CHAPTER
2

THREE-PHASE SYSTEMS

2.1 BALANCED THREE-PHASE CIRCUITS

The generation, transmission and distribution of electric power is accomplished by means of three-phase circuits. At the generating station, three sinusoidal voltages are generated having the same amplitude but displaced in phase by $120°$. This is called a *balanced source*. If the generated voltages reach their peak values in the sequential order ABC, the generator is said to have a *positive phase sequence*, shown in Figure 2.11(a). If the phase order is ACB, the generator is said to have a *negative phase sequence*, as shown in Figure 2.11(b).

In a three-phase system, the instantaneous power delivered to the external loads is constant rather than pulsating as it is in a single-phase circuit. Also, three-phase motors, having constant torque, start and run much better than single-phase motors. This feature of three-phase power, coupled with the inherent efficiency of its transmission compared to single-phase (less wire for the same delivered power), accounts for its universal use.

A power system has Y-connected generators and usually includes both Δ- and Y-connected loads. Generators are rarely Δ-connected, because if the voltages are not perfectly balanced, there will be a net voltage, and consequently a circulating current, around the Δ. Also, the phase voltages are lower in the Y-connected

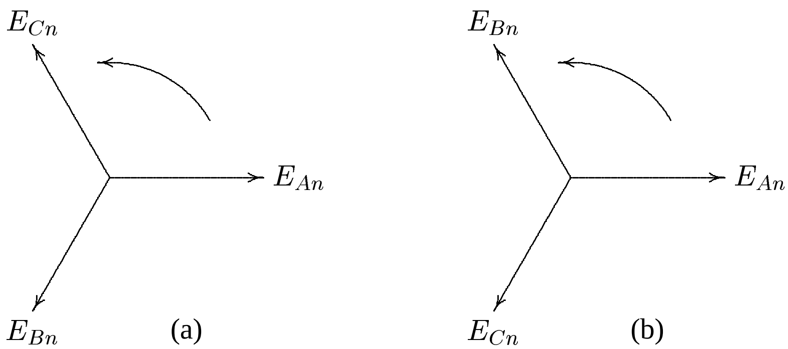

FIGURE 2.1
(a) Positive, or ABC, phase sequence. (b) Negative, or ACB, phase sequence.

generator, and thus less insulation is required. Figure 2.2 shows a Y-connected generator supplying balanced Y-connected loads through a three-phase line. Assuming a positive phase sequence (phase order ABC) the generated voltages are:

$$\begin{aligned} E_{An} &= |E_p|\angle 0^\circ \\ E_{Bn} &= |E_p|\angle -120^\circ \\ E_{Cn} &= |E_p|\angle -240^\circ \end{aligned} \tag{2.1}$$

In power systems, great care is taken to ensure that the loads of transmission lines are balanced. For balanced loads, the terminal voltages of the generator V_{An}, V_{Bn} and V_{Cn} and the phase voltages V_{an}, V_{bn} and V_{cn} at the load terminals are balanced. For "phase A," these are given by

$$V_{An} = E_{An} - Z_G I_a \tag{2.2}$$

$$V_{an} = V_{An} - Z_L I_a \tag{2.3}$$

2.2 Y-CONNECTED LOADS

To find the relationship between the line voltages (line-to-line voltages) and the phase voltages (line-to-neutral voltages), we assume a positive, or ABC, sequence. We arbitrarily choose the line-to-neutral voltage of the a-phase as the reference, thus

$$\begin{aligned} V_{an} &= |V_p|\angle 0^\circ \\ V_{bn} &= |V_p|\angle -120^\circ \\ V_{cn} &= |V_p|\angle -240^\circ \end{aligned} \tag{2.4}$$

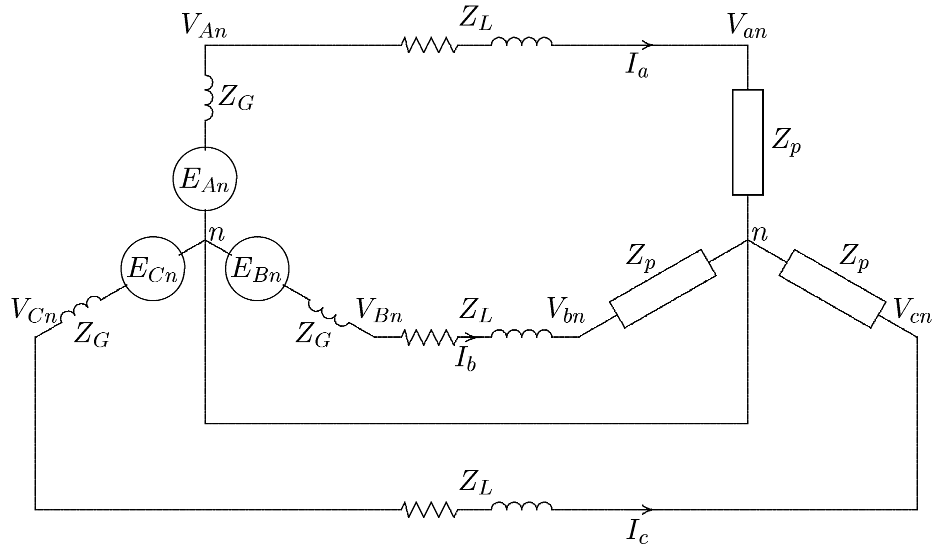

FIGURE 2.2
A Y-connected generator supplying a Y-connected load.

where $|V_p|$ represents the magnitude of the phase voltage (line-to-neutral voltage).

The line voltages at the load terminals in terms of the phase voltages are found by the application of Kirchhoff's voltage law

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} = |V_p|(1\angle 0^\circ - 1\angle{-120^\circ}) = \sqrt{3}|V_p|\angle 30^\circ \\ V_{bc} &= V_{bn} - V_{cn} = |V_p|(1\angle{-120^\circ} - 1\angle{-240^\circ}) = \sqrt{3}|V_p|\angle{-90^\circ} \\ V_{ca} &= V_{cn} - V_{an} = |V_p|(1\angle{-240^\circ} - 1\angle 0^\circ) = \sqrt{3}|V_p|\angle 150^\circ \end{aligned} \tag{2.5}$$

The voltage phasor diagram of the Y-connected loads of Figure 2.2 is shown in Figure 2.3. The relationship between the line voltages and phase voltages is demonstrated graphically.

If the rms value of any of the line voltages is denoted by V_L, then one of the important characteristics of the Y-connected three-phase load may be expressed as

$$V_L = \sqrt{3}\,|V_p|\angle 30^\circ \tag{2.6}$$

Thus in the case of Y-connected loads, the magnitude of the line voltage is $\sqrt{3}$ times the magnitude of the phase voltage, and for a positive phase sequence, the set of line voltages leads the set of phase voltages by 30°.

The three-phase currents in Figure 2.2 also possess three-phase symmetry and are given by

$$I_a = \frac{V_{an}}{Z_p} = |I_p|\angle - \theta$$

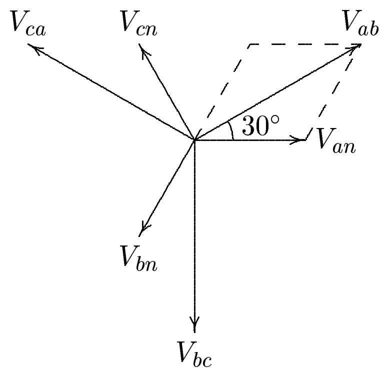

FIGURE 2.3
Phasor diagram showing phase and line voltages.

$$I_b = \frac{V_{bn}}{Z_p} = |I_p|\angle{-120^\circ} - \theta \qquad (2.7)$$
$$I_c = \frac{V_{cn}}{Z_p} = |I_p|\angle{-240^\circ} - \theta$$

where θ is the impedance phase angle.

The currents in lines are also the phase currents (the current carried by the phase impedances). Thus

$$I_L = I_p \qquad (2.8)$$

2.3 Δ-CONNECTED LOADS

A balanced Δ-connected load (with equal phase impedances) is shown in Figure 2.4.

It is clear from the inspection of the circuit that the line voltages are the same as phase voltages.

$$V_L = V_p \qquad (2.9)$$

Consider the phasor diagram shown in Figure 2.5, where the phase current I_{ab} is arbitrarily chosen as reference. we have

$$I_{ab} = |I_p|\angle 0^\circ$$
$$I_{bc} = |I_p|\angle{-120^\circ} \qquad (2.10)$$
$$I_{ca} = |I_p|\angle{-240^\circ}$$

where $|I_p|$ represents the magnitude of the phase current.

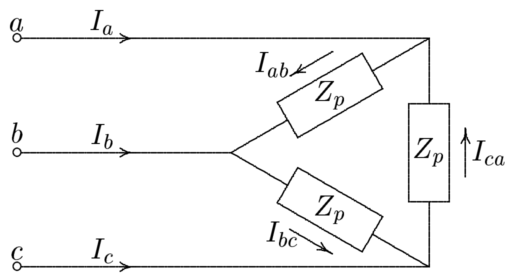

FIGURE 2.4
A Δ-connected load.

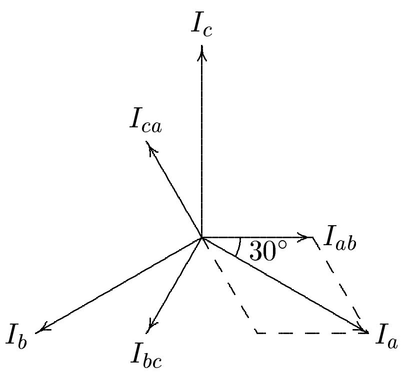

FIGURE 2.5
Phasor diagram showing phase and line currents.

The relationship between phase and line currents can be obtained by applying Kirchhoff's current law at the corners of Δ.

$$\begin{aligned} I_a &= I_{ab} - I_{ca} = |I_p|(1\angle 0° - 1\angle{-240°}) = \sqrt{3}|I_p|\angle{-30°} \\ I_b &= I_{bc} - I_{ab} = |I_p|(1\angle{-120°} - 1\angle 0°) = \sqrt{3}|I_p|\angle{-150°} \\ I_c &= I_{ca} - I_{bc} = |I_p|(1\angle{-240°} - 1\angle{-120°}) = \sqrt{3}|I_p|\angle 90° \end{aligned} \tag{2.11}$$

The relationship between the line currents and phase currents is demonstrated graphically in Figure 2.5.

If the rms of any of the line currents is denoted by I_L, then one of the important characteristics of the Δ-connected three-phase load may be expressed as

$$I_L = \sqrt{3}|I_p|\ \angle{-30°} \tag{2.12}$$

Thus in the case of Δ-connected loads, the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, and with positive phase sequence, the set of line currents lags the set of phase currents by 30°.

2.4 Δ-Y TRANSFORMATION

For analyzing network problems, it is convenient to replace the Δ-connected circuit with an equivalent Y-connected circuit. Consider the fictitious Y-connected circuit of Z_Y Ω/phase which is equivalent to a balanced Δ-connected circuit of Z_Δ Ω/phase, as shown in Figure 2.6.

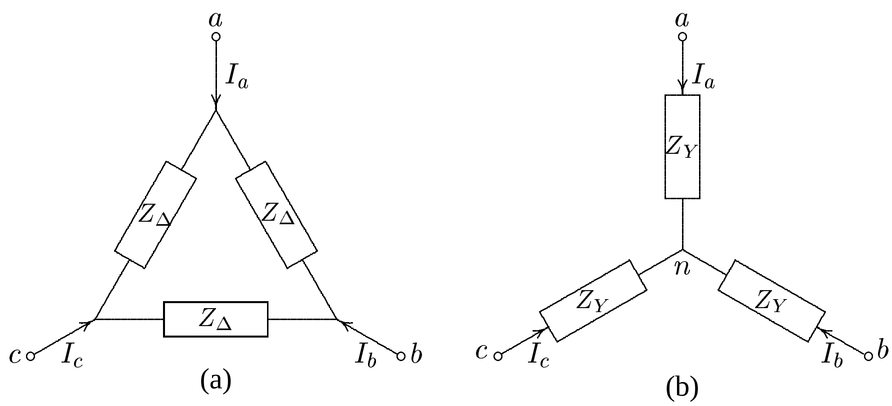

FIGURE 2.6
(a) Δ to (b) Y-connection.

For the Δ-connected circuit, the phase current I_a is given by

$$I_a = \frac{V_{ab}}{Z_\Delta} + \frac{V_{ac}}{Z_\Delta} = \frac{V_{ab} + V_{ac}}{Z_\Delta} \tag{2.13}$$

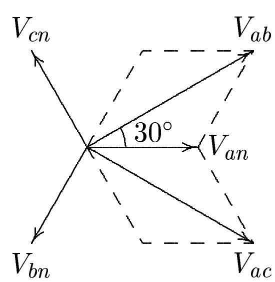

FIGURE 2.7
Phasor diagram showing phase and line voltages.

The phasor diagram in Figure 2.7 shows the relationship between balanced phase and line-to-line voltages. From this phasor diagram, we find

$$V_{ab} + V_{ac} = \sqrt{3}\,|V_{an}|\angle 30^\circ + \sqrt{3}\,|V_{an}|\angle -30^\circ \tag{2.14}$$

$$= 3V_{an} \tag{2.15}$$

Substituting in (2.13), we get

$$I_a = \frac{3V_{an}}{Z_\Delta}$$

or

$$V_{an} = \frac{Z_\Delta}{3} I_a \tag{2.16}$$

Now, for the Y-connected circuit, we have

$$V_{an} = Z_Y I_a \tag{2.17}$$

Thus, from (2.16) and (2.17), we find that

$$Z_Y = \frac{Z_\Delta}{3} \tag{2.18}$$

2.5 PER-PHASE ANALYSIS

The current in the neutral of the balanced Y-connected loads shown in Figure 2.2 is given by

$$I_n = I_a + I_b + I_c = 0 \tag{2.19}$$

Since the neutral carries no current, a neutral wire of any impedance may be replaced by any other impedance, including a short circuit and an open circuit. The return line may not actually exist, but regardless, a line of zero impedance is included between the two neutral points. The balanced power system problems are then solved on a "per-phase" basis. It is understood that the other two phases carry identical currents except for the phase shift.

We may then look at only one phase, say "phase A," consisting of the source V_{An} in series with Z_L and Z_p, as shown in Figure 2.8. The neutral is taken as datum and usually a single-subscript notation is used for phase voltages.

If the load in a three-phase circuit is connected in a Δ, it can be transformed into a Y by using the Δ-to-Y transformation. When the load is balanced, the impedance of each leg of the Y is one-third the impedance of each leg of the Δ, as given by (2.18), and the circuit is modeled by the single-phase equivalent circuit.

2.6 BALANCED THREE-PHASE POWER

Consider a balanced three-phase source supplying a balanced Y- or Δ- connected load with the following instantaneous voltages

$$v_{an} = \sqrt{2}|V_p| \cos(\omega t + \theta_v)$$

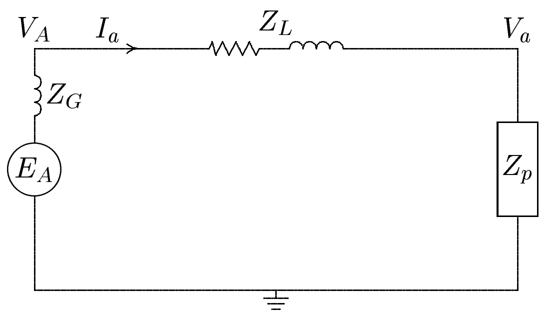

FIGURE 2.8
Single-phase circuit for per-phase analysis.

$$v_{bn} = \sqrt{2}|V_p|\cos(\omega t + \theta_v - 120^\circ) \tag{2.20}$$
$$v_{cn} = \sqrt{2}|V_p|\cos(\omega t + \theta_v - 240^\circ)$$

For a balanced load the phase currents are

$$\begin{aligned} i_a &= \sqrt{2}|I_p|\cos(\omega t + \theta_i) \\ i_b &= \sqrt{2}|I_p|\cos(\omega t + \theta_i - 120^\circ) \\ i_c &= \sqrt{2}|I_p|\cos(\omega t + \theta_i - 240^\circ) \end{aligned} \tag{2.21}$$

where $|V_p|$ and $|I_P|$ are the magnitudes of the rms phase voltage and current, respectively. The total instantaneous power is the sum of the instantaneous power of each phase, given by

$$p_{3\phi} = v_{an}i_a + v_{bn}i_b + v_{cn}i_c \tag{2.22}$$

Substituting for the instantaneous voltages and currents from (2.20) and (2.21) into (2.22)

$$\begin{aligned} p_{3\phi} = {} & 2|V_p||I_p|\cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \\ & +2|V_p||I_p|\cos(\omega t + \theta_v - 120^\circ)\cos(\omega t + \theta_i - 120^\circ) \\ & +2|V_p||I_p|\cos(\omega t + \theta_v - 240^\circ)\cos(\omega t + \theta_i - 240^\circ) \end{aligned}$$

Using the trigonometric identity **(??)**

$$\begin{aligned} p_{3\phi} = {} & |V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \\ & +|V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 240^\circ)] \\ & +|V_p||I_p|[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 480^\circ)] \end{aligned} \tag{2.23}$$

The three double frequency cosine terms in (2.23) are out of phase with each other by 120° and add up to zero, and the three-phase instantaneous power is

$$P_{3\phi} = 3|V_p||I_p|\cos\theta \tag{2.24}$$

$\theta = \theta_v - \theta_i$ is the angle between phase voltage and phase current or the impedance angle.

Note that although the power in each phase is pulsating, the total instantaneous power is constant and equal to three times the real power in each phase. Indeed, this constant power is the main advantage of the three-phase system over the single-phase system. Since the power in each phase is pulsating, the power, then, is made up of the real power and the reactive power. In order to obtain formula symmetry between real and reactive powers, the concept of complex or apparent power (S) is extended to three-phase systems by defining the three-phase reactive power as

$$Q_{3\phi} = 3|V_p||I_p| \sin\theta \tag{2.25}$$

Thus, the complex three-phase power is

$$S_{3\phi} = P_{3\phi} + jQ_3\phi \tag{2.26}$$

or

$$S_{3\phi} = 3V_p I_p^* \tag{2.27}$$

Equations (2.24) and (2.25) are sometimes expressed in terms of the rms magnitude of the line voltage and the rms magnitude of the line current. In a Y-connected load the phase voltage $|V_p| = |V_L|/\sqrt{3}$ and the phase current $I_p = I_L$. In the Δ-connection $V_p = V_L$ and $|I_p| = |I_L|/\sqrt{3}$. Substituting for the phase voltage and phase currents in (2.24) and (2.25), the real and reactive powers for either connection are given by

$$P_{3\phi} = \sqrt{3}|V_L||I_L| \cos\theta \tag{2.28}$$

and

$$Q_{3\phi} = \sqrt{3}|V_L||I_L| \sin\theta \tag{2.29}$$

A comparison of the last two expressions with (2.24) and (2.25) shows that the equation for the power in a three-phase system is the same for either a Y or a Δ connection when the power is expressed in terms of line quantities.

When using (2.28) and (2.29) to calculate the total real and reactive power, remember that θ is the phase angle between the phase voltage and the phase current. As in the case of single-phase systems for the computation of power, it is best to use the complex power expression in terms of phase quantities given by (2.27). The rated power is customarily given for the three-phase and rated voltage is the line-to-line voltage. Thus, in using the per-phase equivalent circuit, care must be taken to use per-phase voltage by dividing the rated voltage by $\sqrt{3}$.

Example 2.7

A three-phase line has an impedance of $2 + j4\ \Omega$ as shown in Figure 2.9.

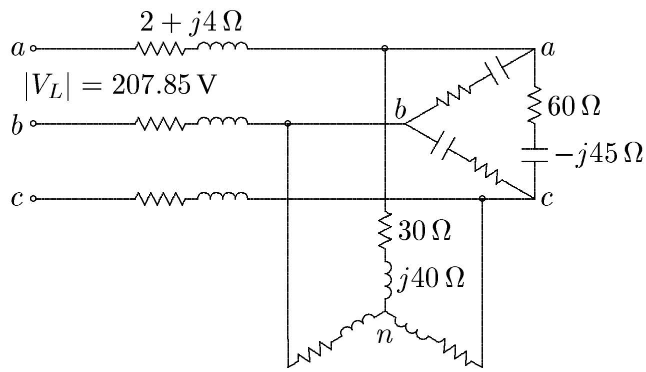

FIGURE 2.9
Three-phase circuit diagram for Example 2.7.

The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of $30+j40\ \Omega$ per phase. The second load is Δ-connected and has an impedance of $60 - j45\ \Omega$. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 V. Taking the phase voltage V_a as reference, determine:

(a) The current, real power, and reactive power drawn from the supply.
(b) The line voltage at the combined loads.
(c) The current per phase in each load.
(d) The total real and reactive powers in each load and the line.

(a) The Δ-connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15\ \ \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120\ \text{V}$$

The single-phase equivalent circuit is shown in Figure 2.10.
The total impedance is

$$\begin{aligned} Z &= 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} \\ &= 2 + j4 + 22 - j4 = 24\ \Omega \end{aligned}$$

With the phase voltage V_{an} as reference, the current in phase a is

$$I = \frac{V_1}{Z} = \frac{120\angle 0^\circ}{24} = 5\ \text{A}$$

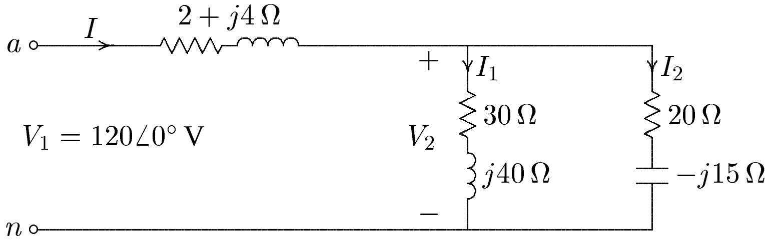

FIGURE 2.10
Single-phase equivalent circuit for Example 2.7.

The three-phase power supplied is

$$S = 3V_1 I^* = 3(120\angle 0^\circ)(5\angle 0^\circ) = 1800 \text{ W}$$

(b) The phase voltage at the load terminal is

$$\begin{aligned} V_2 &= 120\angle 0^\circ - (2 + j4)(5\angle 0^\circ) = 110 - j20 \\ &= 111.8\angle{-10.3^\circ} \text{ V} \end{aligned}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\,\angle 30^\circ\; V_2 = \sqrt{3}\;(111.8)\angle 19.7^\circ = 193.64\angle 19.7^\circ \text{ V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the Δ load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle - 63.4^\circ \text{ A}$$
$$I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56^\circ \text{ A}$$

The phase current in the original Δ-connected load, i.e., I_{ab} is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle{-30^\circ}} = \frac{4.472\angle 26.56^\circ}{\sqrt{3}\angle{-30^\circ}} = 2.582\angle 56.56^\circ \text{ A}$$

(d) The three-phase power absorbed by each load is

$$S_1 = 3V_2 I_1^* = 3(111.8\angle - 10.3^\circ)(2.236\angle 63.4^\circ) = 450 \text{ W} + j600\,\text{var}$$
$$S_2 = 3V_2 I_2^* = 3(111.8\angle - 10.3^\circ)(4.472\angle{-26.56^\circ}) = 1200 \text{ W} - j900\,\text{var}$$

The three-phase power absorbed by the line is

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150 \text{ W} + j300 \text{ var}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$S_1 + S_2 + S_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800 \text{ W} + j0 \text{ var}$$

Example 2.8

A three-phase line has an impedance of $0.4 + j2.7\ \Omega$ per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1 kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 V. Determine:

(a) The magnitude of the line voltage at the source end of the line.
(b) Total real and reactive power loss in the line.
(c) Real power and reactive power supplied at the sending end of the line.

(a) The phase voltage at the load terminals is

$$V_2 = \frac{3810.5}{\sqrt{3}} = 2200 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.11.

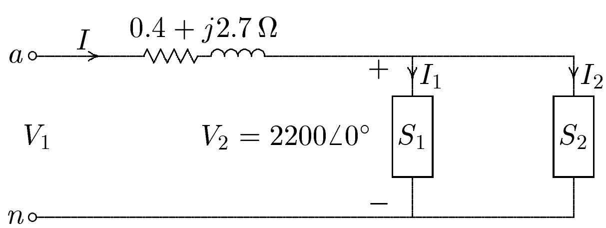

FIGURE 2.11
Single-phase equivalent diagram for Example 2.8.

The total complex power is

$$S_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396$$
$$= 660\angle 36.87^\circ \text{ kVA}$$

With the phase voltage V_2 as reference, the current in the line is

$$I = \frac{S^*_{R(3\phi)}}{3V_2^*} = \frac{660,000\angle -36.87^\circ}{3(2200\angle 0^\circ)} = 100\angle -36.87^\circ \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200\angle 0^\circ + (0.4 + j2.7)100\angle -36.87^\circ = 2401.7\angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is

$$\begin{aligned} S_{L(3\phi)} = 3R|I|^2 + j3X|I|^2 &= 3(0.4)(100)^2 + j3(2.7)(100)^2 \\ &= 12 \text{ kW} + j81 \text{ kvar} \end{aligned}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1 I^* = 3(2401.7\angle 4.58^\circ)(100\angle 36.87^\circ) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$