

Decoders, muxes and intro to sequential circuits

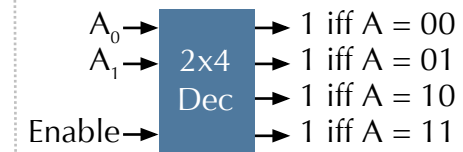
Lecture 4 — § 2.2 - 2.3
Computer Science 218

Mike Feeley

Decoders and encoders

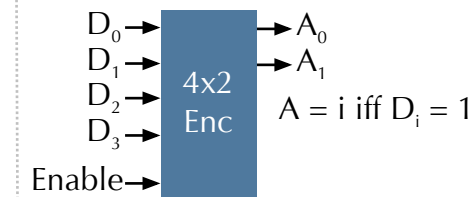
decoder selects one labelled output

- n-bit input pattern selects the output
 - you can think of this pattern as a number (address)
- m outputs labelled with unique patterns
- output is 1 iff its label matches input
- $m \leq 2^n$ outputs
 - its less if some input patterns are ignored



encoder does the opposite

- inputs are labelled with unique pattern
- output encodes pattern of input that = 1
 - there can be only one such input



uses

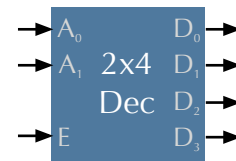
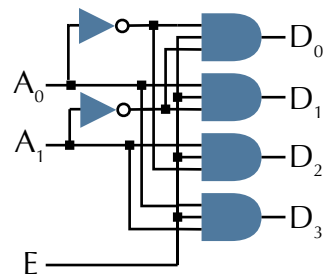
- addressing memory cells
- getting interrupt device address

Implement a decoder ...

a 2x4 decoder

- two bit input; four bit output
- package and truth table look like this

draw the circuit ...

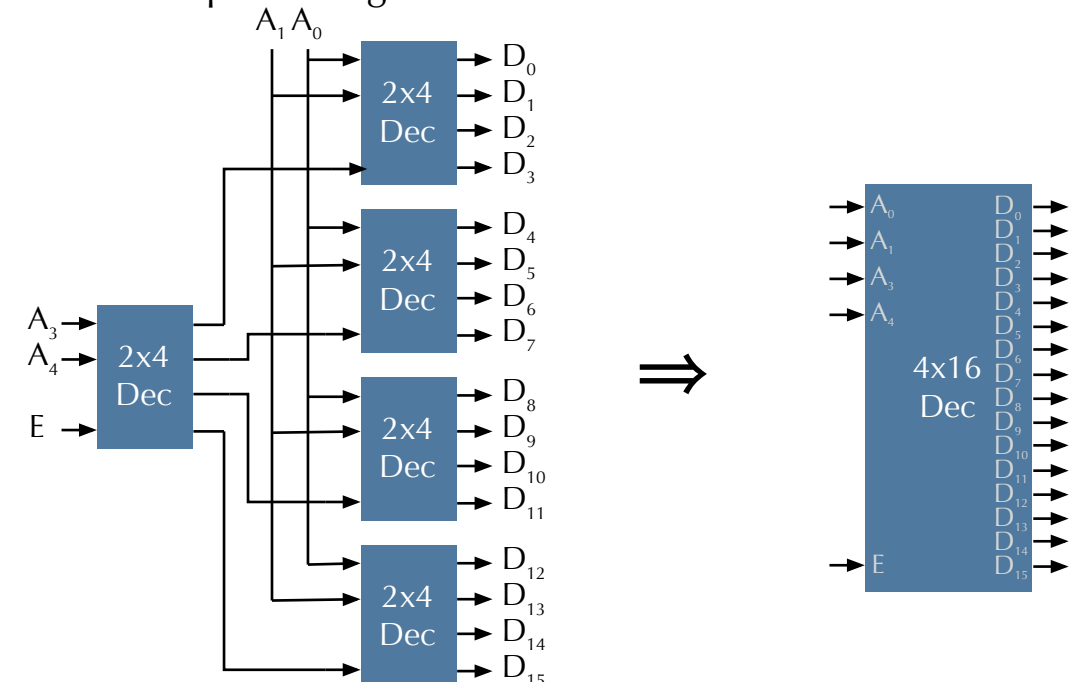


E	A ₁	A ₀	D ₀	D ₁	D ₂	D ₃
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	x	x	0	0	0	0

Composing decoders using enable

build 2n-bit decoder from n+1 n-bit decoders

- high-order inputs to one decoder, low-order inputs to all others
- connect outputs of higher-order decoder to *enables* of each other

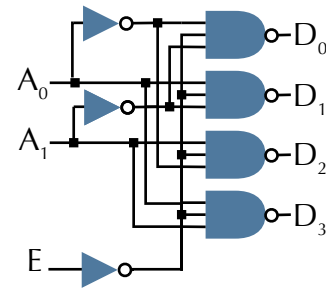


NAND decoders

NAND gates are cheaper than ANDs

- require fewer transistors to implement

a NAND decoder looks like this



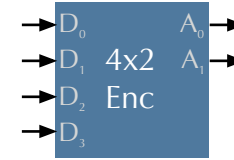
and this inverted truth table

E	A ₁	A ₀	D ₀	D ₁	D ₂	D ₃
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0
1	x	x	1	1	1	1

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Implementing an Encoder

encoder does the opposite of decoder



E	D ₀	D ₁	D ₂	D ₃	A ₁	A ₀
1	1	0	0	0	0	0
1	0	1	0	0	0	1
1	0	0	1	0	1	0
1	0	0	0	1	1	1
0	0	0	0	0	x	x

its implemented like this ...

$$A_0 = ED_1 + ED_3 = E(D_1 + D_3)$$

$$A_1 = ED_2 + ED_3 = E(D_2 + D_3)$$

or gates and an **and** for enable

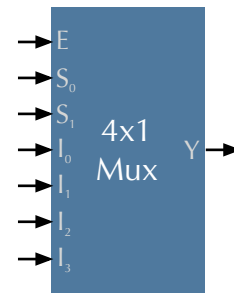
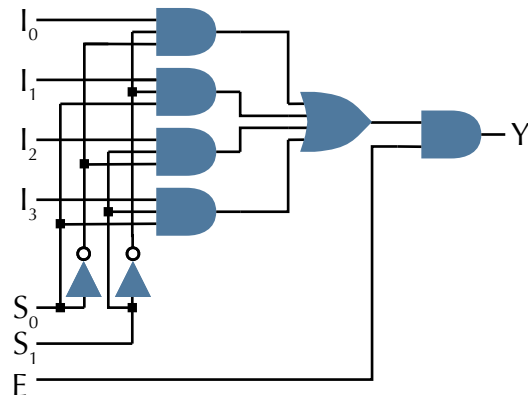
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Multiplexers

select one input to send to output

- 2ⁿ data inputs plus n select inputs
 - data inputs are labelled with unique n-bit number
- one output
 - has value of data input with label matching select

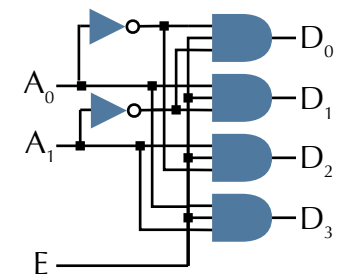
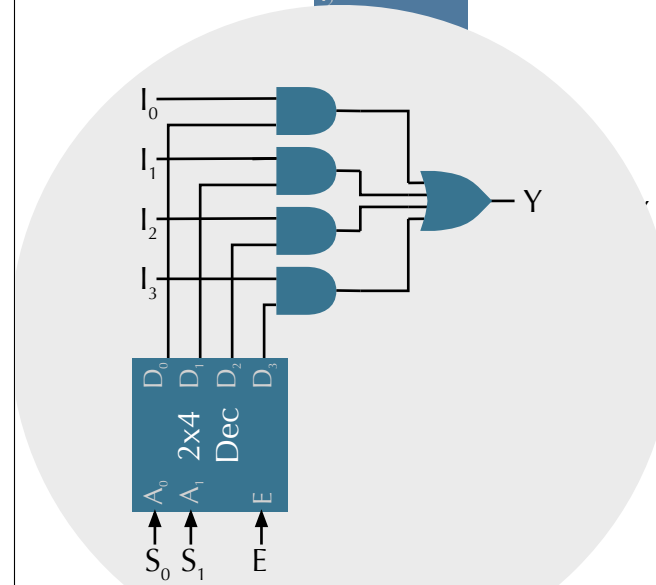
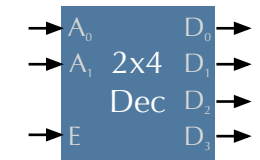
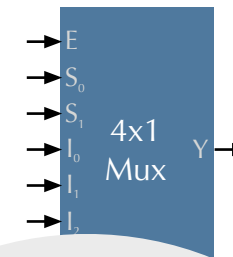
implementation



E	S ₁	S ₀	Y
1	0	0	I ₀
1	0	1	I ₁
1	1	0	I ₂
1	1	1	I ₃
0	x	x	0

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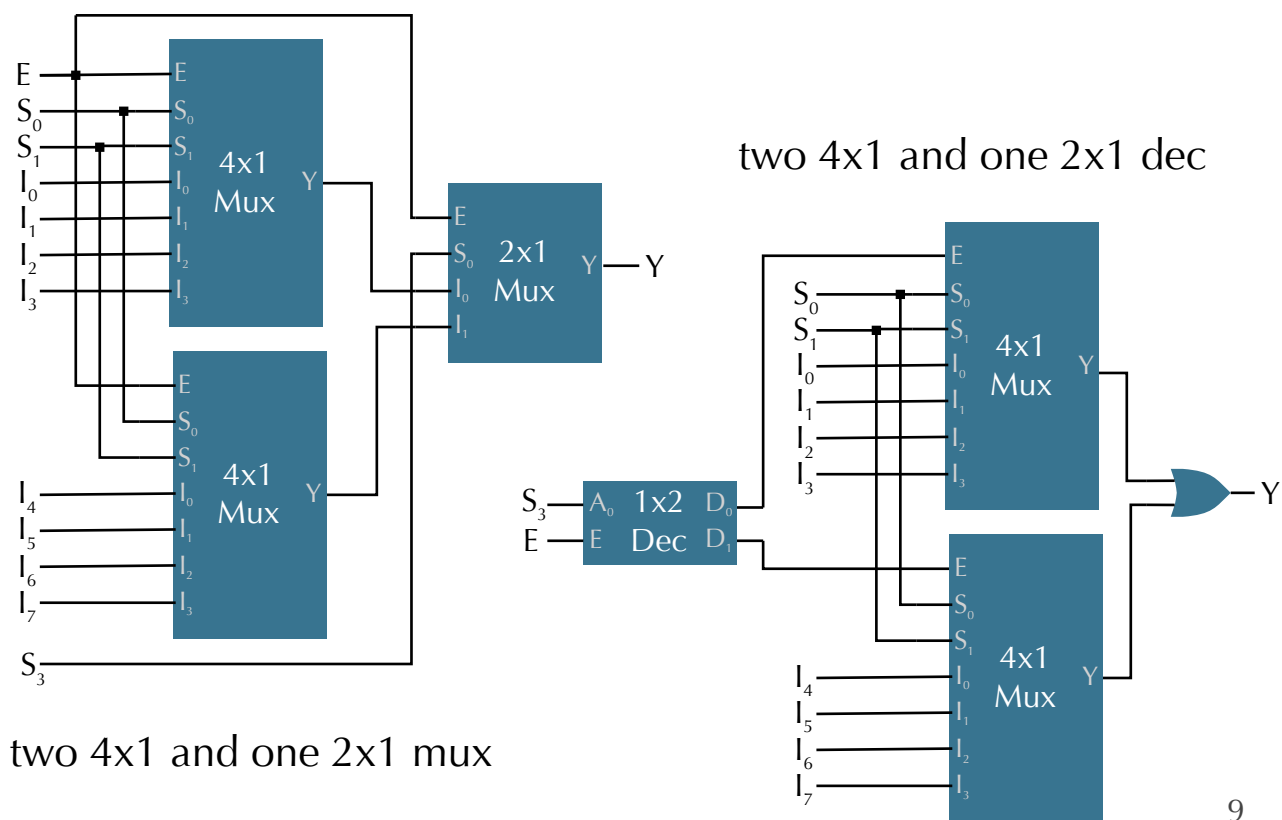
Comparing mux and decoder



Can you implement a mux using a dec? ...

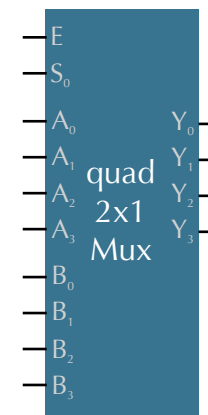
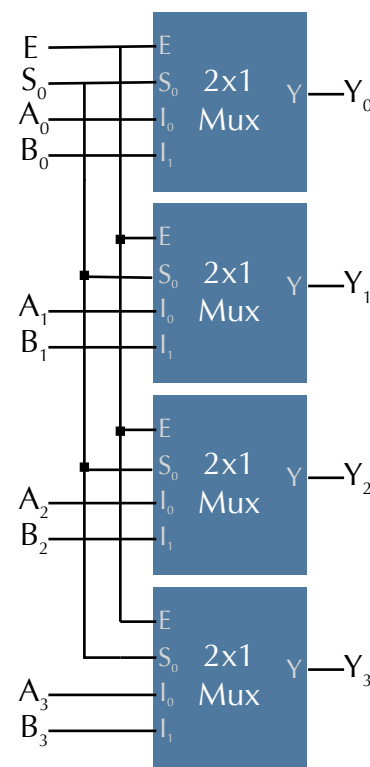
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Composing to form $2n \times 1$ mux



Composing to form *multipole* mux

quadruple mux switches 4 bits at once



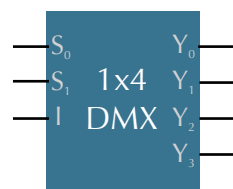
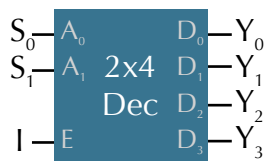
E	S	Y
0	x	all 0's
1	0	A
1	1	B

Demultiplexer

direct input to one of 2^n outputs

- one data input, n select inputs
- 2^n outputs each labelled with number
 - output label named by select = input; others = 0

what does this look like? ...



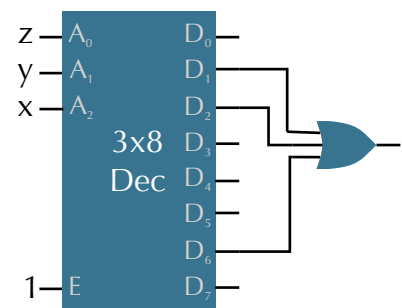
S ₁	S ₀	Y ₀	Y ₁	Y ₂	Y ₃
0	0	1			
0	1		1		
1	0			1	
1	1				1

Fun with decoders and muxes

convert truth table to logic diagram

- not necessarily efficient, but its easy
- decoder
- connect input variables to dec's select
 - one dec output for each row of table
 - **or** dec outputs for row's where f=1

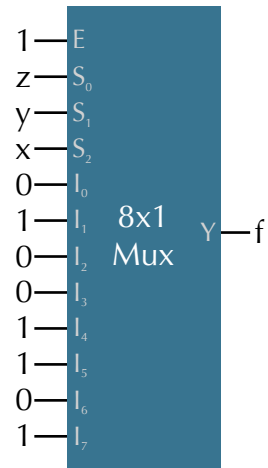
x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0



More fun

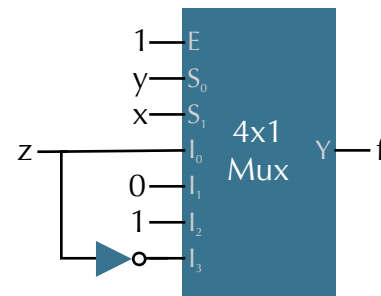
mux

- connect table row values to mux inputs
- connect input variables to mux select
- mux output is function output



can we use a smaller mux?

x	y	z	f	
0	0	0	0	I_0
0	0	1	1	I_1
0	1	0	0	I_2
0	1	1	0	I_3
1	0	0	1	I_4
1	0	1	1	I_5
1	1	0	1	I_6
1	1	1	0	I_7



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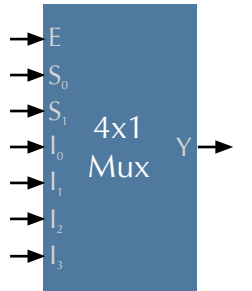
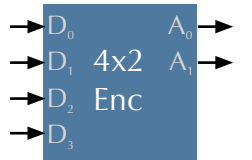
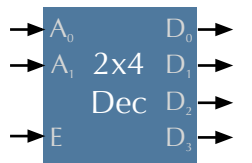
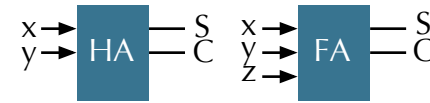
Summary

combinational circuits

- no loops — no state
- basic building block
- describe implementation with
 - boolean algebra
 - truth tables
 - logic diagrams
- abstract as
 - package

combinational circuits you know

- adders: half, full, multi-bit
- decoders, encoders and multiplexers



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Flip flops and sequential circuits

Overview

sequential circuits

- adding state to combinational circuits
- clocks

flip-flops

- a state-holding component
- there are several of them: SR, D, JK and T
- excitation tables

designing a sequential circuit

- state tables and diagrams
- some examples

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Sequential circuits


output depends on **past** as well as current inputs

- assembled by combining combinational circuit with memory
- memory elements are called *flip flops*

uses

- reduce combinational-circuit propagation delay by pipelining
 - divide circuit into multiple “stages” with flip flops between inputs and outputs
- use internal state as addition inputs
 - just about any interesting problem requires this
 - ~ e.g., clock, counter, microwave oven

approaches

- synchronous
 - central clock controls when values are locked into flip-flops 
- asynchronous
 - self timed circuits that do not use a clock

Examples

combinational circuit

time to get one answer = $7 * \text{inverter-gate-delay}$
answers per second = $1 / (7 * \text{inverter-gate-delay})$



pipelined sequential circuit

time to get one answer = $7 * \text{inverter-gate-delay} + 8 * \text{flip-flop-gate-delay}$
answers per second = $1 / (1 * \text{inverter-gate-delay} + 1 * \text{flip-flop-gate-delay})$



sequential circuit

one of inputs is output from previous step

