Measurement of three-phase power with the 2-wattmeter method.

Section from: “Introduction to Electrical Engineering”, M. S. Sarma, Oxford University press, 2001.

4.3 MEASUREMENT OF POWER

A *wattmeter* is an instrument with a potential coil and a current coil so arranged that its deflection is proportional to $VI \cos \theta$, where V is the voltage (rms value) applied across the potential coil, I is the current (rms value) passing through the current coil, and θ is the angle between $\bar{V}$ and $\bar{I}$. By inserting such a single-phase wattmeter to measure the average real power in each phase (with its current coil in series with one phase of the load and its potential coil across the phase of the load), the total real power in a three-phase system can be determined by the sum of the wattmeter readings. However, in practice, this may not be possible due to the nonaccessibility of either the neutral of the wye connection, or the individual phases of the delta connection. Hence it is more desirable to have a method for measuring the total real power drawn by a three-phase load while we have access to only three line terminals.

The three-phase power can be measured by three single-phase wattmeters having current coils in each line and potential coils connected across the given line and any common junction. Since this common junction is completely arbitrary, it may be placed on any one of the three lines, in which case the wattmeter connected in that line will indicate zero power because its potential coil has no voltage across it. Hence, that wattmeter may be dispensed with, and three-phase power can be measured by means of only two single-phase wattmeters having a common potential junction on any of the three lines in which there is no current coil. This is known as the *two-wattmeter method of measuring three-phase power.* In general, m-phase power can be measured by means of $m - 1$ wattmeters. The method is valid for both balanced and unbalanced circuits with either the load or the source unbalanced.

Figure 4.3.1 shows the connection diagram for the two-wattmeter method of measuring three-phase power. The total real power delivered to the load is given by the *algebraic sum* of the two wattmeter readings,

$$P = W_A + W_C \qquad (4.3.1)$$

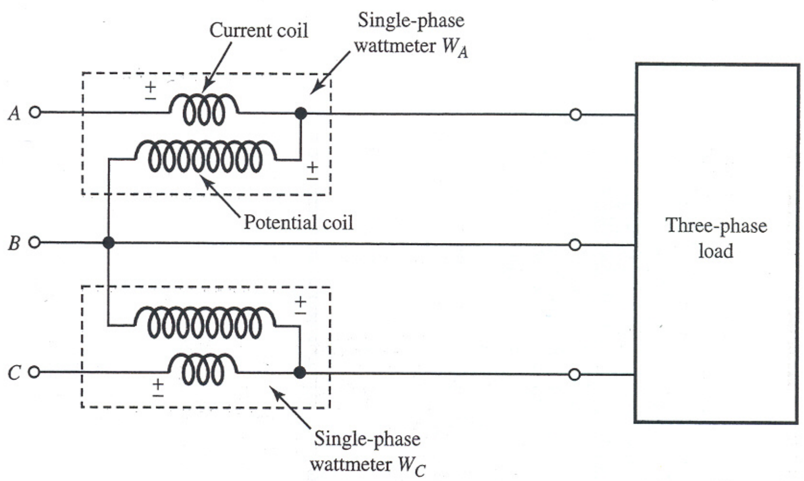

Figure 4.3.1 Connection diagram for two-wattmeter method of measuring three-phase power.

The significance of the algebraic sum will be realized in the paragraphs that follow. Two wattmeters can be connected with their current coils in any two lines, while their potential coils are connected to the third line, as shown in Figure 4.3.1. The wattmeter readings are given by

$$W_A = V_{AB} \cdot I_A \cdot \cos \theta_A \tag{4.3.2}$$

where θ_A is the angle between the phasors $\bar{V}_{AB}$ and $\bar{I}_A$, and

$$W_C = V_{CB} \cdot I_C \cdot \cos \theta_C \tag{4.3.3}$$

where θ_C is the angle between the phasors $\bar{V}_{CB}$ and $\bar{I}_C$.

The two-wattmeter method, when applied to the *balanced* loads, yields interesting results. Considering either balanced wye- or delta-connected loads, with the aid of the corresponding phasor diagrams drawn earlier for the phase sequence *A–B–C* (Figures 4.2.2. and 4.2.3), it can be seen that the angle between $\bar{V}_{AB}$ and $\bar{I}_A$ is $(30° + \phi)$ and that between $\bar{V}_{CB}$ and $\bar{I}_C$ is $(30 - \phi)$, where ϕ is the load power factor angle, or the angle associated with the load impedance. Thus, we have

$$W_A = V_L I_L \cos (30° + \phi) \tag{4.3.4}$$

and

$$W_C = V_L I_L \cos (30° - \phi) \tag{4.3.5}$$

where V_L and I_L are the magnitudes of the line-to-line voltage and line current, respectively. Simple manipulations yield

$$W_A + W_C = \sqrt{3}\, V_L I_L \cos \phi \tag{4.3.6}$$

and

$$W_C - W_A = V_L I_L \sin \phi \tag{4.3.7}$$

from which,

$$\tan \phi = \sqrt{3}\, \frac{W_C - W_A}{W_C + W_A} \tag{4.3.8}$$

When the load power factor is unity, corresponding to a purely resistive load, both wattmeters will indicate the same wattage. In fact, both of them should read positive; if one of the wattmeters has a below-zero indication in the laboratory, an upscale deflection can be obtained by simply reversing the leads of either the current or the potential coil of the wattmeter. The sum of the wattmeter readings gives the total power absorbed by the load.

At zero power factor, corresponding to a purely reactive load, both wattmeters will again have the same wattage indication but with the opposite signs, so that their algebraic sum will yield zero power absorbed, as it should. The transition from a negative to a positive value occurs when the load power factor is 0.5 (i.e., ϕ is equal to 60°). At this power factor, one wattmeter reads zero while the other one reads the total real power delivered to the load.

For power factors (leading or lagging) greater than 0.5, both wattmeters read positive, and the sum of the two readings gives the total power. For a power factor less than 0.5 (leading or lagging), the smaller reading wattmeter should be given a negative sign and the total real power absorbed by the load (which has to be positive) is given by the difference between the two wattmeter readings. Figure 4.3.2 shows a plot of the load power factor versus the ratio W_l/W_h, where W_l and W_h are the lower and higher readings of the wattmeters, respectively.

Another method that is sometimes useful in a laboratory environment for determining whether the total power is the sum or difference of the two wattmeter readings is described here. To begin, make sure that both wattmeters have an upscale deflection. To perform the test, remove the lead of the potential coil of the lower reading wattmeter from the common line that has no current coil, and touch the lead to the line that has the current coil of the higher reading wattmeter. If the pointer of the lower reading wattmeter deflects upward, the two wattmeter readings should be added; if the pointer deflects in the below-zero direction, the wattage reading of the lower reading wattmeter should be subtracted from that of the higher reading wattmeter.

Given the two wattmeter readings from the two-wattmeter method used on a three-phase *balanced* load, it is possible to find the tangent of the phase impedance angle as $\sqrt{3}$ times the ratio of the difference between the two wattmeter readings and their sum, based on Equation (4.3.8). If one knows the system sequence and the lines in which the current coils of the wattmeters are located, the sign for the angle can be determined with the aid of the following expressions. For sequence *A–B–C*,

$$\tan\phi = \sqrt{3}\,\frac{W_C - W_A}{W_C + W_A} = \sqrt{3}\,\frac{W_A - W_B}{W_A + W_B} = \sqrt{3}\,\frac{W_B - W_C}{W_B + W_C} \qquad (4.3.9)$$

and for sequence *C–B–A*,

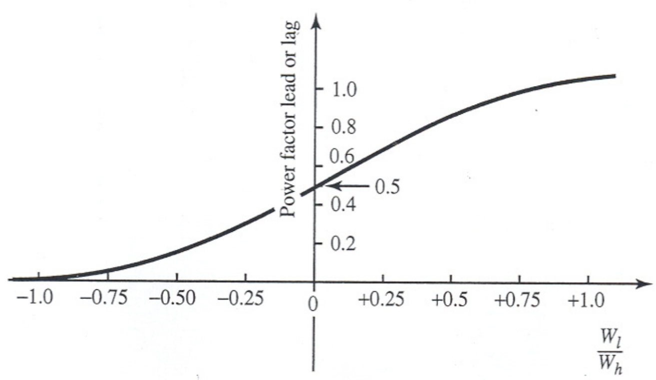

Figure 4.3.2 Plot of load power factor versus W_l/W_h.

$$\tan\phi = \sqrt{3}\,\frac{W_A - W_C}{W_A + W_C} = \sqrt{3}\,\frac{W_B - W_A}{W_B + W_A} = \sqrt{3}\,\frac{W_C - W_B}{W_C + W_B} \tag{4.3.10}$$

The two-wattmeter method discussed here for measuring three-phase power makes use of single-phase wattmeters. It may be noted, however, that three-phase wattmeters are also available, which, when connected appropriately, indicate the total real power absorbed. The total reactive power associated with the three-phase *balanced* load is given by

$$Q = \sqrt{3}\,V_L I_L\,\sin\phi = \sqrt{3}\,(W_C - W_A) \tag{4.3.11}$$

based on the two wattmeter readings of the two-wattmeter method.

With the generator action of the source assumed, $+P$ for the real power indicates that the source is supplying real power to the load; $+Q$ for the reactive power shows that the source is delivering inductive VARs while the current lags the voltage (i.e., the power factor is lagging); and $-Q$ for the reactive power indicates that the source is delivering capacitive VARs or absorbing inductive VARs, while the current leads the voltage (i.e., the power factor is leading).

EXAMPLE 4.3.1

Considering Figure 4.3.1, let balanced positive-sequence, three-phase voltages with $\bar{V}_{AB} = 100\sqrt{3}\angle 0°$ V (rms) be applied to terminals *A*, *B*, and *C*. The three-phase wye-connected balanced load consists of a per-phase impedance of $(10 + j10)\,\Omega$. Determine the wattmeter readings of W_A and W_C. Then find the total three-phase real and reactive powers delivered to the load. Based on the wattmeter readings of W_A and W_C, compute the load power factor and check the sign associated with the power factor angle.

Solution

$$\bar{V}_{AB} = 100\sqrt{3}\,\angle 0°\text{ V};\qquad \bar{V}_{BC} = 100\sqrt{3}\,\angle -120°;\qquad \bar{V}_{CA} = 100\sqrt{3}\,\angle 120°$$

$$\bar{V}_{AN} = 100\angle -30°\text{ V};\qquad \bar{V}_{BN} = 100\angle -150°;\qquad \bar{V}_{CN} = 100\angle 90°$$

$$\bar{I}_A = \frac{\bar{V}_{AN}}{\bar{Z}} = \frac{100\angle -30°}{10\sqrt{2}\,\angle 45°} = 5\sqrt{2}\,\angle -75°\text{ A (rms)}$$

$$\bar{I}_C = \frac{\bar{V}_{CN}}{\bar{Z}} = \frac{100\angle 90°}{10\sqrt{2}\,\angle 45°} = 5\sqrt{2}\,\angle 45°\text{ A (rms)}$$

The load power factor angle $\phi = 45°$, and it is a case of lagging power factor with the inductive load.

$$W_A = V_{AB} I_A\,\cos(30° + \phi) = 100\sqrt{3}\left(5\sqrt{2}\right)\,\cos 75° = 317\text{ W}$$

$$W_C = V_{CB} I_C\,\cos(30° - \phi) = 100\sqrt{3}\left(5\sqrt{2}\right)\,\cos 15° = 1183\text{ W}$$

The total three-phase real power delivered to the load

$$W_A + W_C = 317 + 1183 = 1500\text{ W}$$

which checks with

$$\sqrt{3}\,V_L I_L\,\cos\phi = \sqrt{3}\,100\sqrt{3}\left(5\sqrt{2}\right)\,\cos 45° = 1500\text{ W}$$

The total three-phase reactive power delivered to the load is

$$\sqrt{3}(W_C - W_A) = \sqrt{3}(1183 - 317) = 1500 \text{ VAR}$$

which checks with

$$\sqrt{3}V_L I_L \sin \phi = \sqrt{3}(100\sqrt{3})(5\sqrt{2}) \sin 45° = 1500 \text{ VAR}$$

For positive-sequence *A–B–C*,

$$\tan \varnothing = \sqrt{3}\left(\frac{W_C - W_A}{W_C + W_A}\right) = \sqrt{3}\left(\frac{1183 - 317}{1183 + 317}\right) = 1.0$$

or $\varnothing = 45°$ with a positive sign, implying thereby that it corresponds to an inductive or lagging load, which checks with the given load specification. The load power factor is given by

$$\cos \varnothing = 0.707$$

and is lagging in this case.