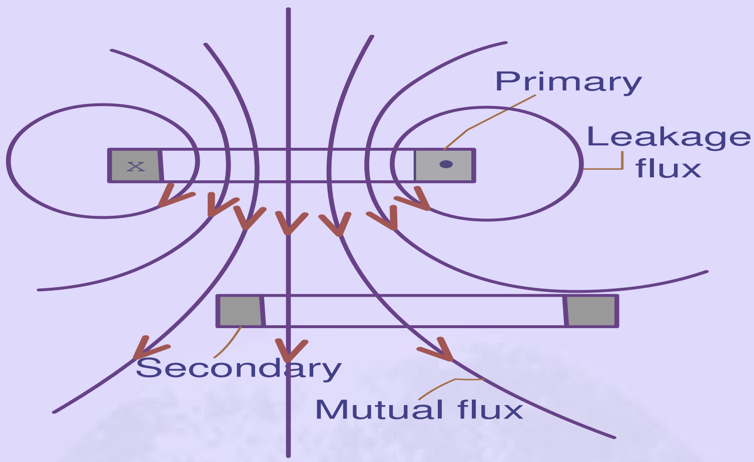
4 Ideal Transformer

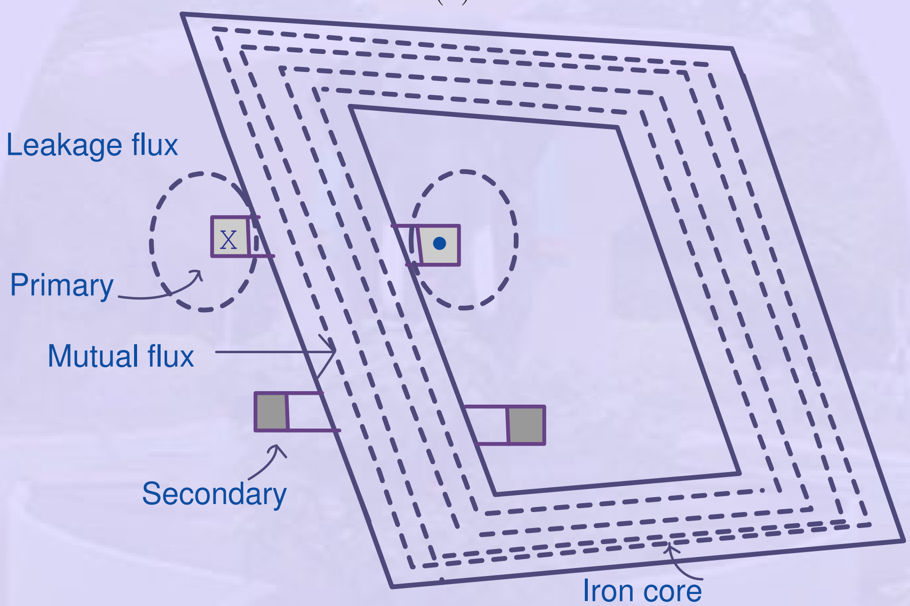
Earlier it is seen that a voltage is induced in a coil when the flux linkage associated with the same changed. If one can generate a time varying magnetic field any coil placed in the field of influence linking the same experiences an induced emf. A time varying field can be created by passing an alternating current through an electric coil. This is called mutual induction. The medium can even be air. Such an arrangement is called air cored transformer. Indeed such arrangements are used in very high frequency transformers. Even though the principle of transformer action is not changed, the medium has considerable influence on the working of such devices. These effects can be summarized as the followings.

1. The magnetizing current required to establish the field is very large, as the reluctance of the medium is very high.
2. There is linear relationship between the mmf created and the flux produced.
3. The medium is non-lossy and hence no power is wasted in the medium.
4. Substantial amount of leakage flux exists.
5. It is very hard to direct the flux lines as we desire, as the whole medium is homogeneous.

If the secondary is not loaded the energy stored in the magnetic field finds its way back to the source as the flux collapses. If the secondary winding is connected to a load then part of the power from the source is delivered to the load through the magnetic field as a link. The medium does not absorb and lose any energy. Power is required to create the field and not to maintain the same. As the winding losses can be made very small by proper choice of material, the ideal efficiency of a transformer approaches 100%. The large magnetizing



(a)



(b)

Figure 10: Mutual Induction a) air core b) iron core

current requirement is a major deterrent. However if now a piece of magnetic material is introduced to form the magnetic circuit Fig. 10(b) the situation changes dramatically. These can be enumerated as below.

1. Due to the large value for the permeance (μ_r of the order of 1000 as compared to air) the magnetizing current requirement decreases dramatically. This can also be visualized as a dramatic increase in the flux produced for a given value of magnetizing current.

2. The magnetic medium is linear for low values of induction and exhibits saturation type of non-linearity at higher flux densities.

3. The iron also has hysteresis type of non-linearity due to which certain amount of power is lost in the iron (in the form of hysteresis loss), as the B-H characteristic is traversed.

4. Most of the flux lines are confined to iron path and hence the mutual flux is increased very much and leakage flux is greatly reduced.

5. The flux can be easily 'directed' as it takes the path through steel which gives great freedom for the designer in physical arrangement of the excitation and output windings.

6. As the medium is made of a conducting material eddy currents are induced in the same and produce losses. These are called 'eddy current losses'. To minimize the eddy current losses the steel core is required to be in the form of a stack of insulated laminations.

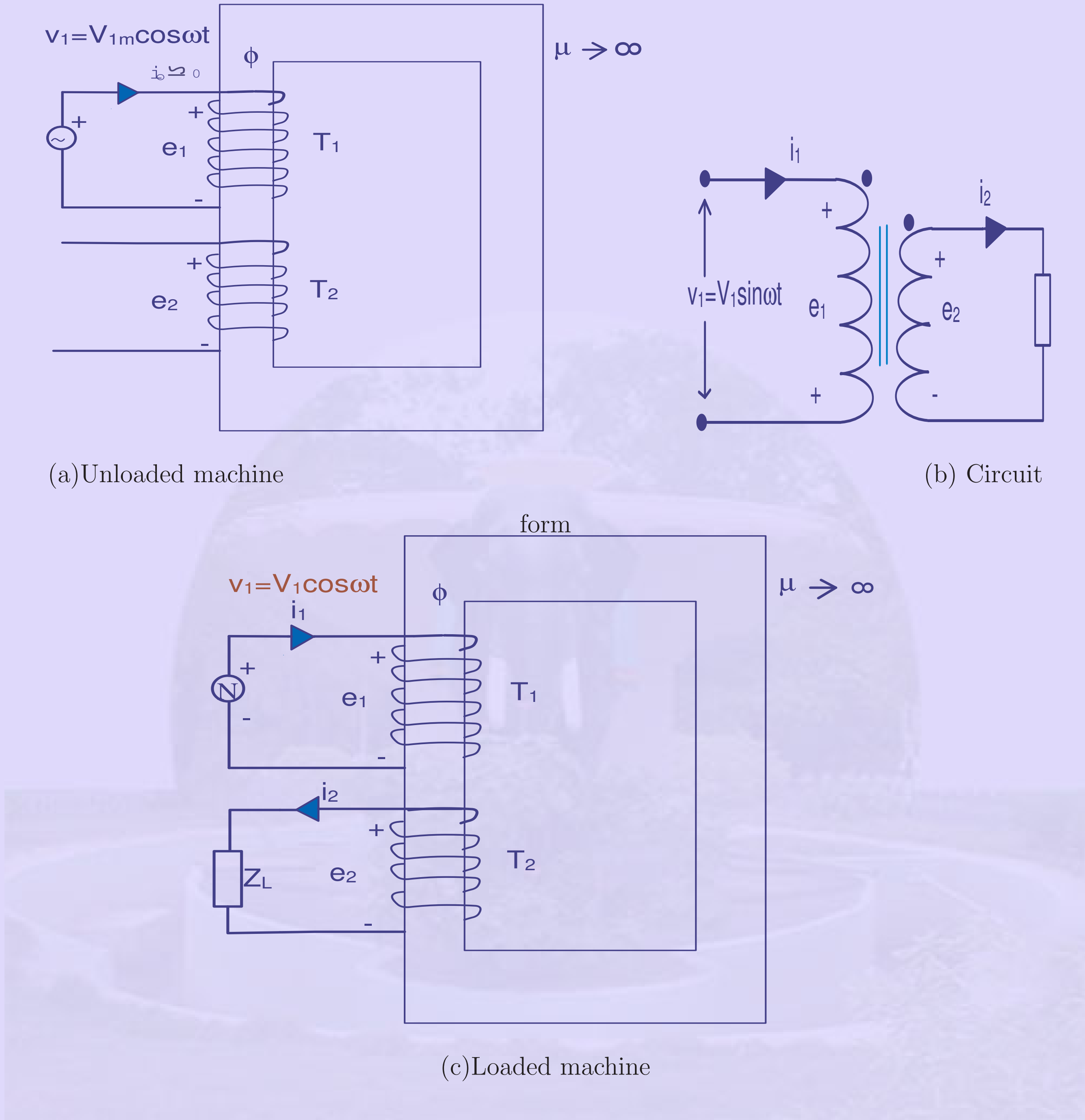
From the above it is seen that the introduction of magnetic core to carry the flux introduced two more losses. Fortunately the losses due to hysteresis and eddy current for the available grades of steel is very small at power frequencies. Also the copper losses in the winding due to magnetization current is reduced to an almost insignificant fraction of the full load losses. Hence steel core is used in power transformers.

In order to have better understanding of the behavior of the transformer, initially certain idealizations are made and the resulting 'ideal' transformer is studied. These idealizations are as follows:

1. Magnetic circuit is linear and has infinite permeability. The consequence is that a vanishingly small current is enough to establish the given flux. Hysteresis loss is negligible. As all the flux generated confines itself to the iron, there is no leakage flux.
2. Windings do not have resistance. This means that there are no copper losses, nor there is any ohmic drop in the electric circuit.

In fact the practical transformers are very close to this model and hence no major departure is made in making these assumptions.
Fig. 11 shows a two winding ideal transformer. The primary winding has T_1 turns and is connected to a voltage source of V_1 volts. The secondary has T_2 turns. Secondary can be connected to a load impedance for loading the transformer. The primary and secondary are shown on the same limb and separately for clarity.

As a current I_0 amps is passed through the primary winding of T_1 turns it sets up an mmf of I_0T_1 ampere which is in turn sets up a flux ϕ through the core. Since the reluctance of the iron path given by $R = l/\mu A$is zero as $\mu \longrightarrow \infty$, a vanishingly small value of current I_0 is enough to setup a flux which is finite. As I_0 establishes the field inside the transformer



(a)Unloaded machine (b) Circuit

(c)Loaded machine

Figure 11: Two winding Ideal Transformer unloaded and loaded

it is called the magnetizing current of the transformer.

$$Flux \quad \phi = \frac{mmf}{Reluctance} = \frac{I_0 T_1}{\frac{l}{\mu A}} = \frac{I_0 T_1 A\mu}{l}. \tag{9}$$

This current is the result of a sinusoidal voltage V applied to the primary. As the current through the loop is zero (or vanishingly small), at every instant of time, the sum of the voltages must be zero inside the same. Writing this in terms of instantaneous values we have,

$$v_1 - e_1 = 0 \tag{10}$$

where v_1 is the instantaneous value of the applied voltage and e_1 is the induced emf due to Faradays principle. The negative sign is due to the application of the Lenz's law and shows that it is in the form of a voltage drop. Kirchoff's law application to the loop will result in the same thing.

This equation results in $v_1 = e_1$ or the induced emf must be same in magnitude to the applied voltage at every instant of time. Let $v_1 = V_{1peak} \cos \omega t$ where V_{1peak} is the peak value and $\omega = 2\pi f\ t$. f is the frequency of the supply. As $v_1 = e_1$; $e_1 = d\psi_1/dt$ but $e_1 = E_{1peak} \cos \omega t \therefore E_1 = V_1$. It can be easily seen that the variation of flux linkages can be obtained as $\psi_1 = \psi_{1peak} \sin \omega t$. Here ψ_{1peak} is the peak value of the flux linkages of the primary.

Thus the RMS primary induced emf is

$$e_1 = \frac{d\psi_1}{dt} = \frac{d(\psi_{1peak} \sin \omega t)}{dt} \tag{11}$$

$$= \psi_{1peak}.\omega.\cos \omega t \quad \text{or the rms value} \tag{12}$$

$$E_1 = \frac{\psi_{1peak}.\omega}{\sqrt{2}} = \frac{2\pi f T_1 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_1 \quad \text{volts}$$

Here ψ_{1peak} is the peak value of the flux linkages of the primary. The same mutual flux links the secondary winding. However the magnitude of the flux linkages will be $\psi_{2peak} = T_2.\phi_m$. The induced emf in the secondary can be similarly obtained as ,

$$e_2 = \frac{d\psi_2}{dt} = \frac{d(\psi_{2peak}\sin\omega t)}{dt} \quad (13)$$

$$= \psi_{2peak}.\omega.\cos\omega t \quad \text{or the rms value} \quad (14)$$

$$E_2 = \frac{2\pi f T_2 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_2 \quad \text{volt}$$

which yields the voltage ratio as

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} \quad (15)$$

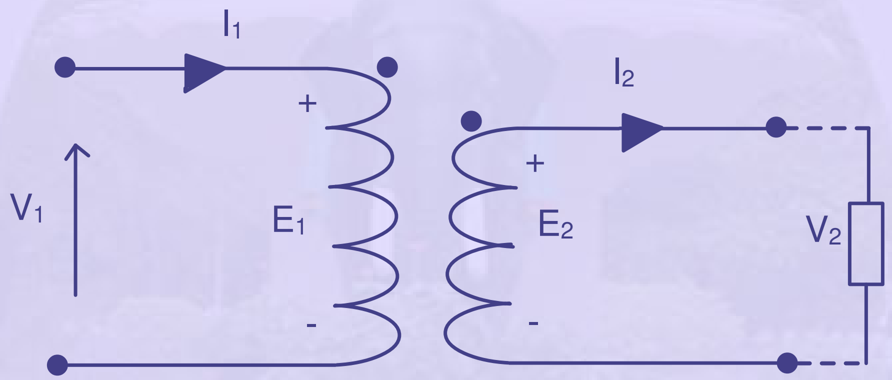


Figure 12: Dot Convention

The voltages E_1 and E_2 are obtained by the same mutual flux and hence they are in phase. If the winding sense is opposite i.e., if the primary is wound in clockwise sense and the secondary counter clockwise sense then if the top terminal of the first winding is at maximum potential the bottom terminal of the second winding would be at the peak potential. Similar problem arises even when the sense of winding is kept the same, but the two windings are on opposite limbs (due to the change in the direction of flux). Hence in the circuit representation of transformers a dot convention is adopted to indicate the terminals of the windings that go high (or low) together. (Fig. 12). This can be established experimentally by means of a polarity test on the transformers. At a particular instant of time if the current enters the terminal marked with a dot it magnetizes the core. Similarly a current leaving the terminal with a dot demagnetizes the core.

So far, an unloaded ideal transformer is considered. If now a load impedance Z_L is connected across the terminals of the secondary winding a load current flows as marked in Fig. 11(c). This load current produces a demagnetizing mmf and the flux tends to collapse. However this is detected by the primary immediately as both E_2 and E_1 tend to collapse. The current drawn from supply increases up to a point the flux in the core is restored back to its original value. The demagnetizing mmf produced by the secondary is neutralized by additional magnetizing mmf produces by the primary leaving the mmf and flux in the core as in the case of no-load. Thus the transformer operates under constant induced emf mode. Thus,

$$i_1T_1 - i_2T_2 = i_0T_1 \quad \text{but} \quad i_0 \to 0 \tag{16}$$

$$i_2T_2 = i_1T_1 \quad \text{and the rms value} \quad I_2T_2 = I_1T_1. \tag{17}$$

If the reference directions for the two currents are chosen as in the Fig. 12, then the above equation can be written in phasor form as,

$$\bar{I}_1T_1 = \bar{I}_2T_2 \quad \text{or} \quad \bar{I}_1 = \frac{T_2}{T_1}.\bar{I}_2 \tag{18}$$

$$\text{Also} \quad \frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{I_2}{I_1} \quad E_1I_1 = E_2I_2 \tag{19}$$

Thus voltage and current transformation ratio are inverse of one another. If an impedance of Z_L is connected across the secondary,

$$\bar{I}_2 = \frac{\bar{E}_2}{\bar{Z}_L} \quad \text{or} \quad \bar{Z}_L = \frac{\bar{E}_2}{\bar{I}_2} \tag{20}$$

The input impedance under such conditions is

$$\bar{Z}_i = \frac{\bar{E}_1}{\bar{I}_1} = (\frac{T_1}{T_2})^2.\frac{\bar{E}_2}{\bar{I}_2} = (\frac{T_1}{T_2})^2.\bar{Z}_L \tag{21}$$

An impedance of Z_L when viewed 'through' a transformer of turns ratio $(\frac{T_1}{T_2})$ is seen as $(\frac{T_1}{T_2})^2.Z_L$. Transformer thus acts as an impedance converter. The transformer can be interposed in between a source and a load to 'match' the impedance.

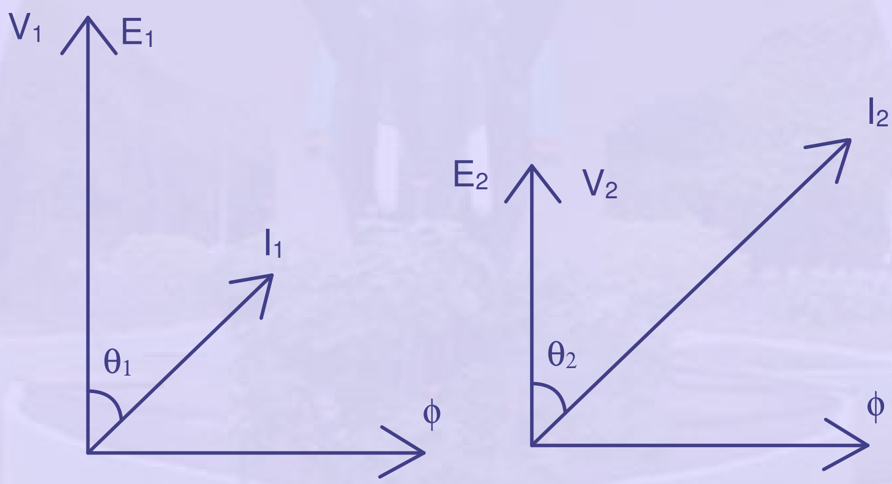


Figure 13: Phasor diagram of Operation of an Ideal Transformer

Finally, the phasor diagram for the operation of the ideal transformer is shown in Fig. 13 in which θ_1 and θ_2 are power factor angles on the primary and secondary sides. As the transformer itself does not absorb any active or reactive power it is easy to see that $\theta_1 = \theta_2$.

Thus, from the study of the ideal transformer it is seen that the transformer provides electrical isolation between two coupled electric circuits while maintaining power invariance at its two ends. However, grounding of loads and one terminal of the transformer on the secondary/primary side are followed with the provision of leakage current detection devices to safe guard the persons working with the devices. Even though the isolation aspect is a desirable one its utility cannot be over emphasized. It can be used to step up or step down the voltage/current at constant volt-ampere. Also, the transformer can be used for impedance matching. In the case of an ideal transformer the efficiency is 100% as there are no losses inside the device.