 By Howard Johnson, PhD

Modeling skin effect

Why does high-frequency current flow only on the outer surface of a printed-circuit trace?—Dipak Patel

Magnetic fields cause the behavior you describe. The technical name for this property is the *skin effect*. It happens in all conductors. If you really like mathematics, the following section will help you to better understand *why* the skin effect happens. If not, this might be a good time to step out for a cup of tea.

I'll start the discussion with a perfect coaxial cable. **Figure 1** divides the center conductor of this cable into a series of three concentric rings with radii r_0, r_1, and r_2. A lumped-element model of this simple circuit demonstrates that high-frequency signal current flows only on ring 2.

At dc, the longitudinal-voltage drop per inch across each conductor n equals the current, I_n, times its resistance per inch. You can express this relation in matrix terms by defining a square matrix, R, with elements

$$R_{n,n}=\frac{\rho}{\pi\left(r_n^2-r_{n-1}^{\;2}\right)}$$

along the diagonal and 0 elsewhere. Then, write $V=RI$, where V is a vector representing the longitudinal voltage per inch across the ends of each conductor, I is a vector representing the currents, and ρ is the resistivity of the center conductor in units of ohm inches.

At high frequencies, the magnetic interactions between conductors become significant. **Figure 1** illustrates the pattern of magnetic fields between the center conductor and shield. The magnetic lines of force (B-field) form concentric circles around the conductive rings. The drawing plots the field intensity, $|B|$, versus radial position, r, assuming a positive signal current of 1A flowing in ring 0 with the return current flowing in the shield. The field strength is zero within the interior of ring 0 and zero outside the shield and varies with $1/r$ (Ampere's law) in between. The exact field intensity for a current of 1A on conductor m is

$$B_m=\left(\frac{\mu}{2\pi}\right)\left(\frac{1}{r}\right)$$

for $r_m<r<d/2$, where μ is the magnetic permeability of the dielectric material (usually 3.192×10^{-8} webers/amp-in.).

You calculate the mutual inductance per inch between conductors n and m (for $n\geq m$) using Faraday's law as the integral of the magnetic-field strength, B_m, taken over the range from conductor n (at radial position r_n) and the shield (at radial position $d/2$). Integrating $1/r$ yields $\ln(r)$ and the following matrix equation for mutual inductance:

$$L_{n,m}=\left(\frac{\mu}{2\pi}\right)\ln\left(\frac{d}{2r_n}\right).$$

To find values for $n<m$, use symmetry: $L_{n,m}=L_{m,n}$.

The Laplace system equation for the whole coaxial circuit sums both resistive and inductive terms as $V=(R+sL)I$. The following is the inductance matrix for an RG-58/U coaxial cable:

$$L=\begin{bmatrix}12.1 & 8.6 & 6.5\\ 8.6 & 8.6 & 6.5\\ 6.5 & 6.5 & 6.5\end{bmatrix}\text{nH/in.}$$

Now comes the main point of this article: The terms in the right-hand column of L are all the same. Why? Because ring 2 concentrates *all its flux* into the space between ring 2 and the shield. Therefore, all other rings couple 100% to this flux.

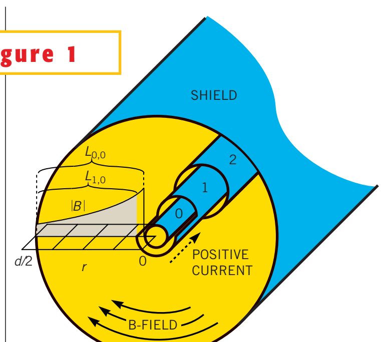
The constancy of the right-hand column greatly simplifies the solution to the system equation. To solve this equation, you must find a pattern of currents I such that $(R+sL)I$ generates the same longitudinal voltage across every ring. You need the same voltage across every ring because the rings are all connected together at their ends. If you operate at a frequency so high that the R term becomes insignificant compared with sL, the solution is simple. Just fill in the last element of I, leaving all others zero. This solution peels off only the right-hand column of L, properly generating the same voltage for every ring. This is one of the few matrix problems you can solve by inspection.

The simple solution says that at high frequencies, the signal current flows only on the outer ring, as governed by matrix L. At dc, the current distributes itself more evenly, according to matrix R. At middle frequencies, you get a mixture of both effects. That's the nature of the skin effect.

Continuous conductors behave in a similar manner, as if they were made from a continuum of infinitely thin rings. At higher and higher frequencies, the current squeezes more and more tightly against the surface of the conductor, progressively decreasing the useful current-carrying cross section of the conductor and raising its effective resistance.

You can view all of the mathematical details in a completely worked 40-ring example at http://signalintegrity.com/articles/misc/skineffect.htm.□

Figure 1



The total magnetic flux within the shaded region equals $L_{1,0}$.

Howard Johnson, PhD, the author of High-Speed Digital Design: A Handbook of Black Magic *(Prentice-Hall, 1993), conducts technical workshops for digital engineers worldwide. Comments invited! www.sigcon.com, howiej@sigcon.com.*

