Lecture 27: Mixers. Gilbert Cell

Mixers shift the frequency spectrum of an input signal. This is an essential component in electrical communications (wireless or otherwise) if we wish to use RF signals to convey audio or data signals over long distances, for example.

The circuit symbol for a mixer has three ports (Fig. 12.1):

![Circuit symbol for a mixer](image)

Notice that all three ports have signals at different frequencies!

“Mixing” has a couple of connotations. One is that of combining (by summing) signals from different channels (or sources), as in the recording industry. Fig. 15.1 from the *ARRL Handbook* on the next page illustrates this principle.

Clearly, this is not the type of “mixing” that’s needed in communications. We need to shift the frequency. This type of mixing is the result of multiplying signals in the time domain, as shown in Fig. 15.2 of the *ARRL Handbook*. 
There are two general types of mixing circuits, those involving:

1. **Nonlinear** components, such as diodes, etc.
2. **Linear**, but **time-varying** circuits. These circuits can shift the frequency spectrum of a signal, in contrast to linear and time **invariant** circuits (which cannot).

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**Gilbert Cell**

All mixers in the NorCal 40A are based on the **Gilbert cell**. The Gilbert cell uses a **linear, time-varying circuit** to achieve time-domain multiplication, and hence, frequency shifting.
A Gilbert cell is shown in Fig. 12.2. The RF signal is input to a long-tailed differential amplifier, which we studied in Sec. 9.8. The collectors of Q1 and Q2 have a cross connected set of four transistors, which are driven by a local oscillator (LO).

To see how the Gilbert cell operates, first consider what happens when the voltage $V_{\text{LO1}}$ is large enough so that Q3 and Q5 turn on and $V_{\text{LO2}}$ is small enough, that Q4 and Q6 turn off:

Here we see that Q3 and Q5 act as closed switches so that:
- Q1 is connected to R1, and
- Q2 is connected to R2

as in a “typical” differential amplifier configuration with the output taken at, what we will call here, the “IF” terminals.
Second, consider what happens when the opposite input occurs. Specifically, suppose $V_{LO2}$ is large enough so that Q4 and Q6 are on while $V_{LO1}$ is small enough that Q3 and Q5 are off:

![Diagram](image)

Now we see that Q4 and Q6 act as closed switches so that
- Q1 is connected to R2, and
- Q2 is connected to R1.

This also is a differential amplifier configuration, but with the outputs interchanged wrt to the previous case. In other words, the output (the IF) is almost the same as before. It has just been multiplied by the factor −1.

The overall function of the Gilbert cell is to multiply in the *time domain* the input RF signal (at the RF frequency) by a square wave with value +1 or −1 at the LO frequency! This is mixing.
The Gilbert cell is also an active mixer in that the IF output signal is amplified because of the differential amplifier gain. (Active mixers are very nice in the sense that they accomplish two jobs at once: they mix and they amplify.)

A Gilbert cell is the active mixer inside the SA602AN IC used in the NorCal 40A (see the datasheet on p. 415). As we’ve already seen, this IC also has part of an oscillator circuit built inside. What a versatile IC!

From Fig. 4 of the SA602AN datasheet we can see that certain subsystems are internally biased. Consequently, we don’t need to construct an external bias circuit. However, we must capacitively couple to the SA602AN so we don’t disturb this biasing (examples of this are C4, C5, C13, C15, C31 and C33).

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**Mixer Mathematics**

It is helpful to study the mathematic basis behind the Gilbert cell. This will help us understand this circuit better, as well as develop an appreciation of the mixing process in general.

Let’s define the RF input voltage as

\[ V_{rf}(t) = V_{rf} \cos(\omega_{rf}t) \]  \hspace{1cm} (12.2)
and define the mixing signal (which is not the LO signal, as is stated in the text) by the square wave:

\[ V_{\text{mix}}(t) = \begin{cases} +1 & \text{for } t \in [0, T_{\text{LO}}) \\ -1 & \text{for } t \in [T_{\text{LO}}, 2T_{\text{LO}}) \end{cases} \]

Since this mixing signal is a periodic waveform, we can expand it in this Fourier series

\[ V_{\text{mix}}(t) = \frac{4}{\pi} \left[ \cos(\omega_0 t) - \frac{1}{3} \cos(3\omega_0 t) + \frac{1}{5} \cos(5\omega_0 t) - \cdots \right] \quad (12.3) \]

as derived in Sec. 2 of Appendix B.

The Gilbert cell effectively multiplies both of these signals (12.2) and (12.3) in the time domain as

\[ V(t) = V_{\text{rf}}(t) \cdot V_{\text{mix}}(t) \quad (12.4) \]

giving

\[ V(t) = \frac{2V_{\text{rf}}}{\pi} \left[ \cos(\omega_- t) - \frac{1}{3} \cos(\omega_3 t) + \frac{1}{5} \cos(\omega_5 t) - \cdots \right] + \frac{2V_{\text{rf}}}{\pi} \left[ \cos(\omega_+ t) - \frac{1}{3} \cos(\omega_3 t) + \frac{1}{5} \cos(\omega_5 t) - \cdots \right] \quad (12.5) \]

where

\[ \omega_- = \omega_0 - \omega_{\text{rf}} \quad \omega_+ = \omega_0 + \omega_{\text{rf}} \]
\[ \omega_{3+} = 3\omega_{lo} + \omega_{rf} \quad \omega_{3-} = 3\omega_{lo} - \omega_{rf} \]
\[ \omega_{5+} = 5\omega_{lo} + \omega_{rf} \quad \omega_{5-} = 5\omega_{lo} - \omega_{rf} \]
\[ \vdots \quad \vdots \]

Note in (12.5) that we have the sum and difference signals present in the output (IF) voltage signal:

\[ V_+(t) = \frac{2V_{rf}}{\pi} \cos(\omega_+ t) \quad (12.6) \]
and
\[ V_-(t) = \frac{2V_{rf}}{\pi} \cos(\omega_- t) \quad (12.7) \]

as well as the third-harmonic terms:

\[ V_{3+}(t) = -\frac{2V_{rf}}{3\pi} \cos(\omega_{3+} t) \quad (12.10) \]
and
\[ V_{3-}(t) = -\frac{2V_{rf}}{3\pi} \cos(\omega_{3-} t) \quad (12.11) \]

and the fifth-harmonic terms:

\[ V_{5+}(t) = \frac{2V_{rf}}{5\pi} \cos(\omega_{5+} t) \]
and
\[ V_{5-}(t) = \frac{2V_{rf}}{5\pi} \cos(\omega_{5-} t) \]

and all higher-ordered odd harmonics.

Observe that the amplitudes of these harmonics are decreasing with increasing harmonic number.
Also note that the RF signal, the LO signal and the even mixer harmonics are not present in the output. Nice! This occurs because the Gilbert cell is a balanced mixer. However, in reality some (or all) of these signal components will be present in the output since we won’t have a perfectly balanced mixer.

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**NorCal 40A Mixers**

There are three mixers in the NorCal 40A. You’ll install the:

1. RF Mixer in Prob. 28,
2. Product Detector in Prob. 29, and
3. Transmit Mixer in Prob. 30.

In addition, using the measured spectrum from the output of the Transmit Mixer shown in Fig. 12.15, you will identify the various harmonics using

\[ f_{mn} = mf_{vfo} \pm nf_{to} \]  \hspace{1cm} (12.46)

There is a misprint of this equation in the text.

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**Conversion Gain**

Gain (or loss) of a mixer is characterized with a power gain expression similar to any amplifier
\[ G = \frac{P}{P_+} \]  \hspace{1cm} (12.1)

where \( G \) is called the conversion gain, \( P \) is the output IF power and \( P_+ \) is the available power from the RF source.

Here, however, the input and output frequencies of the two signals are different.