Induction Motor
Induction Motors

• The single-phase induction motor is the most frequently used motor in the world

• Most appliances, such as washing machines and refrigerators, use a single-phase induction machine

• Highly reliable and economical

Figure 1 Single-phase induction motor.
Induction Motors

- For industrial applications, the three-phase induction motor is used to drive machines
- Figure 2 Large three-phase induction motor. (Courtesy Siemens).
Figure 3
Induction motor components.
Induction Motors

- The motor housing consists of three parts:
  - The cylindrical middle piece that holds the stator iron core,
  - The two bell-shaped end covers holding the ball bearings.
  - This motor housing is made of cast aluminum or cast iron. Long screws hold the three parts together.
  - The legs at the middle section permit the attachment of the motor to a base.
  - A cooling fan is attached to the shaft at the left-hand side. This fan blows air over the ribbed stator frame.
Induction Motors

Figure 4  Stator of a large induction motor.  (Courtesy Siemens).
Induction Motors

- The iron core has cylindrical shape and is laminated with slots.
- The iron core on the figure has paper liner insulation placed in some of the slots.
- In a three-phase motor, the three phase windings are placed in the slots.
- A single-phase motor has two windings: the main and the starting windings.
- Typically, thin enamel insulated wires are used.

Figure 5  Stator iron core without windings
Induction Motors

- A single-phase motor has two windings: the main and the starting windings.
- The elements of the laminated iron core are punched from a silicon iron sheet.
- The sheet has 36 slots and 4 holes for the assembly of the iron core.

Figure 6 Single-phase stator with main windings.
Induction Motors

• The elements of the laminated iron core are punched from a silicon iron sheet.

• The sheet has 36 slots and 4 holes for the assembly of the iron core

Figure 7 Stator iron core sheet.
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Figure 8 Stator and rotor magnetic circuit
Induction Motors

Squirrel cage rotor.

- This rotor has a laminated iron core with slots, and is mounted on a shaft.
- Aluminum bars are molded in the slots and the bars are short circuited with two end rings.
- The bars are slanted on a small rotor to reduce audible noise.
- Fins are placed on the ring that shorts the bars. These fins work as a fan and improve cooling.
Induction Motors

Rotor bars (slightly skewed)

End ring

Figure 9 Squirrel cage rotor concept.
Induction Motors

Figure 10 Squirrel cage rotor.
Induction Motors

Wound rotor.

- Most motors use the squirrel-cage rotor because of the robust and maintenance-free construction.
- However, large, older motors use a wound rotor with three phase windings placed in the rotor slots.
- The windings are connected in a three-wire wye.
- The ends of the windings are connected to three slip rings.
- Resistors or power supplies are connected to the slip rings through brushes for reduction of starting current and speed control.
Figure 11 Rotor of a large induction motor. (Courtesy Siemens).
Operating principle
Induction Motors

- This two-pole motor has three stator phase windings, connected in three-wire wye.
- Each phase has $2 \times 3 = 6$ slots. The phases are shifted by $120^\circ$.
- The squirrel cage rotor has short-circuited bars.
- The motor is supplied by balanced three-phase voltage at the terminals.
- The stator three-phase windings can also be connected in a delta configuration.

Figure 11 Connection diagram of a two-pole induction motor with squirrel cage rotor.
Induction Motors

Operation Principle

• The three-phase stator is supplied by balanced three-phase voltage that drives an ac magnetizing current through each phase winding.

• The magnetizing current in each phase generates a pulsating ac flux.

• The flux amplitude varies sinusoidally and the direction of the flux is perpendicular to the phase winding.
Induction Motors

Operation Principle

• The three-phase stator is supplied by balanced three-phase voltage that drives an ac magnetizing current through each phase winding.
• The magnetizing current in each phase generates a pulsating ac flux.
• The total flux in the machine is the sum of the three fluxes.
• The summation of the three ac fluxes results in a rotating flux, which turns with constant speed and has constant amplitude.
Induction Motors

Operation Principle

• The rotating flux induces a voltage in the short-circuited bars of the rotor. This voltage drives current through the bars.

• The induced voltage is proportional with the difference of motor and synchronous speed. Consequently the motor speed is less than the synchronous speed

• The interaction of the rotating flux and the rotor current generates a force that drives the motor.

• The force is proportional with the flux density and the rotor bar current
Induction Motors

• The figure shows the three components of the magnetic field at a phase angle of –60°.

• Each phase generates a magnetic field vector.

• The vector sum of the component vectors $\Phi_a$, $\Phi_b$, $\Phi_c$ gives the resulting rotating field vector $\Phi_{rot}$.

• The amplitude is 1.5 times the individual phase vector amplitudes, and $\Phi_{rot}$ rotates with constant speed.

Figure 12 Three-phase winding-generated rotating magnetic field.
Induced Voltage Generation
Induction Motors

Faraday’s law

- Voltage is induced in a conductor that moves perpendicular to a magnetic field,
- The induced voltage is:

![Diagram showing voltage induced in a conductor moving through a magnetic field.](image)
Induction Motors

• The three-phase winding on the stator generates a rotating field.

• The rotor bar cuts the magnetic field lines as the field rotates.

• The rotating field induces a voltage in the short-circuited rotor bars.

• The induced voltage is proportional to the speed difference between the rotating field and the spinning rotor.

\[ V = B L (v_{syn} - v_m) \]
Induction Motors

- The speed of flux cutting is the difference between the magnetic field speed and the rotor speed.

- The two speeds can be calculated by using the radius at the rotor bar location and the rotational speed.

\[
\begin{align*}
\nu_{\text{syn}} &= 2\pi r_{\text{rot}} n_{\text{syn}} \\
\nu_{\text{mot}} &= 2\pi r_{\text{rot}} n_{m} \\
V_{\text{bar}} &= 2\pi r_{\text{rot}} B \ell_{\text{rot}} (n_{\text{syn}} - n_{m})
\end{align*}
\]
Induction Motors

• The voltage and current generation in the rotor bar require a speed difference between the rotating field and the rotor.
• Consequently, the rotor speed is always less than the magnetic field speed.
• The relative speed difference is the slip, which is calculated using

\[ S = \frac{n_{sy} - n_{m}}{n_{sy}} = \frac{\omega_{sy} - \omega_{m}}{\omega_{sy}} \]

The synchronous speed is

\[ n_{sy} = \frac{f}{p/2} \]
Motor Force Generation
Induction Motors

• The interaction between the magnetic field $B$ and the current generates a force

$$F = B L I$$

Figure 15 Force direction on a current-carrying conductor placed in a magnetic field ($B$) (current into the page).
Induction Motors

Force generation in a motor

• The three-phase winding generates a rotating field;

• The rotating field induces a current in the rotor bars;

• The current generation requires a speed difference between the rotor and the magnetic field;

• The interaction between the field and the current produces the driving force.

Figure 16 Rotating magnetic field generated driving force.
Equivalent circuit
Induction Motors

- An induction motor has two magnetically coupled circuits: the stator and the rotor. The latter is short-circuited.
- This is similar to a transformer, whose secondary is rotating and short-circuited.
- The motor has balanced three-phase circuits; consequently, the single-phase representation is sufficient.
- Both the stator and rotor have windings, which have resistance and leakage inductance.
- The stator and rotor winding are represented by a resistance and leakage reactance connected in series.
Induction Motors

- A transformer represents the magnetic coupling between the two circuits.

- The stator produces a rotating magnetic field that induces voltage in both windings.
  - A magnetizing reactance (Xm) and a resistance connected in parallel represent the magnetic field generation.
  - The resistance (Rc) represents the eddy current and hysteresis losses in the iron core

- The induced voltage is depend on the slip and the turn ratio
Induction Motors

Figure 17 Single-phase equivalent circuit of a three-phase induction motor.
Induction Motors

- In this circuit, the magnetizing reactance generates a flux that links with both the stator and the rotor and induces a voltage in both circuits.

- The magnetic flux rotates with constant amplitude and synchronous speed.

- This flux cuts the stationary conductors of the stator with the synchronous speed and induces a 60 Hz voltage in the stator windings.

- The rms value of the voltage induced in the stator is:

$$V_{sta} = \frac{N_{sta} \Phi_{max} \omega_{sy}}{\sqrt{2}}$$
Induction Motors

- The flux rotates with the synchronous speed and the rotor with the motor speed.
- Consequently, the flux cuts the rotor conductors with the speed difference between the rotating flux and the rotor.
- The speed difference is calculated using the slip equation:
  \[
  (\omega_{sy} - \omega_m) = \omega_{sy} \cdot s
  \]
- The induced voltage is:
  \[
  V_{rot} = \frac{N_{rot} \Phi_{\text{max}} (\omega_{sy} - \omega_m)}{\sqrt{2}} = \frac{N_{rot} \Phi_{\text{max}} \omega_{sy} s}{\sqrt{2}}
  \]
Induction Motors

• The division of the rotor and stator induced voltage results in:

\[ V_{rot} = \frac{N_{rot}}{N_{sta}} V_{sta} s = V_{rot-s} s \]

• This speed difference determines the frequency of the rotor current

\[ f_{rot} = \frac{\omega_{rot}}{2\pi} = \frac{\omega_{sy} - \omega_{m}}{2\pi} = \frac{\omega_{sy}}{2\pi} = s f_{sy} \]

• The rotor circuit leakage reactance is:

\[ X_{rot-m} = L_{rot} \omega_{rot} = L_{rot} \omega_{sy} s = X_{rot} s \]
Induction Motors

• The relation between rotor current and the rotor-induced voltage is calculated by the loop voltage equation:

\[ V_{rot} = V_{rot\_s} \cdot s = I_{rot} \left( R_{rot} + j \cdot X_{rot} \cdot s \right) \]

• The division of this equation with the slip yields

\[ V_{rot\_s} = I_{rot} \left( \frac{R_{rot}}{s} + j \cdot X_{rot} \right) \]

• The implementation of this equation simplifies the equivalent circuit
Figure 18  Modified equivalent circuit of a three-phase induction motor.

The rotor impedance is transferred to the stator side. This eliminates the transformer.
Figure 19 Simplified equivalent circuit of a three-phase induction motor.
Induction Motors

- The last modification of the equivalent circuit is the separation of the rotor resistance into two parts:

\[
\frac{R_{rot\_t}}{S} = R_{rot\_t} + \frac{[1 - S]}{S} R_{rot\_t}
\]

- The obtained resistance represents the outgoing mechanical power

\[
\frac{[1 - S]}{S} R_{rot\_t}
\]
Figure 20 Final single-phase equivalent circuit of a three-phase induction motor.
Motor performance
Induction Motors

- Figure 21 shows the energy balance in a motor.

- The supply power is:

\[ P_{\text{sup}} = \text{Re}(S_{\text{sup}}) = \text{Re}\left(3 V_{\text{sup}} I_{\text{sta}}^* \right) \]

- The power transferred through the air gap by the magnetic coupling is the input power \( P_{\text{sup}} \) minus the stator copper loss and the magnetizing (stator iron) loss.

- The electrically developed power \( P_{\text{dv}} \) is the difference between the air gap power \( P_{\text{ag}} \) and rotor copper loss.
Induction Motors

- The electrically developed power can be computed from the power dissipated in the second term of rotor resistance:

\[ P_{dv} = 3 |I_{rot_t}|^2 \left( R_{rot_t} \frac{1 - S}{S} \right) \]

- The subtraction of the mechanical ventilation and friction losses (\( P_{mloss} \)) from the developed power gives the mechanical output power

\[ P_{out} = P_{dv} - P_{mloss} \]
Induction Motors

- The motor efficiency:

\[ \eta = \frac{P_{\text{out}}}{P_{\text{sup}}} \]

- Motor torque:

\[ M = \frac{P_{\text{out}}}{\omega_m} \]
Induction Motors

Input power $P_{\text{sup}}$

Air gap power $P_{\text{ag}}$

Developed power $P_{dv} = 3 I_{\text{rot}}^2 R_{\text{rot}} (1-s)/s$

Output power $P_{\text{out}}$

Stator Copper loss $3 I_{\text{sta}}^2 R_{\text{sta}}$

Stator Iron loss $3 V_{\text{sta}}^2 / R_c$

Rotor Copper loss $3 I_{\text{rot}}^2 R_{\text{rot}}$

Ventilation and friction losses

Figure 21 Motor energy balance flow diagram.
7.3.4 Motor performance analysis
Induction Motors

1) Motor impedance

\[ Z_{\text{rot}_t}(s) := j \cdot X_{\text{rot}_t} + R_{\text{rot}_t} + R_{\text{rot}_t} \cdot \frac{(1 - s)}{s} \]

\[ Z_m := \frac{j \cdot X_m \cdot R_c}{j \cdot X_m + R_c} \]

\[ Z_{\text{sta}} := j \cdot X_{\text{sta}} + R_{\text{sta}} \]

\[ Z_{\text{mot}}(s) := Z_{\text{sta}} + \frac{Z_m \cdot Z_{\text{rot}_t}(s)}{Z_m + Z_{\text{rot}_t}(s)} \]

Figure 7.22 Simplified motor equivalent circuit.
Induction Motors

2) Motor Current

\[ V_{\text{sup}} := \frac{V_{\text{mot}}}{\sqrt{3}} \]

\[ I_{\text{sta}}(s) := \frac{V_{\text{sup}}}{Z_{\text{mot}}(s)} \]

\[ I_{\text{rot\_t}}(s) := I_{\text{sta}}(s) \cdot \frac{Z_{m}}{Z_{m} + Z_{\text{rot\_t}}(s)} \]

Figure 7.22 Simplified motor equivalent circuit.
Induction Motors

3) Motor Input Power

\[ S_{\text{sup}}(s) := 3 \cdot V_{\text{sup}} \cdot I_{\text{sta}}(s) \quad P_{\text{sup}}(s) := \text{Re}(S_{\text{sup}}(s)) \]

\[ P_{f_{\text{sup}}}(s) := \frac{P_{\text{sup}}(s)}{|S_{\text{sup}}(s)|} \quad Q_{\text{sup}}(s) := \text{Im}(S_{\text{sup}}(s)) \]

4) Motor Output Power and efficiency

\[ P_{\text{dev}}(s) := 3 \cdot \left( |I_{\text{rot}_t}(s)| \right)^2 \cdot R_{\text{rot}_t} \cdot \frac{(1 - s)}{s} \]

\[ P_{\text{mech}}(s) := P_{\text{dev}}(s) - P_{\text{mech\_loss}} \quad \eta(s) := \frac{P_{\text{mech}}(s)}{P_{\text{sup}}(s)} \]
Induction Motors

\[ s := 0.1 \cdot \% , 0.2 \cdot \% \ldots 100 \cdot \% \]

![Graph](image)

**Figure 24** Mechanical output power versus slip.
Induction Motors

5. Motor Speed

\[ \text{rpm} := \frac{1}{\text{min}} \]

\[ n_{\text{sy}} := \frac{f}{p} \]

\[ n_{m}(s) := n_{\text{sy}} \cdot (1 - s) \]

\[ \omega_{m}(s) := 2 \cdot \pi \cdot n_{m}(s) \]

6. Motor Torque

\[ T_{m}(s) := \frac{P_{\text{mech}}(s)}{\omega_{m}(s)} \]
Induction Motors

\( s := 0.5\% , 0.6\% \ldots 80\% \)

**Figure 25  Torque versus slip.**
Induction Motors

$s := 0.5\%, 0.6\% .. 80\%$

Figure 26 Torque versus speed.
Induction Motors
Induction Motors

Motor Starting torque

• When the motor starts at $s = 1$,
• The ventilation losses are zero and the friction loss is passive. The negative friction loss does not drive the motor backwards.
• The mechanical losses are zero when $s = 1$
• This implies that the starting torque is calculated from the developed power instead of the mechanical output power.
Induction Motors

Motor Starting torque

\[
M_{m\_start}(s) := \frac{3 \cdot (|I_{rot\_t}(s)|)^2 \cdot R_{rot\_t}}{2 \cdot \pi \cdot n_{sy} \cdot (1 - s)} \cdot \frac{(1 - s)}{s}
\]

\[
M_{m\_start}(s) := \frac{3 \cdot (|I_{rot\_t}(s)|)^2 \cdot R_{rot\_t}}{2 \cdot \pi \cdot n_{sy}} \cdot \frac{s}{s}
\]
Induction Motors

• Kloss formula

\[ M = \frac{2 M_{\text{MAX}}}{S_{\text{MAX}}} + \frac{S}{S_{\text{MAX}}} \]
Induction Motors

Circular diagram

Měřítko proudů: $m_A \ldots \text{A/cm}$
Měřítko výkonů: $m_p = \sqrt{3} U_n m_A \ldots \text{W/cm}$
Měřítko momentů: $m_M = \frac{m_p}{\omega_S} \ldots \text{Nm/cm}$

$\Delta P_{j1} = 3 \cdot I_{kn}^2 \cdot R_{175}$
Induction Motors

The resistances and reactance in the equivalent circuit for an induction motor can be determined by a series of measurements. The measurements are:

- **No-load test.** This test determines the magnetizing reactance and core loss resistance.
- **Blocked-rotor test.** This test gives the combined value of the stator and rotor resistance and reactance.
- **Stator resistance measurement.**
Induction Motors

No-load test

• The motor shaft is free
• The rated voltage supplies the motor.
• In the case of a three-phase motor:
  – the line-to-line voltages,
  – line currents
  – three-phase power using two wattmeters are measured
**Induction Motors**

**No-load test**

Fig. 29 Equivalent motor circuit in no-load test

Fig. 30 Simplified equivalent motor circuit in no-load test
Induction Motors

No-load test

\[ V_{\text{no-load,ln}} := \frac{V_{\text{no-load}}}{\sqrt{3}} \]

\[ P_{\text{no-load, A}} := \frac{P_{\text{no-load}}}{3} \]

\[ R_c := \frac{V_{\text{no-load, ln}}^2}{P_{\text{no-load, A}}} \]

\[ S_{\text{no-load, A}} := V_{\text{no-load, ln}} \cdot I_{\text{no-load}} \]

\[ Q_{\text{no-load, A}} := \sqrt{S_{\text{no-load, A}}^2 - P_{\text{no-load, A}}^2} \]

\[ X_m := \frac{V_{\text{no-load, ln}}^2}{Q_{\text{no-load, A}}} \]
Induction Motors

Blocked-rotor test

• The rotor is blocked to prevent rotation
• The supply voltage is reduced until the motor current is around the rated value.
• The motor is supplied by reduced voltage and reduced frequency. The supply frequency is typically 15 Hz.
• In the case of a three-phase motor:
  – the line-to-line voltages,
  – line currents
  – three-phase power using two wattmeters are measured
Induction Motors

Blocked-Rotor test

Figure 31 Equivalent motor circuit for blocked-rotor test

Figure 32 Simplified equivalent motor circuit for blocked-rotor test
Induction Motors

Blocked-Rotor test

\[ V_{\text{blocked\_ln}} := \frac{V_{\text{blocked}}}{\sqrt{3}} \]

\[ P_{\text{blocked\_A}} := \frac{P_{\text{blocked}}}{3} \]

\[ R_e := \frac{P_{\text{blocked\_A}}}{I_{\text{blocked}}^2} \]

The stator resistance was measured directly

\[ R_{\text{rot\_t}} := R_e - R_{\text{sta}} \]
**Induction Motors**

**Blocked-Rotor test**

The magnitude of the motor impedance

\[ Z_{\text{blocked}} := \frac{V_{\text{blocked}_\text{ln}}}{I_{\text{blocked}}} \]

The leakage reactance at 15 Hz

\[ X_{e_{15\text{Hz}}} := \sqrt{Z_{\text{blocked}}^2 - R_e^2} \]

The leakage reactance at 60 Hz

\[ X_e := X_{e_{15\text{Hz}}} \cdot \frac{60\text{Hz}}{15\text{Hz}} \]
Numerical Example
A three-phase 30hp, 208V, 4 pole, 60Hz, wye connected induction motor was tested, the obtained results are:

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Voltage (V)</th>
<th>Power (W)</th>
<th>Current (A)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No load test, 60 Hz</td>
<td>V_nL := 208</td>
<td>P_nL := 1600</td>
<td>I_nL := 22</td>
<td></td>
</tr>
<tr>
<td>Blocked Rotor test, 15Hz</td>
<td>V_br := 21</td>
<td>P_br := 2100</td>
<td>I_br := 71</td>
<td>f_br := 15Hz</td>
</tr>
<tr>
<td>DC test</td>
<td>V_dc := 12</td>
<td>I_dc := 75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Motor rating:
- P_mot_rated := 30hp
- V_mot_II := 208V
- p := 4
- f := 60Hz

Calculate:
  a) The equivalent circuit parameters
  b) Motor rated current and synchronous speed

Draw the equivalent circuit
Induction Motors

**No load test**, Determine the core losses $R_c$ and magnetizing reactance $X_m$

Single phase values:

$$R_c := \frac{(V_{nL_{1n}})^2}{P_{nL_{1F}}} \quad P_{nL_{1F}} := \frac{P_{nL}}{3} \quad V_{nL_{1n}} := \frac{V_{nL}}{\sqrt{3}}$$

$$R_c = 27.04\Omega$$

$$Y_{nL} := \frac{I_{nL}}{V_{nL_{1n}}} \quad Y_{nL} = 0.183\text{S}$$

$$X_m := i \frac{1}{\sqrt{Y_{nL}^2 - \left(\frac{1}{R_c}\right)^2}} \quad X_m = 5.573i\Omega$$
Induction Motors

**Block Rotor test**,  
Neglect the magnetizing branch. Consider only the \(X_{\text{sta}}+X_{\text{rot}}\) and \(R_{\text{sta}}+R_{\text{rot}}\)

\[
X_{\text{br}} = X_{\text{sta}} + X_{\text{rot}} \quad \text{and} \quad R_{\text{br}} = R_{\text{sta}} + R_{\text{rot}}
\]

Single phase values:

\[
P_{\text{br\_1F}} := \frac{P_{\text{br}}}{3} \quad \text{and} \quad V_{\text{br\_in}} := \frac{V_{\text{br}}}{\sqrt{3}}
\]

Resistance value is:

\[
R_{\text{br}} := \frac{P_{\text{br\_1F}}}{I_{\text{br}}^2} \quad \text{and} \quad R_{\text{br}} = 0.139\Omega
\]
Induction Motors

\[ Z_{br} := \frac{V_{br\_ln}}{I_{br}} \quad Z_{br} = 0.171 \Omega \]

\[ X_{br\_15Hz} := i\sqrt{Z_{br}^2 - R_{br}^2} \quad X_{br\_15Hz} = 0.099i \Omega \]

The reactance at 60 Hz is:

\[ X_{br\_60Hz} := X_{br\_15Hz} \cdot \frac{60}{15} \quad X_{br\_60Hz} = 0.398i \Omega \]

\[ X_{br} := i\sqrt{\left(\frac{Z_{br}^2 - R_{br}^2}{f_{br}}\right) \cdot \frac{60Hz}{f_{br}}} \quad X_{br} = 0.398i \Omega \]
Induction Motors

Determination of R1 and R2 and X1 and X2

\[ X_{sta} := \frac{X_{br}}{2} \quad X_{rot} := X_{sta} \quad X_{sta} = 0.199\Omega \]

Y connected motor

\[ R_{sta} := \frac{V_{dc}}{2I_{dc}} \quad R_{rot} := R_{br} - R_{sta} \]

\[ R_{sta} = 0.08\Omega \quad R_{rot} = 0.059\Omega \]
Induction Motors

a) The equivalent circuit parameters

\[ V_{\text{sup ln}} \]

\[ I_{\text{sta}} \]

\[ R_{\text{sta}} \]

\[ jX_{\text{sta}} \]

\[ I_c \]

\[ R_c \]

\[ jX_m \]

\[ I_m \]

\[ I_{\text{rot}} \]

\[ R_{\text{rot}} \]

\[ jX_{\text{rot}} \]

\[ R_{\text{rot}} (1-s) / s \]
Induction Motors

C) Motor rated current and synchronous speed

\[ S_{\text{rated}} := \frac{P_{\text{mot rated}}}{\text{pf}_{\text{mot}}} \]

\[ I_{\text{mot rated}} := \frac{S_{\text{rated}}}{\sqrt{3} \cdot V_{\text{mot ll}}} \]

\[ n_{\text{synch}} := \frac{f}{\frac{p}{2}} \]

\[ S_{\text{rated}} = 27.964 \text{kV}.A \]

\[ I_{\text{mot rated}} = 77.62 \text{A} \]

\[ n_{\text{synch}} = 30 \text{Hz} \]

\[ \text{rpm} := \frac{1}{\text{min}} \]

\[ n_{\text{synch}} = 1800 \text{rpm} \]