm{E}}\,\coth\frac{W_{\mathrm{B}}}{L_{\mathrm{P}}} - \Delta p_{\mathrm{C}}\,\mathrm{cosech}\frac{W_{\mathrm{B}}}{L_{\mathrm{P}}}\right) \qquad (40)$$

Collector current

$$I_{\mathrm{C}} = I_{\mathrm{p}}\left(x_{\mathrm{n}} = W_{\mathrm{B}}\right) = e\,A\,\frac{D_{\mathrm{p}}}{L_{\mathrm{p}}}\left(C_2\,\mathrm{e}^{-W_{\mathrm{B}}/L_{\mathrm{P}}} - C_1\,\mathrm{e}^{W_{\mathrm{B}}/L_{\mathrm{P}}}\right) \qquad (41)$$

$$I_{\mathrm{C}} = e\,A\,\frac{D_{\mathrm{p}}}{L_{\mathrm{p}}}\left(\Delta p_{\mathrm{E}}\,\mathrm{cosech}\frac{W_{\mathrm{B}}}{L_{\mathrm{P}}} - \Delta p_{\mathrm{C}}\,\coth\frac{W_{\mathrm{B}}}{L_{\mathrm{P}}}\right) \qquad (42)$$

Base current

$$I_B = I_E - I_C \approx I_{Ep} - I_C \tag{43}$$

$$I_B = e\,A\,\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C\right)\tanh\frac{W_B}{2L_p}\right] \tag{44}$$

Eqs. (40), (42), and (44) are ***generally valid***, *i.e.* for any bias configuration and bias condition of the transistor. The equations can be simplified for a transistor under regular operating conditions, which are

➔ V_{BE} = forward bias

➔ V_{CB} = reverse bias

Appropriate E, B, and C currents

V_{BE} = forward bias ➔ $\Delta p_E \neq 0$

V_{CB} = reverse bias ➔ $\Delta p_C = 0$

From Eqs. (40), (42), and (44) it follows that

$$I_{Ep} = e\,A\,\frac{D_p}{L_p}\,\Delta p_E\,\coth\frac{W_B}{L_p} \tag{45}$$

Using $\coth x \approx (1/x) + (x/3)$, one obtains

$$I_{Ep} = e\,A\,\frac{D_p}{L_p}\,\Delta p_E\left(\frac{L_p}{W_B} + \frac{W_B}{3L_p}\right) \tag{46}$$

Furthermore

$$I_\mathrm{C} \quad = \quad e\,A\,\frac{D_\mathrm{p}}{L_\mathrm{p}}\;\Delta p_\mathrm{E}\;\mathrm{cosech}\,\frac{W_\mathrm{B}}{L_\mathrm{p}} \tag{47}$$

Using $\mathrm{cosech}\,x \;\approx\; (1/x) - (x/6)$, one obtains

$$I_\mathrm{C} \quad = \quad e\,A\,\frac{D_\mathrm{p}}{L_\mathrm{p}}\;\Delta p_\mathrm{E}\left(\frac{L_\mathrm{p}}{W_\mathrm{B}} - \frac{W_\mathrm{B}}{6L_\mathrm{p}}\right) \tag{48}$$

Finally

$$I_B = I_E - I_C \approx I_{Ep} - I_C \qquad (49)$$

$$= e\,A\,\frac{D_p}{L_p}\,\Delta p_E \left(\frac{1}{3}\frac{W_B}{L_p} + \frac{1}{6}\frac{W_B}{L_p} \right) \qquad (50)$$

It follows that

$$I_B = e\,A\,\frac{D_p}{2L_p^2}\,W_B\,\Delta p_E = e\,A\,\frac{W_B}{2\tau_p}\,\Delta p_E \qquad (51)$$

Base transport factor

Using Eqs. (45) and (47) we calculate

$$B = \frac{I_C}{I_{Ep}} = \frac{\text{cosech}\,(W_B / L_p)}{\coth\,(W_B / L_p)} = \text{sech}\,\frac{W_B}{L_p} \qquad (52)$$

Using $\text{sech}\,x \approx 1 - (1/2)\,x^2$, one obtains

$$B = 1 - \frac{1}{2}\left(\frac{W_B}{L_p}\right)^2 \qquad (53)$$

We now have a good expression for *B*.

Compare this to Eq. (27)! (Recall: Do ***not*** use Eq. 27)

We now have γ (*i. e*. the emitter efficiency, see Eq. 22) and *B* (*i. e*. the base transport factor, see Eq. 53).

Since $\alpha = \gamma B$, we can calculate the current amplification of a transistor:

$$\alpha = \gamma B = \left(1 - \frac{D_n W_B N_D}{D_p L_n N_A}\right)\left(1 - \frac{W_B{}^2}{2 L_p{}^2}\right) \qquad (54)$$

Example

Problem: Calculate the Base Transport Factor for W_B = 0.1 μm and for the following diffusion lengths:

(1) L_p = 0.1 μm and (2) L_p = 1 μm.

Solution: Calculating the base transport factor using

$$B = 1 - (1/2)\left(W_B / L_p\right)^2$$

yields

(1) B = 0.5 and (2) B = 0.995

Summary of operation regimes

Cutoff

V_{BE} is too low to provide significant injection

Example:
Given is a transistor with

$$L_p = 1\,\mu m, \quad W_B = 0.1\,\mu m$$

$$D_p = 10\,cm^2/s, \quad N_{D,\,Base} = 10^{17}\,cm^{-3}$$

$$n_i = 10^{10}\,cm^{-3}, \quad A = 100 \times 100\,\mu m^2$$

Calculate I_{Ep} for V_{BE} = 0.3 V!

$$\Delta p_E = p_{n_0} \, e^{eV/kT} = \frac{n_i^{\,2}}{N_D} \, e^{eV/kT} = 10^8 \text{ cm}^{-3}$$

$$I_{Ep} \approx e \, A \, \frac{D_P}{L_P} \, \Delta p_E \, \frac{L_P}{W_B} = 1.6 \times 10^{-9} \text{ A}$$

Calculate I_{EP} for V_{BE} = 0.7 V!

$$\Delta p_E = 5.6 \times 10^{14} \text{ cm}^{-3}$$

$$I_{Ep} = 8.9 \text{ mA}$$

Forward active

Forward biased EB junction $\Delta p_E \neq 0$

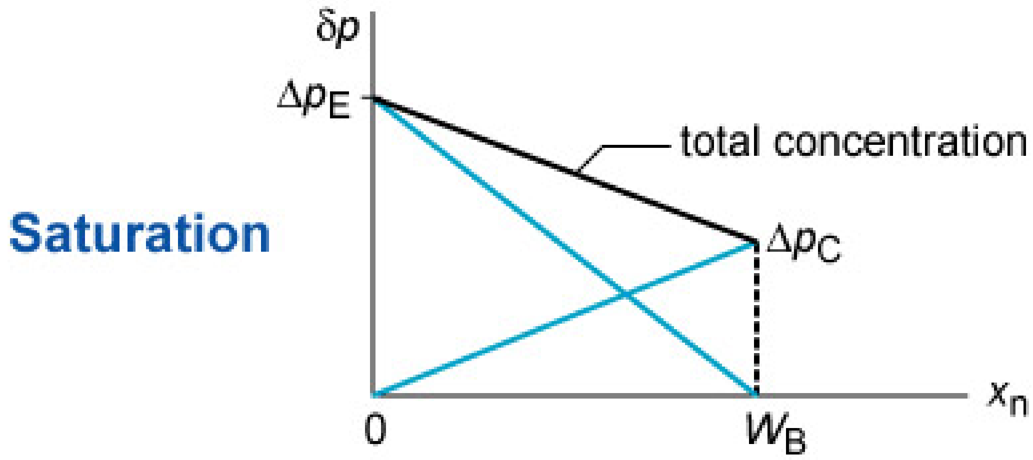
Reverse biased CB junction $\Delta p_C = 0$

Diffusion triangle in base

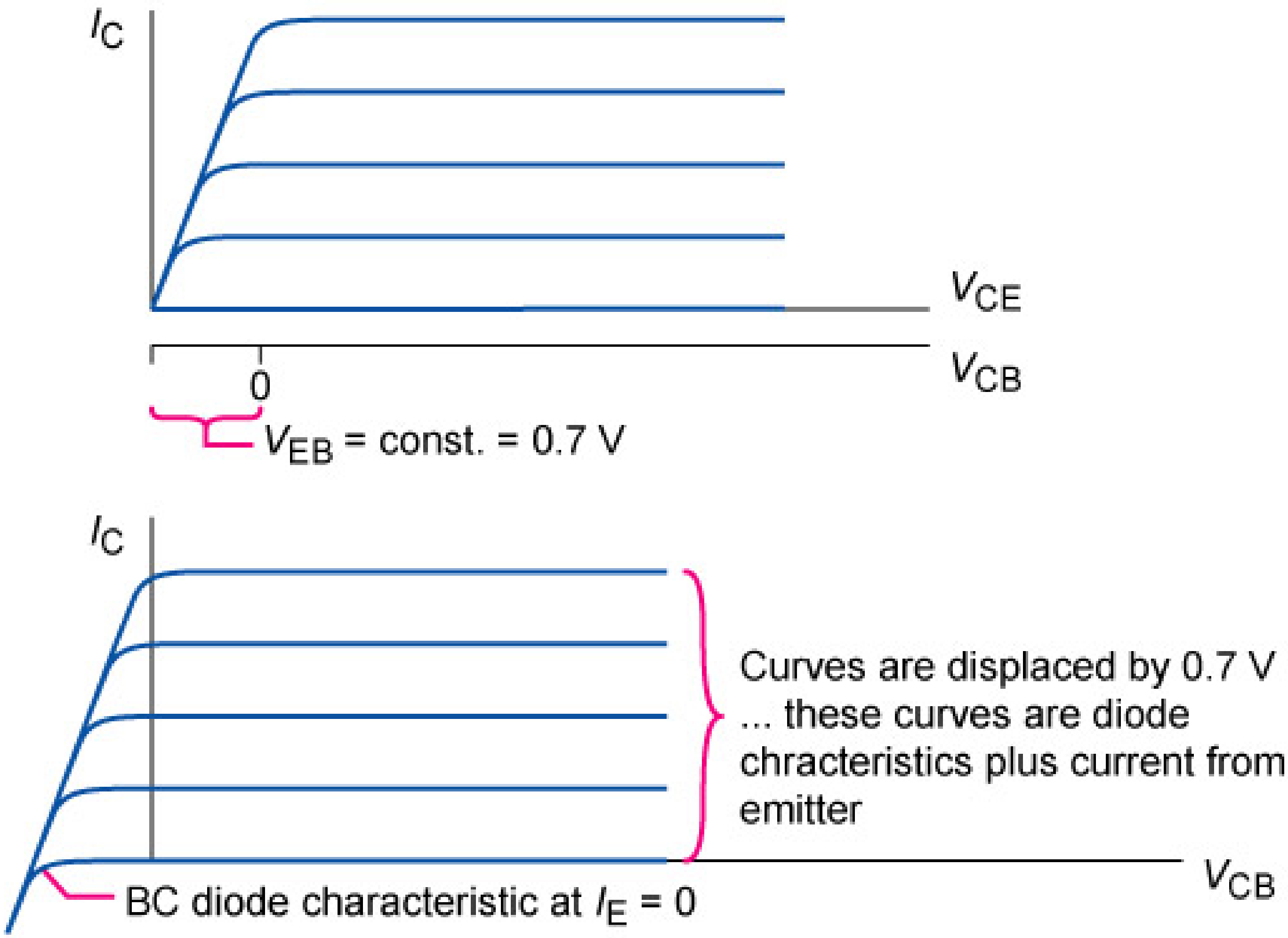
Saturation

CB junction is forward biased as well. Simultaneous transistor action in both directions, *i. e*. both diodes are forward biased.

If | V_{BE} | > | V_{CB} |, one obtains the following hole distribution in the base:



It is useful to consider the following thought experiment: Consider a transistor with V_{EB} = 0.7 V = const..



Curves are displaced by 0.7 V.

➔ The *I-V* curves can be identified as a diode characteristic plus a current from emitter.

Bridge between device physics and electrical circuit

Material parameters

Mobilities
Lifetimes
Diffusion constants
Doping concentrations
Physical constants
Material constants

Device physics

Emitter efficiency (γ)

Base transport factor (B)

Circuit parameters

Current amplification in common base configuration (α)

Current amplification in common emitter configuration (β)