

## 5 Practical Transformer

An ideal transformer is useful in understanding the working of a transformer. But it cannot be used for the computation of the performance of a practical transformer due to the non-ideal nature of the practical transformer. In a working transformer the performance aspects like magnetizing current, losses, voltage regulation, efficiency etc are important. Hence the effects of the non-idealization like finite permeability, saturation, hysteresis and winding resistances have to be added to an ideal transformer to make it a practical transformer. Conversely, if these effects are removed from a working transformer what is left behind is an ideal transformer.

Finite permeability of the magnetic circuit necessitates a finite value of the current to be drawn from the mains to produce the mmf required to establish the necessary flux. The current and mmf required is proportional to the flux density  $B$  that is required to be established in the core.

$$B = \mu H; \quad B = \frac{\phi}{A} \quad (22)$$

where  $A$  is the area of cross section of the iron core  $m^2$ .  $H$  is the magnetizing force which is given by,

$$H = i \cdot \frac{T_1}{l} \quad (23)$$

where  $l$  is the length of the magnetic path, m. or

$$\phi = B \cdot A = \frac{A\mu(iT_1)}{l} = \text{permeance} * \text{mmf}(\text{here that of primary}) \quad (24)$$

The magnetizing force and the current vary linearly with the applied voltage as long as the magnetic circuit is not saturated. Once saturation sets in, the current has to vary in

a nonlinear manner to establish the flux of sinusoidal shape. This non-linear current can be resolved into fundamental and harmonic currents. This is discussed to some extent under harmonics. At present the effect of this non-linear behavior is neglected as a secondary effect. Hence the current drawn from the mains is assumed to be purely sinusoidal and directly proportional to the flux density of operation. This current can be represented by a current drawn by an inductive reactance in the circuit as the net energy associated with the same over a cycle is zero. The energy absorbed when the current increases is returned to the electric circuit when the current collapses to zero. This current is called the magnetizing current of the transformer. The magnetizing current  $I_m$  is given by  $I_m = E_1/X_m$  where  $X_m$  is called the magnetizing reactance. The magnetic circuit being lossy absorbs and dissipates the power depending upon the flux density of operation. These losses arise out of hysteresis, eddy current inside the magnetic core. These are given by the following expressions:

$$P_h \propto B^{1.6} f \quad (25)$$

$$P_e \propto B^2 f^2 t^2 \quad (26)$$

$P_h$  -Hysteresis loss, Watts

$B$ - Flux density of operation Tesla.

$f$  - Frequency of operation, Hz

$t$  - Thickness of the laminations of the core, m.

For a constant voltage, constant frequency operation  $B$  is constant and so are these losses. An active power consumption by the no-load current can be represented in the input circuit as a resistance  $R_c$  connected in parallel to the magnetizing reactance  $X_m$ . Thus the no-load current  $I_0$  may be made up of  $I_c$ (loss component) and  $I_m$  (magnetizing component)

as )

$$\bar{I}_0 = \bar{I}_c - j\bar{I}_m \quad (27)$$

$I_c^2 R_c$  - gives the total core losses (i.e. hysteresis + eddy current loss)

$I_m^2 X_m$  - Reactive volt amperes consumed for establishing the mutual flux.

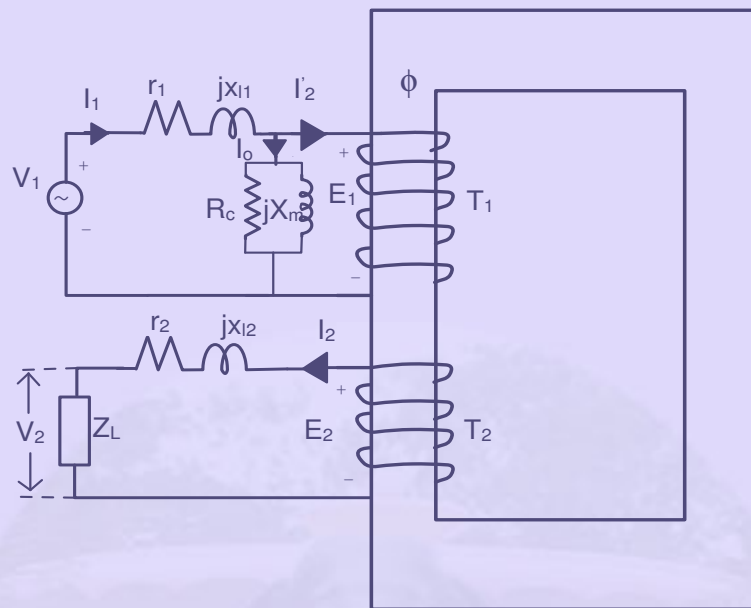
Finite  $\mu$  of the magnetic core makes a few lines of flux take to a path through the air. Thus these flux lines do not link the secondary winding. It is called as leakage flux. As the path of the leakage flux is mainly through the air the flux produced varies linearly with the primary current  $I_1$ . Even a large value of the current produces a small value of flux. This flux produces a voltage drop opposing its cause, which is the current  $I_1$ . Thus this effect of the finite permeability of the magnetic core can be represented as a series inductive element  $jx_{l1}$ . This is termed as the reactance due to the primary leakage flux. As this leakage flux varies linearly with  $I_1$ , the flux linkages per ampere and the primary leakage inductance are constant (This is normally represented by  $l_{l1}$  Henry). The primary leakage reactance therefore becomes

$$x_{l1} = 2\pi f l_{l1} \quad \text{ohm} \quad (28)$$

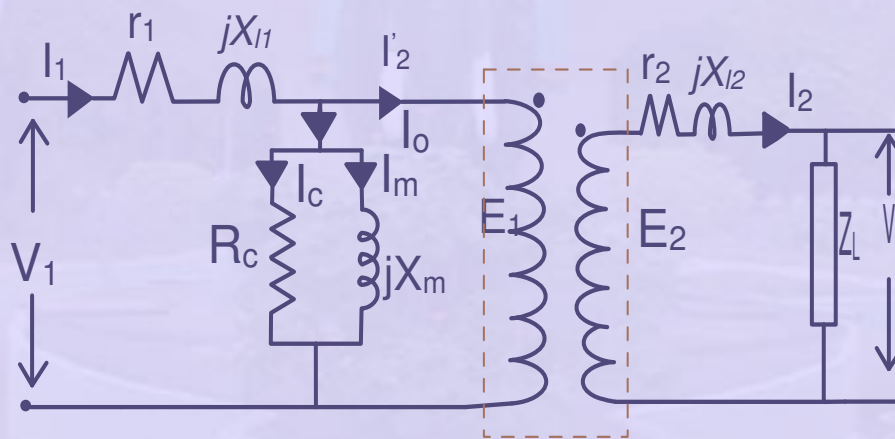
A similar effect takes place on the secondary side when the transformer is loaded. The secondary leakage reactance  $jx_{l2}$  arising out of the secondary leakage inductance  $l_{l2}$  is given by

$$x_{l2} = 2\pi f l_{l2} \quad (29)$$

Finally, the primary and secondary windings are wound with copper (sometimes aluminium in small transformers) conductors; thus the windings have a finite resistance (though



(a) Physical arrangement



(b) Equivalent circuit

Figure 14: A Practical Transformer

small). This is represented as a series circuit element, as the power lost and the drop produced in the primary and secondary are proportional to the respective currents. These are represented by  $r_1$  and  $r_2$  respectively on primary and secondary side. A practical transformer sans these imperfections (taken out and represented explicitly in the electric circuits) is an ideal transformer of turns ratio  $T_1 : T_2$  (voltage ratio  $E_1 : E_2$ ). This is seen in Fig. 14.  $I_2'$  in the circuit represents the primary current component that is required to flow from the mains in the primary  $T_1$  turns to neutralize the demagnetizing secondary current  $I_2$  due to the load in the secondary turns. The total primary current

$$\text{vectorially is } \bar{I}_1 = \bar{I}_2' + \bar{I}_0 \quad (30)$$

$$\text{Here } I_2' T_1 = I_2 T_2 \quad \text{or} \quad I_2' = I_2 \frac{T_2}{T_1} \quad (31)$$

$$\text{Thus } \bar{I}_1 = \bar{I}_2 \frac{T_2}{T_1} + \bar{I}_0 \quad (32)$$

By solving this circuit for any load impedance  $Z_L$  one can find out the performance of the loaded transformer.

The circuit shown in Fig. 14(b). However, it is not very convenient for use due to the presence of the ideal transformer of turns ratio  $T_1 : T_2$ . If the turns ratio could be made unity by some transformation the circuit becomes very simple to use. This is done here by replacing the secondary by a 'hypothetical' secondary having  $T_1$  turns which is 'equivalent' to the physical secondary. The equivalence implies that the ampere turns, active and reactive power associated with both the circuits must be the same. Then there is no change as far as their effect on the primary is considered. Thus

$$V_2' = aV_2, \quad I_2' = \frac{I_2}{a}, \quad r_2' = a^2 r_2, \quad x_{l2}' = a^2 x_{l2} \quad Z_L' = a^2 Z_L.$$

where  $a$  -turns ratio  $\frac{T_1}{T_2}$

This equivalent circuit is as shown in Fig. ??(a). As the ideal transformer in this case has a turns ratio of unity the potentials on either side are the same and hence they may be conductively connected dispensing away with the ideal transformer. This particular equivalent circuit is as seen from the primary side. It is also possible to refer all the primary parameters to secondary by making the hypothetical equivalent primary winding on the input side having the number of turns to be  $T_2$ . Such an equivalent circuit having all the parameters referred to the secondary side is shown in fig. 15.

The equivalent circuit can be derived, with equal ease, analytically using the Kirchoff's equations applied to the primary and secondary. Referring to fig. 14(a), we have (by neglecting the shunt branch)

$$V_1 = E_1 + I_1(r_1 + jx_{l1}) \quad (33)$$

$$E_2 = V_2 + I_2(r_2 + jx_{l2}) \quad (34)$$

$$T_1 I_0 = T_1 I_1 + T_2 I_2 \quad \text{or} \quad I_1 = -\frac{I_2}{a} + I_0 \quad (35)$$

$$= -\frac{I_2}{a} + I_c + I_m$$

$$a = \frac{T_1}{T_2}.$$

Multiply both sides of Eqn.34 by 'a' [This makes the turns ratio unity and retains the power invariance].

$$aE_2 = aV_2 + aI_2(r_2 + jx_{l2}) \quad \text{but} \quad aE_2 = E_1 \quad (36)$$

Substituting in Eqn.33 we have

$$\begin{aligned}
 V_1 &= aV_2 + aI_2(r_2 + jx_{l2}) + I_1(r_1 + jx_{l1}) \\
 &= V_2' + I_1(a^2r_2 + ja^2x_{l2}) + I_1(r_1 + jx_{l1}) \\
 &= V_2' + I_1(\overline{r_1 + r_2'} + \overline{jx_{l1} + x_{l2}'})
 \end{aligned} \tag{37}$$

A similar procedure can be used to refer all parameters to secondary side. (Shown in fig. 15.)

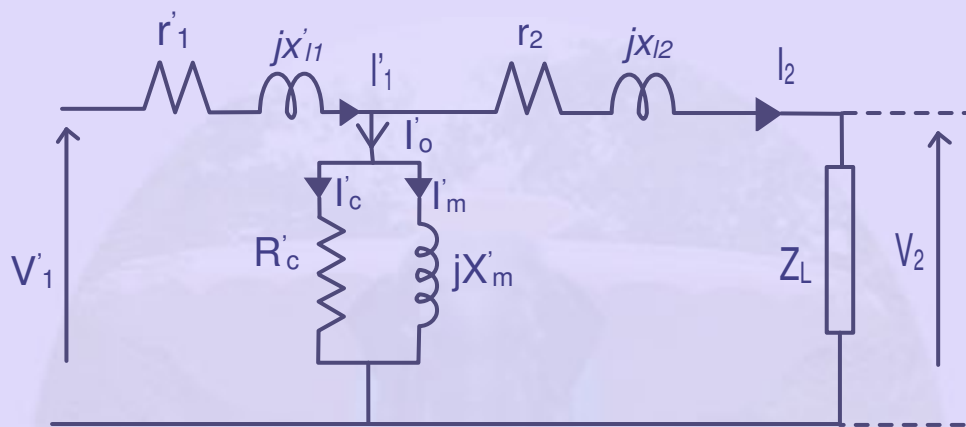


Figure 15: Equivalent Circuit Referred to the Secondary Side