11 Auto Transformer

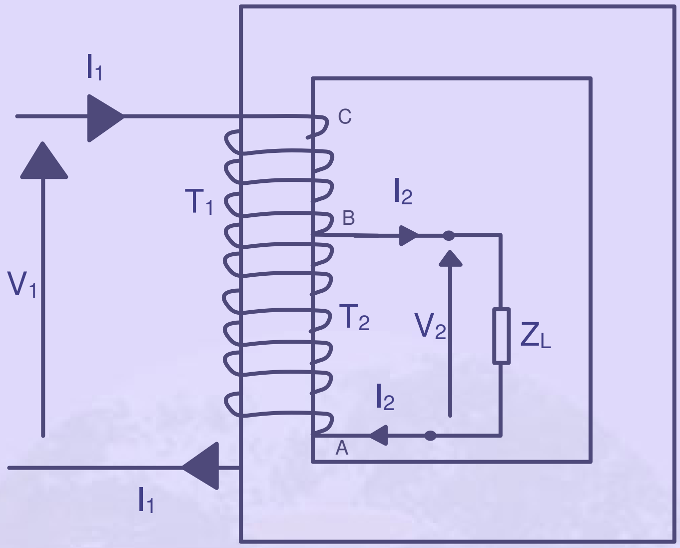

Figure 28: Autotransformer - Physical Arrangement

The primary and secondary windings of a two winding transformer have induced emf in them due to a common mutual flux and hence are in phase. The currents drawn by these two windings are out of phase by 180°. This prompted the use of a part of the primary as secondary. This is equivalent to fusing the secondary turns into primary turns. The fused section need to have a cross sectional area of the conductor to carry $(I_2 - I_1)$ ampere! This ingenious thought led to the invention of an auto transformer. Fig. 28 shows the physical arrangement of an auto transformer. Total number of turns between A and C are T_1. At point B a connection is taken. Section AB has T_2 turns. As the volts per turn, which is proportional to the flux in the machine, is the same for the whole winding,

$$V_1 : V_2 = T_1 : T_2 \tag{76}$$

For simplifying analysis, the magnetizing current of the transformer is neglected.

When the secondary winding delivers a load current of I_2 ampere the demagnetizing ampere turns is I_2T_2 . This will be countered by a current I_1 flowing from the source through the T_1 turns such that,

$$I_1T_1 = I_2T_2 \tag{77}$$

A current of I_1 ampere flows through the winding between B and C . The current in the winding between A and B is $(I_2 - I_1)$ ampere. The cross section of the wire to be selected for AB is proportional to this current assuming a constant current density for the whole winding. Thus some amount of material saving can be achieved compared to a two winding transformer. The magnetic circuit is assumed to be identical and hence there is no saving in the same. To quantify the saving the total quantity of copper used in an auto transformer is expressed as a fraction of that used in a two winding transformer as,

$$\frac{copper\ in\ auto\ transformer}{copper\ in\ two\ winding\ transformer} = \frac{(T_1 - T_2)I_1 + T_2(I_2 - I_1)}{T_1I_1 + T_2I_2} \tag{78}$$

$$= 1 - \frac{2T_2I_1}{T_1I_1 + T_2I_2}$$

$$\text{But } T_1I_1 = T_2I_2 \tag{79}$$

$$\therefore \text{The Ratio} = 1 - \frac{2T_2I_1}{2T_1I_1} = 1 - \frac{T_2}{T_1} \tag{80}$$

This means that an auto transformer requires the use of lesser quantity of copper given by the ratio of turns. This ratio therefore denotes the savings in copper. As the space for the second winding need not be there, the window space can be less for an auto transformer, giving some saving in the lamination weight also. The larger the ratio of the voltages, smaller is the savings. As T_2 approaches T_1 the savings become significant. Thus auto transformers become ideal choice for close ratio transformations. The savings in material is obtained, however, at a price. The electrical isolation between primary and secondary

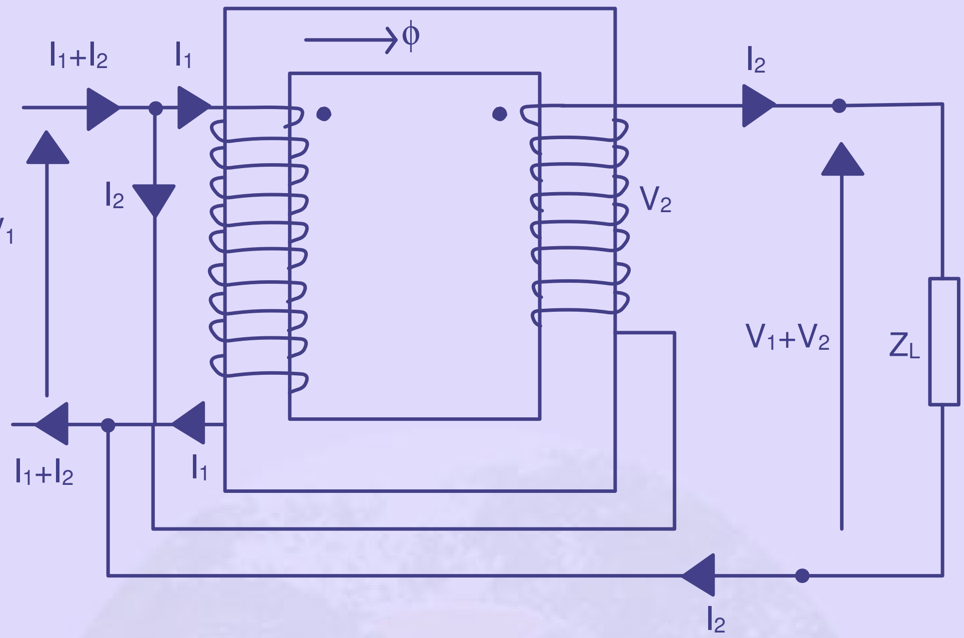

Figure 29: Two Winding Transformer used as auto transformer

has to be sacrificed.

If we are not looking at the savings in the material, even then going in for the auto transformer type of connection can be used with advantage, to obtain higher output. This can be illustrated as follows. Fig. 29 shows a regular two winding transformer of a voltage ratio $V_1 : V_2$, the volt ampere rating being $V_1I_1 = V_2I_2 = S$. If now the primary is connected across a supply of V_1 volt and the secondary is connected in series addition manner with the primary winding, the output voltage becomes $(V_1 + V_2)$ volt. The new output of this auto transformer will now be

$$I_2(V_1 + V_2) = I_2V_2(1 + \frac{V_1}{V_2}) = S(1 + \frac{V_1}{V_2}) \quad (81)$$

$$= V_1(I_1 + I_2) = S(1 + \frac{I_2}{I_1}) \quad (82)$$

Thus an increased rating can be obtained compared to a two winding transformer with the same material content. The windings can be connected in series opposition fashion also. Then the new output rating will be

$$I_2(V_1 - V_2) = I_2 V_2 (\frac{V_1}{V_2} - 1) = S(\frac{V_1}{V_2} - 1) \tag{83}$$

The differential connection is not used as it is not advantageous as the cumulative connection.

11.1 Equivalent circuit

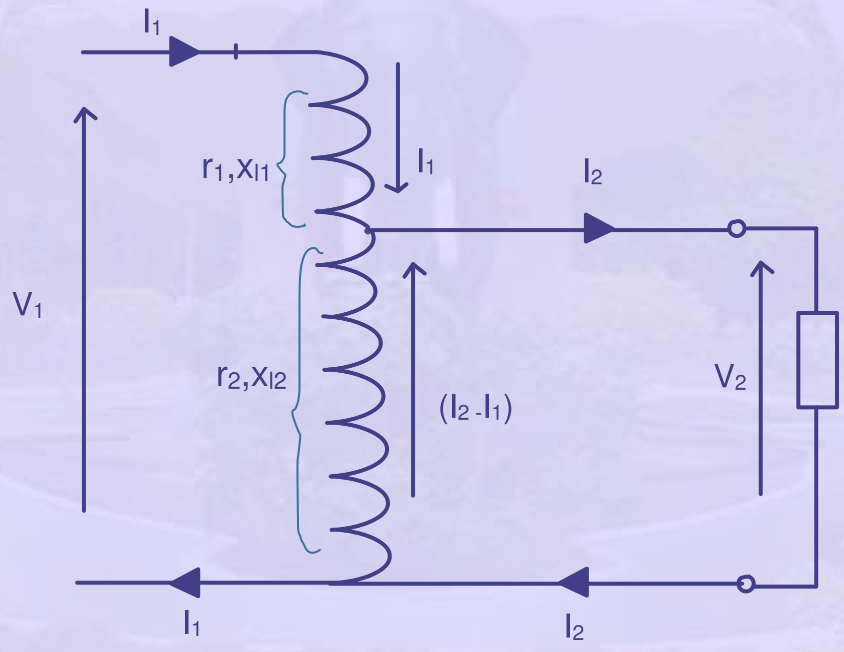

Figure 30: Kirchoff's Law Application to auto transformer

As mentioned earlier the magnetizing current can be neglected, for simplicity. Writing the Kirchoff's equation to the primary and secondary of Fig. 30 we have

$$V_1 = E_1 + I_1(r_1 + jx_{l1}) - (I_2 - I_1)(r_2 + jx_{l2}) \tag{84}$$

Note that the resistance r_1 and leakage reactance x_{l1} refer to that part of the winding where only the primary current flows. Similarly on the load side we have,

$$E_2 = V_2 + (I_2 - I_1)(r_2 + jx_{l2}) \tag{85}$$

The voltage ratio $V_1 : V_2 = E_1 : E_2 = T_1 : T_2 = a$ where T_1 is the total turns of the primary.
Then $E_1 = aE_2$ and $I_2 = aI_1$
multiplying equation(84) by 'a' and substituting in (83) we have

$$\begin{aligned}
V_1 &= aV_2 + a(I_2 - I_1)(r_2 + jx_{l2}) + I_1(r_1 + jx_{l1}) - (I_2 - I_1)(r_2 + jx_{l2}) \\
&= aV_2 + I_1(r_1 + jx_{l1} + r_2 + jx_{l2} - ar_2 - ajx_l2) + I_2(ar_2 + jax_{l2} - r_2 - jx_{l2}) \\
&= aV_2 + I_1(r_1 + jx_{l1} + r_2 + jx_{l2} + a^2r_2 + ja^2x_{l2} - ar_2 - ajx_{l2} - ar_2 - jax_{l2}) \\
&= aV_2 + I_1(r_1 + r_2(1 + a^2 - 2a) + jx_{l1} + x_{l2}(1 + a^2 - 2a)) \\
&= aV_2 + I_1(r_1 + (a-1)^2r_2 + jx_{l1} + (a-1)^2x_{l2}) \qquad (86)
\end{aligned}$$

Equation (85) yields the equivalent circuit of Fig. 31 where $R_e = r_1 + (a-1)^2r_2$ and $X_e = x_{l1} + (a-1)^2x_{l2}$.

The magnetization branch can now be hung across the mains for completeness. The above equivalent circuit can now be compared with the approximate equivalent circuit of a two winding case $R_e = r_1 + a^2r_2$ and $X_e = x_{l1} + a^2x_{l2}$. Thus in the case of an auto transformer total value of the short circuit impedance is lower and so also the percentage resistance and reactance. Thus the full load regulation is lower. Having a smaller value of short circuit impedance is sometimes considered to be a disadvantage. That is because

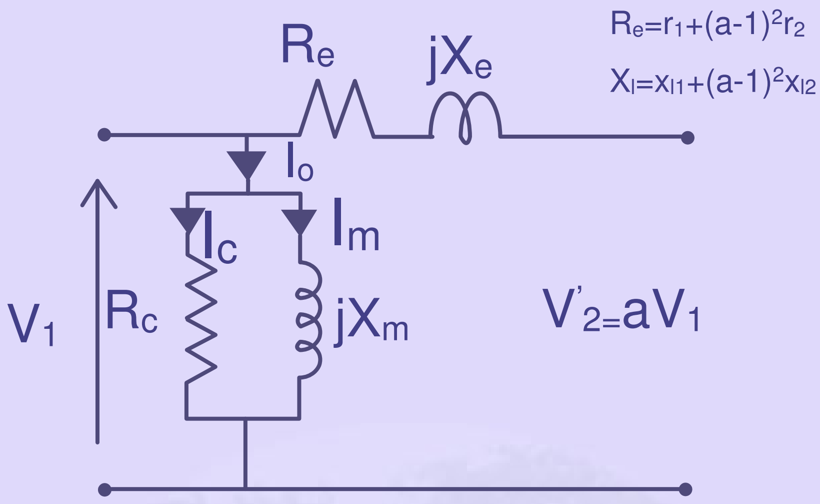

Figure 31: Equivalent Circuit of auto transformers

the short circuit currents become very large in those cases. The efficiency is higher in auto transformers compared to their two winding counter part at the same load. The phasor diagram of operation for the auto transformer drawing a load current at a lagging power factor angle of θ_2 is shown in Fig. 32. The magnetizing current is omitted here again for simplicity.

From the foregoing study it is seen that there are several advantages in going in for the autotransformer type of arrangement. The voltage/current transformation and impedance conversion aspects of a two winding transformer are retained but with lesser material (and hence lesser weight) used. The losses are reduced increasing the efficiency. Reactance is reduced resulting in better regulation characteristics. All these benefits are enhanced as the voltage ratio approaches unity. The price that is required to be paid is loss of electrical isolation and a larger short circuit current (and larger short circuit forces on the winding).

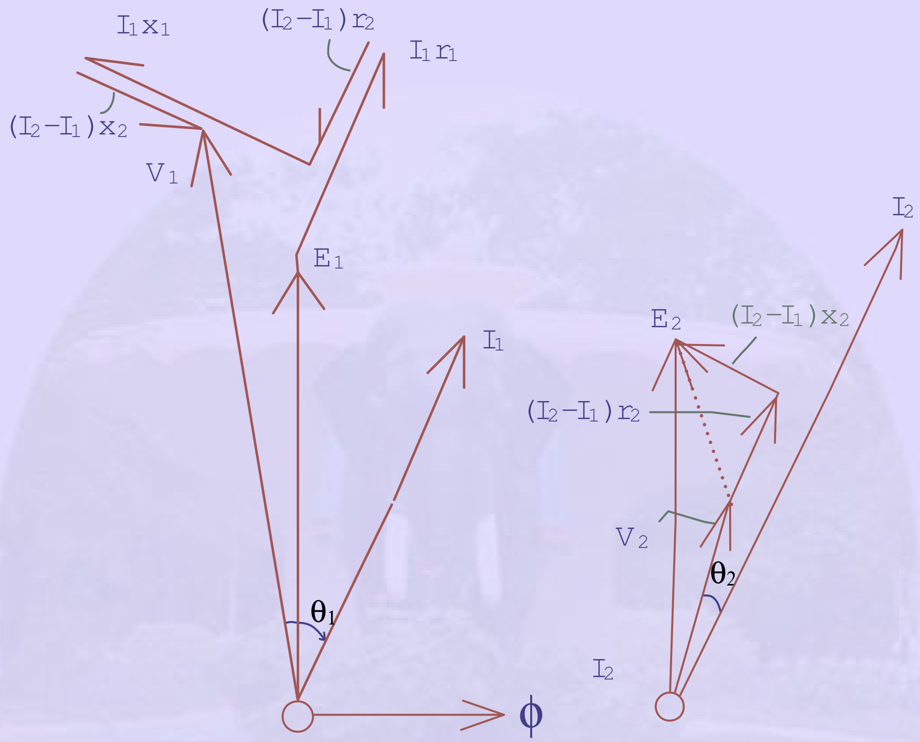

Figure 32: Phasor Diagram of Operation of an autotransformer

Auto transformers are used in applications where electrical isolation is not a critical requirement. When the ratio $V_2 : V_1$ is 0.3 or more they are used with advantage. The normal applications are motor starters, boosters or static balancers.

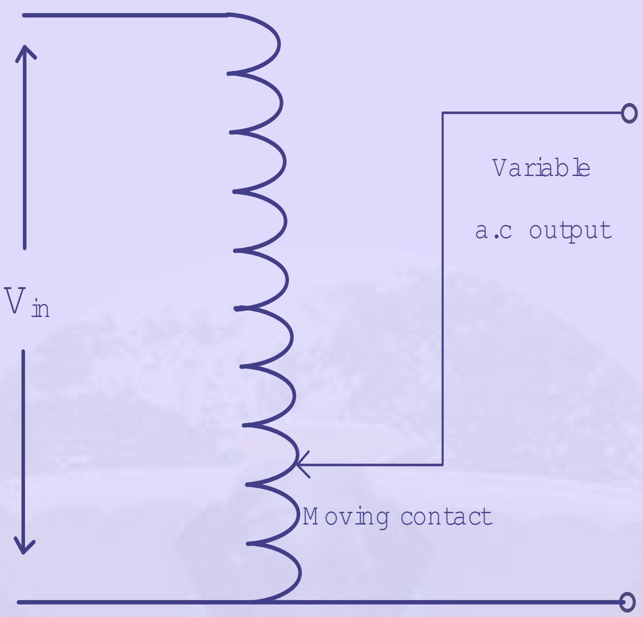

Figure 33: Variable Secondary Voltage Arrangement

Another wide spread application of auto transformer type of arrangement is in obtaining a variable a.c. voltage from a fixed a.c. voltage supply. Here only one winding is used as in the auto transformer. The secondary voltage is tapped by a brush whose position and hence the output voltage is variable. The primary conductor is bared to facilitate electrical contact Fig. 33. Such arrangement cannot exploit the savings in the copper as the output voltage is required right from zero volts upwards.

The conductor is selected based on the maximum secondary current that could be drawn as the output voltage varies in practically continuous manner. These are used in voltage stabilizers, variable d.c. arrangements (with a diode bridge) in laboratories, motor starters, dimmers etc.