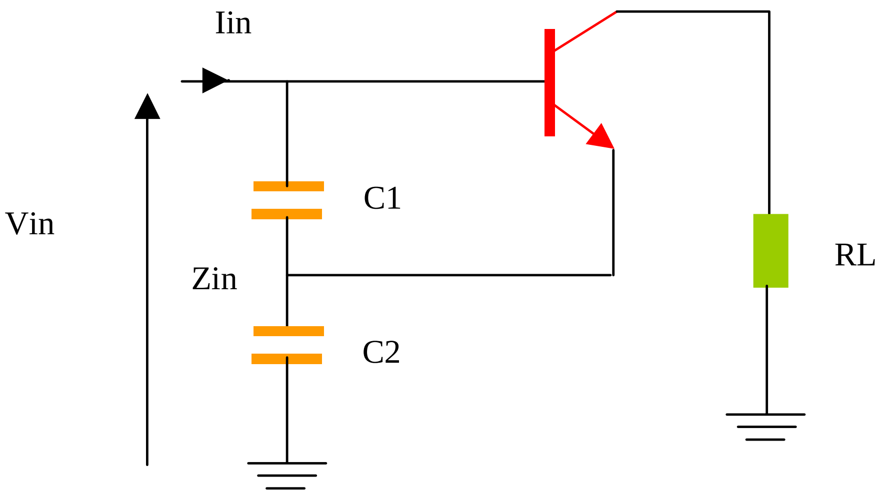
Derivation of Colpitts feedback capacitor values.



Simplified schematic diagram showing the reflection amplifier part of the Colpitts oscillator. The capacitor network of C1 and C2 would provide positive feedback from the emitter to base.

$$V_{in} = I_{in}(XC_1 + XC_2) - I_b(XC_1 - \beta XC_2) \quad - \quad (1)$$

$$0 = -I_{in}(XC_1) + I_b(XC_1 + hie) \quad - \quad (2)$$

rearrange to give Ib and sub into equation (1) ie $I_b = \dfrac{I_{in}.XC_1}{(XC_1 + hie)}$

$$V_{in} = I_{in}(XC_1 + XC_2) - \frac{I_{in}.XC_1}{(XC_1 + hie)}(XC_1 - \beta XC_2)$$ and multiply out

$$V_{in} = I_{in}.XC_1 + I_{in}.XC_2 - \frac{I_{in}.XC_1}{(XC_1 + hie)}.XC_1 + \frac{I_{in}.XC_1}{(XC_1 + hie)}.\beta XC_2$$ x both sides by $XC_1 + hie$

$$V_{in}(XC_1 + hie) = (XC_1 + hie)I_{in}.XC_1 + (XC_1 + hie)I_{in}.XC_2 -$$
$$(XC_1 + hie).\frac{I_{in}.XC_1}{(XC_1 + hie)}.XC_1 + (XC_1 + hie)\frac{I_{in}.XC_1}{(XC_1 + hie)}.\beta XC_2$$ and multiply out again.

$$V_{in}(XC_1 + hie) = I_{in}XC_1^2 + hie.I_{in}.XC_1 + XC_1.I_{in}.XC_2 + hie.I_{in}.XC_2 - I_{in}XC_1^2 + XC_1.I_{in}.XC_2.\beta$$

$$V_{in}(XC_1 + hie) = hie.I_{in}.XC_1 + XC_1.I_{in}.XC_2 + hie.I_{in}.XC_2 + XC_1.I_{in}.XC_2.\beta \quad \text{Rearrange for } I_{in}$$

$$V_{in}(XC_1 + hie) = I_{in}\left[hie.(XC_1 + XC_2) + XC_1.XC_2.I_{in}(1+\beta)\right]$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = \frac{hie.(XC_1 + XC_2) + XC_1.XC_2(1+\beta)}{XC_1 + hie}$$

If we assume that $XC_1 << hie$ then :-

$$\frac{V_{in}}{I_{in}} = Z_{in} = \frac{hie.(XC_1 + XC_2) + XC_1.XC_2(1+\beta)}{hie} \Rightarrow \frac{hie.(XC_1 + XC_2)}{hie} + \frac{XC_1.XC_2(1+\beta)}{hie}$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = XC_1 + XC_2 + \frac{XC_1.XC_2(1+\beta)}{hie} \quad \text{Let } gm = \frac{(1+\beta)}{hie} \quad \text{expand reactances :-}$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = \left(gm.\frac{1}{j\omega.C_1}.\frac{1}{j\omega.C_2}\right) + \frac{1}{j\omega.C_1} + \frac{1}{j\omega.C_2} \quad \text{as } j^2 = -1 \text{ then}$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = -gm.\frac{1}{\omega\ .C_1.C_2} + \frac{1}{j\omega.[C_1.C_2(C_1 + C_2)]}$$

Input impedance (negative) **Parallel combination of C1 & C2**