EE – 3410 Electric Power

Fall 2003

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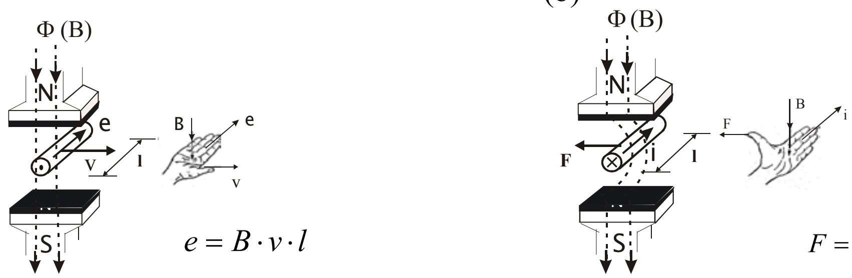
Generation of Electrical Energy

1.Fundamentals on generation of the voltage and mechanical force (torque)

1.1 Linear system

Fig.1.1 illustrates the generation of electromotive force (EMF) voltage ***e***, and mechanical force ***F***. Symbols: ***B*** – magnetic flux density, ***Φ*** – magnetic flux, ***i*** – current, ***v*** - speed

(a) (b)

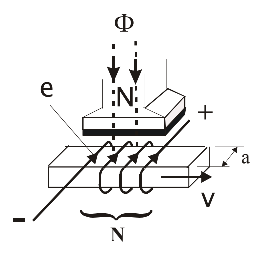


$$e = B \cdot v \cdot l \qquad F = B \cdot i \cdot l$$

Fig.1.1 Generation of: (a) EMF ***e*** (**right** hand rule), (b) force ***F*** (**left** hand rule)

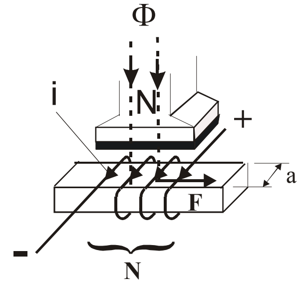
Fig.1.2 shows linear machines where in Fig.1.2.b is illustrated the generation of EMF ***e***, in Fig.1.2.b – generation of mechanical force ***F***, and in Fig.1.2.c – linear generator where the both ***e*** and ***F*** are produced if the winding circuit consisting of ***N*** turns is closed through the load impedance Z_l.

(a)



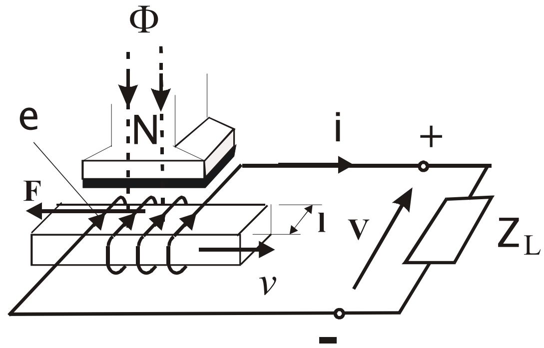
$$E = B \cdot v \cdot l \cdot N = e \cdot N$$

(b)



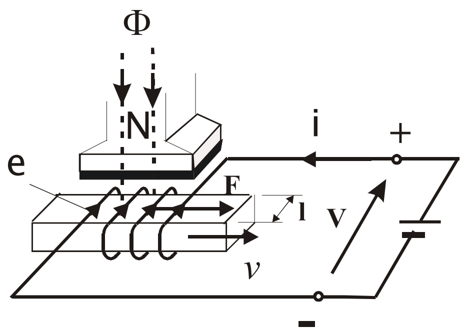
$$F = B \cdot i \cdot l \cdot N$$

(c)



$$E = e \cdot N$$
$$E > V$$
generator

(d)



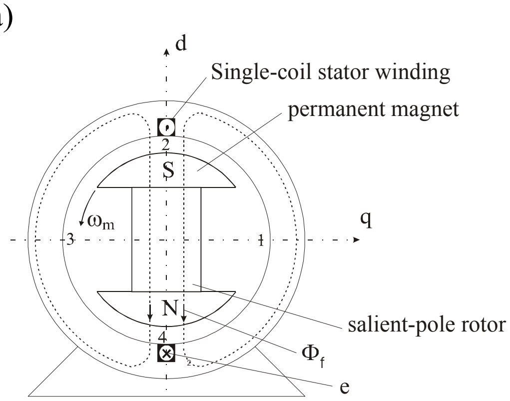
$$E < V$$
motor

Fig.1.2 Linear electric machines: (a) generation of voltage ***e***, (b) generation of force ***F***, (c) linear generator, (d) linear motor

1.2. Rotary system

Two-pole machine with a single coil (phase) is shown in Fig.1.3. The rotor with permanent magnets rotates at speed ω_m. It induces the electromotive force ***e (EMF)***

(a)



(b)

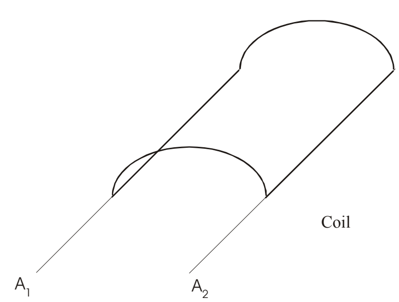


Fig.1.3 Cylindrical machine with a single-coil and permanent magnet rotor: a) machine, b) coil

changing sinusoidally with respect to the rotor position φ (Fig.1.4.a) and sinusoidally in time (Fig.1.4.b) according to the function:

$$e = E_{fm} \sin \omega t \qquad (1)$$

where: $\omega = 2\pi f$, and $\omega = 2p \cdot \omega_m$

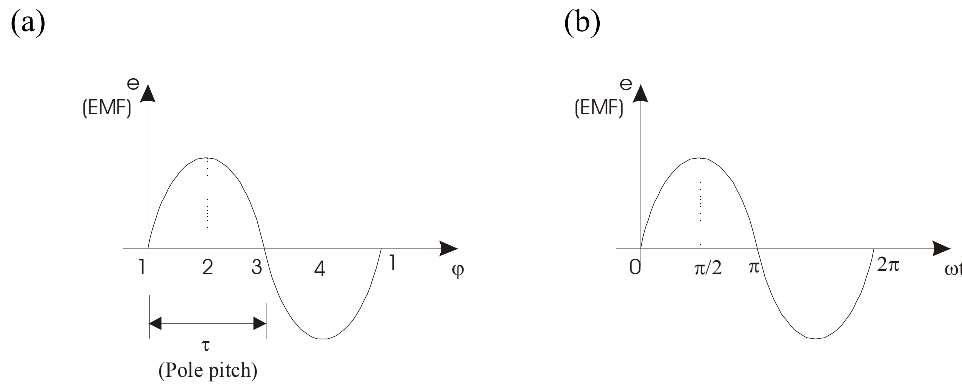


Fig.1.4 Voltage induced in the coil: (a) as a function of rotational angle, (b) as a function of time

If the coil is supplied by the current i the force (electromagnetic torque T_{em}) is produced. The direction of force acting on the coil is opposite to the force (electromagnetic torque T_{em}) acting on the rotor.

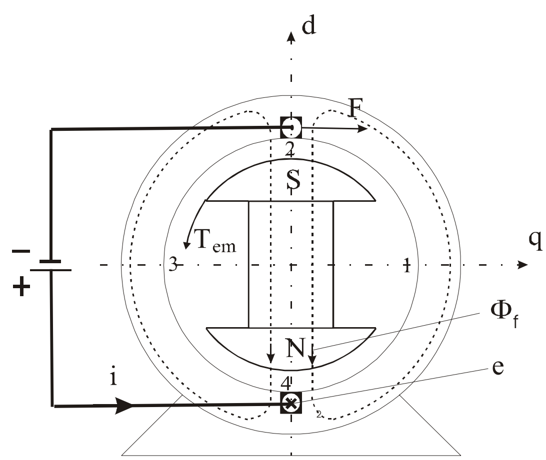


Fig.1.5 Force F (electromagnetic torque T_{em}) developed in the motor

2. Three-phase synchronous generator

2.1 Construction and principle of operation

The 3-phase synchronous machine is shown in Fig.2.1. It consists of two parts: stator and rotor. Both, stator and rotor have windings. The stator winding is a 3-phase winding and is sometimes called the armature winding. The rotor winding is called the field winding, which is connected to dc supply through the slip rings and brushes. There are two types of rotors:

- Salient-pole rotor (Fig.2.1.a) – for low-speed machines (e.g. hydro-generators)
- Cylindrical rotor (Fig.2.1.b) – for high-speed machines (e.g. turbo-generators).

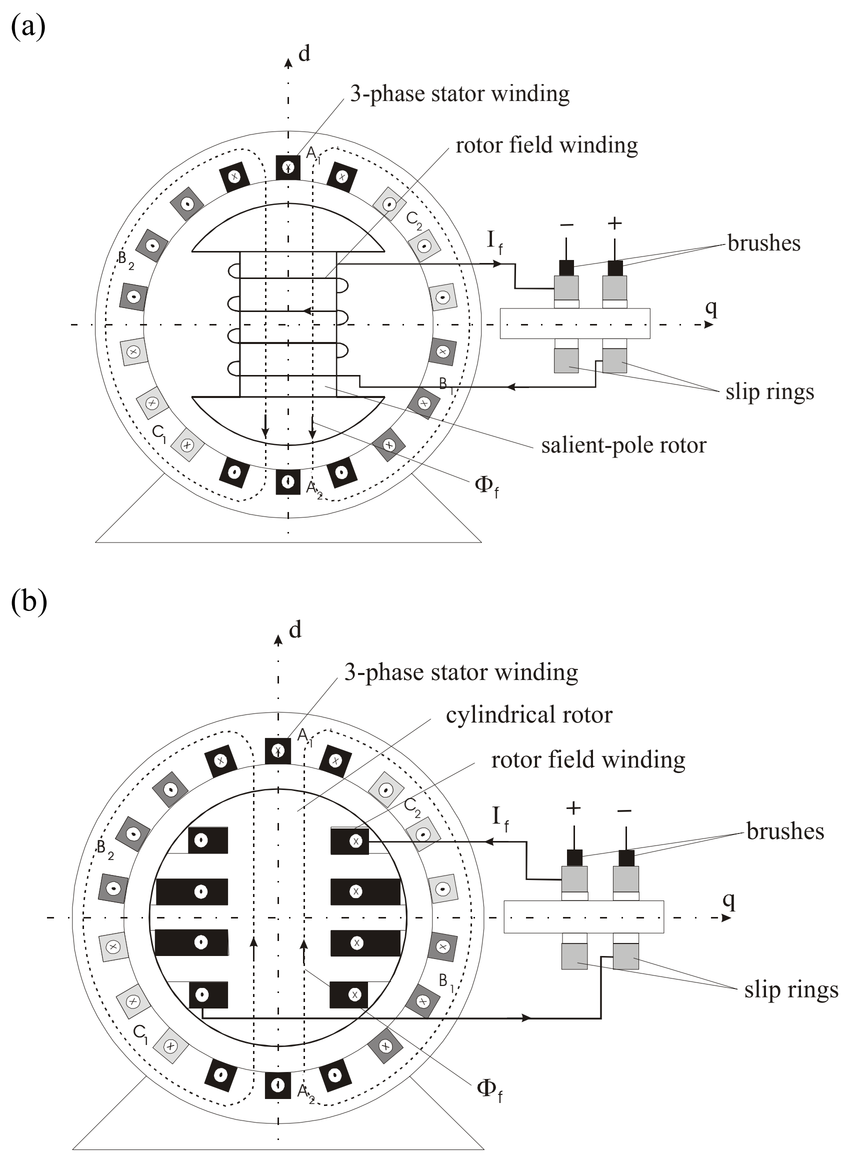


Fig.2.1 Construction scheme of synchronous machine with: (a) salient-pole rotor, (b) cylindrical rotor

The cylindrical rotor has one distributed winding and an essentially uniform air gap. The salient pole rotors have concentrated windings on the poles and a nonuniform air gap.

2.1.1 Synchronous generators

When the field current flows through the rotor field winding it establishes a sinusoidally distributed flux in the air gap. If the rotor rotates the rotating magnetic field induces voltages in the stator windings. Since the three-phase windings are shifted by $\boldsymbol{120^o}$ angle from one another the induced, so called, excitation voltages are shifted in time from one another by the angle of $\boldsymbol{120^o}$ (Fig.2.2):

$$\begin{aligned} e^A &= E_{fm}\sin(\omega t) \\ e^B &= E_{fm}\sin(\omega t - 120^o) \\ e^C &= E_{fm}\sin(\omega t - 240^o) \end{aligned} \tag{2.1}$$

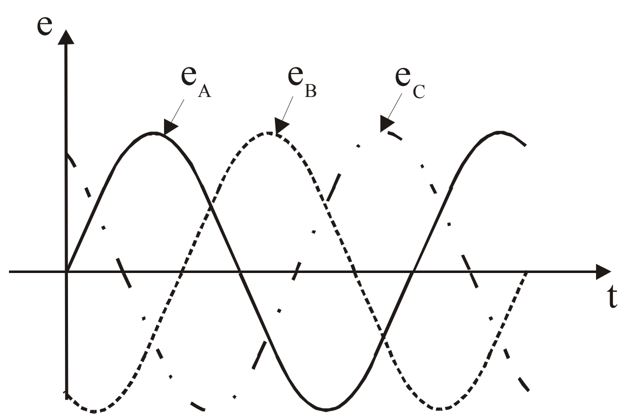


Fig.2.2 Waveforms of 3-phase voltages induced in the armature winding of the synchronous generator

The rms excitation voltage in each phase is

$$E_f = \frac{E_{fm}}{\sqrt{2}} = 4.44 f \Phi_f N K_w \tag{2.2}$$

where:

$\boldsymbol{\Phi_f}$ is the magnetic flux due to the excitation current,
$\boldsymbol{N}$ is the number of turns in each phase,
$\boldsymbol{K_w}$ is the winding factor.

The frequency of the induced voltage is related to the rotor speed by:

$$n = \frac{120f}{p} \tag{2.3}$$

$$f = \frac{n \cdot p}{120} \tag{2.4}$$

where: ***n*** is the rotor speed in rpm,
p is the number of poles

2.1.2 Synchronous motors

If the synchronous machine operates as a motor the 3-phase armature winding is connected to 3-phase ac supply. The stator currents produce the rotating magnetic flux. The field winding connected to dc source produces the magnetic flux, steady with respect to the rotor. To produce the torque these two magnetic fluxes cannot move with respect to one another. It means, that the rotor should rotate with the same speed as the rotating flux produced by the stator (Fig.2.3). When the machine operates as a generator the rotor is driven by the external machine and the stator rotating field follows the rotor being shifted with respect to the rotor by the angle δ (Fig.2.3a). When operating as the synchronous motor the rotor follows the stator rotating field by the angle ***(-*** δ***)*** (Fig.2.3.b). The motor at zero rotor speed does not develop any torque. To make the motor operate the rotor should reach first the synchronous speed. The methods of starting the synchronous motor will be discussed later.

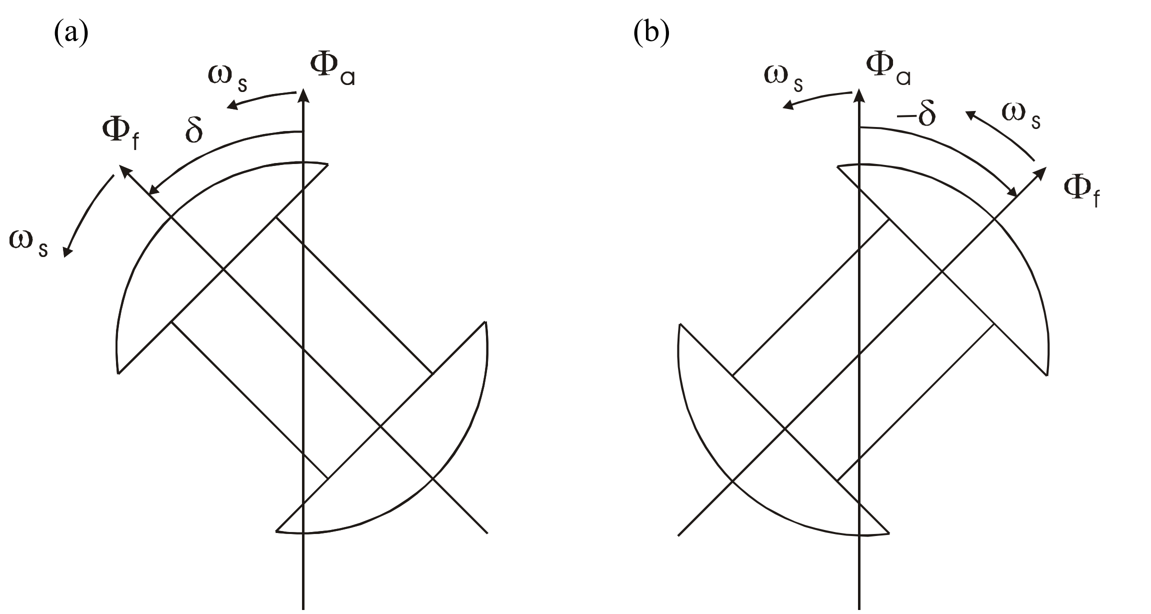


Fig.2.3 Explanation to the operation of: (a) synchronous generator, and (b) synchronous motor

In the following sections, first the steady-state performance of the synchronous machine **with cylindrical rotor and unsaturated magnetic circuit** will be studied. Then, the effect of saliency in the rotor poles will be considered.

2.2 Equivalent circuit model

The equivalent circuit will be derived on a per-phase basis. The current $\boldsymbol{I_f}$ in the field winding produces the $\boldsymbol{\Phi_f}$ flux. The current $\boldsymbol{I_a}$ in the stator produces flux $\boldsymbol{\Phi_a}$**.** Part of it $\boldsymbol{\Phi_{as}}$, known as the leakage flux does not links with the field winding. A major part $\boldsymbol{\Phi_{ar}}$, known as the armature reaction flux links with the field winding. The resultant air gap flux $\boldsymbol{\Phi}$ is therefore due to the two components fluxes $\boldsymbol{\Phi_f}$ and $\boldsymbol{\Phi_a}$. Each component flux induces a component voltage in the stator winding:

$\Phi_f \to E_f$,

$\Phi_{ar} \to E_{ar}$,

$\Phi_{as} \to E_{as}$,

and the resultant flux: $\Phi \to E$.

The excitation voltage $\boldsymbol{E_f}$ can be found from the open circuit curve of Fig.2.4. However, the voltage $\boldsymbol{E_{ar}}$ (known as the armature reaction voltage), and the voltage $\boldsymbol{E_{as}}$ depend on armature current. Therefore they can be presented as the voltage drops across the reactances:

$\boldsymbol{X_{ar}}$ – reactance of armature reaction and

$\boldsymbol{X_{as}}$ – leakage reactance.

This is shown in the equivalent circuit of a synchronous machine (only stator circuit is considered) in Fig.2.5.a.

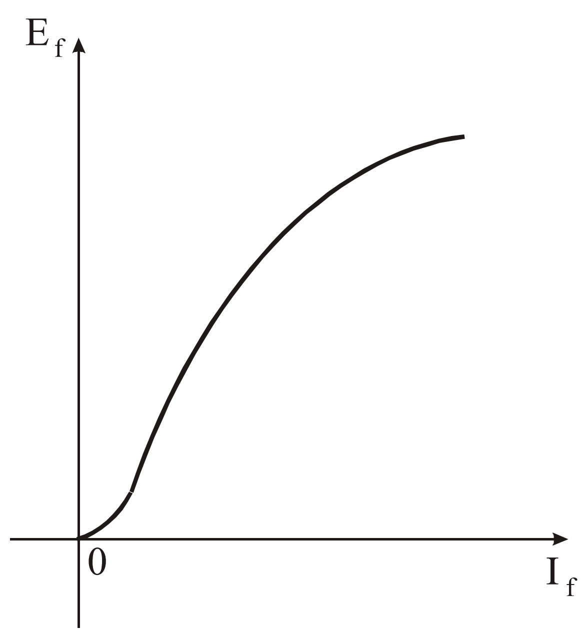


Fig.2.4 Open-circuit characteristic of the synchronous generator

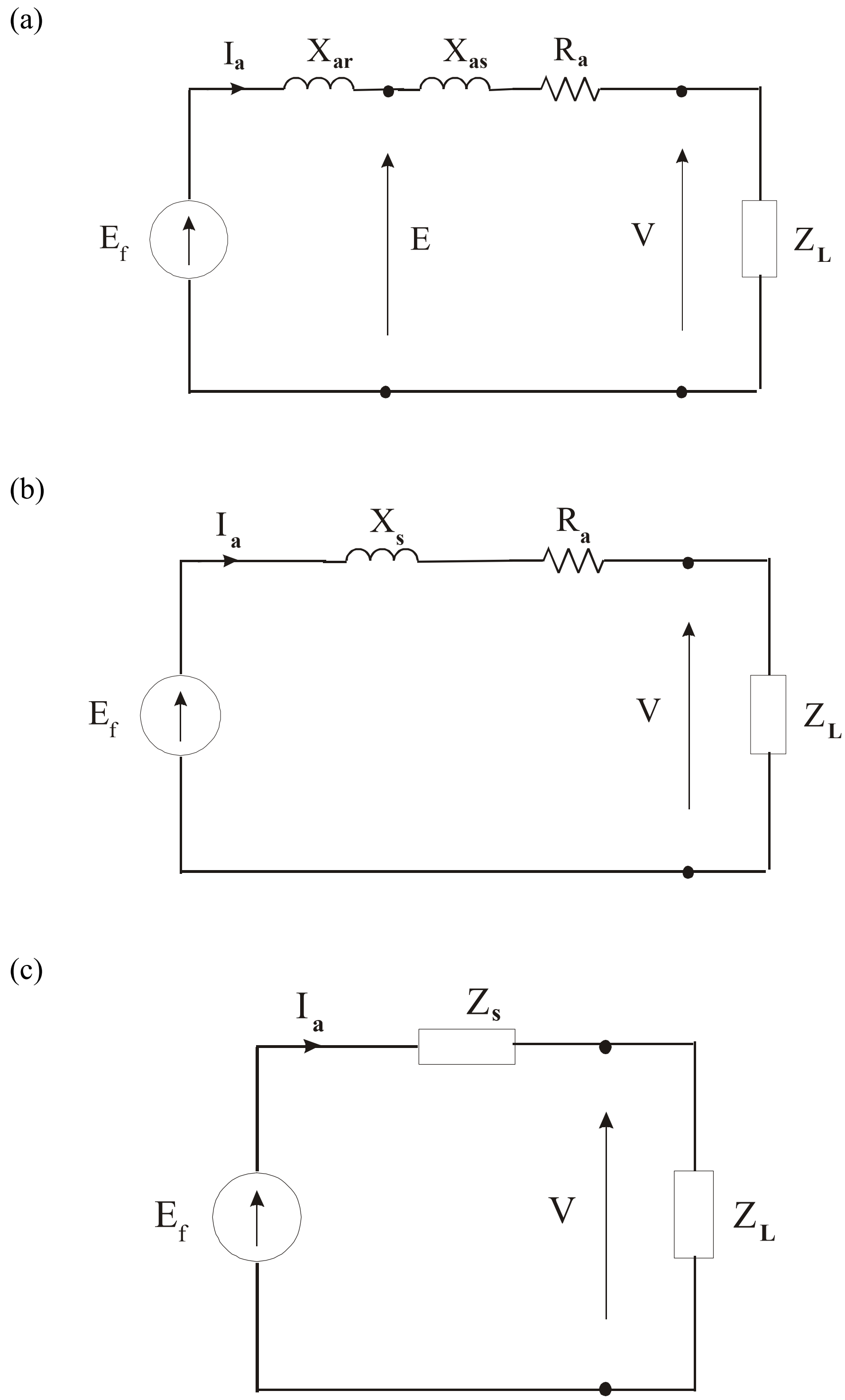


Fig.2.5 Synchronous machine equivalent circuit: (a) armature reaction reactance $\boldsymbol{X_{ar}}$, armature leakage reactance $\boldsymbol{X_{as}}$, (b) synchronous reactance $\boldsymbol{X_s}$, (c) armature impedance $\boldsymbol{Z_s}$

The relations between these voltages are as follows:

$$\underline{E} = \underline{E}_f - \underline{E}_{ar} \tag{2.5}$$

or

$$\underline{E} = \underline{E}_f - jX_{ar}\underline{I}_a \tag{2.6}$$

The voltage equation for the whole circuit is (Fig.2.5.a):

$$\underline{V} = \underline{E}_f - R_a\underline{I}_a - jX_{as}\underline{I}_a - jX_{ar}\underline{I}_a \tag{2.7}$$

or (Fig.2.5.b)

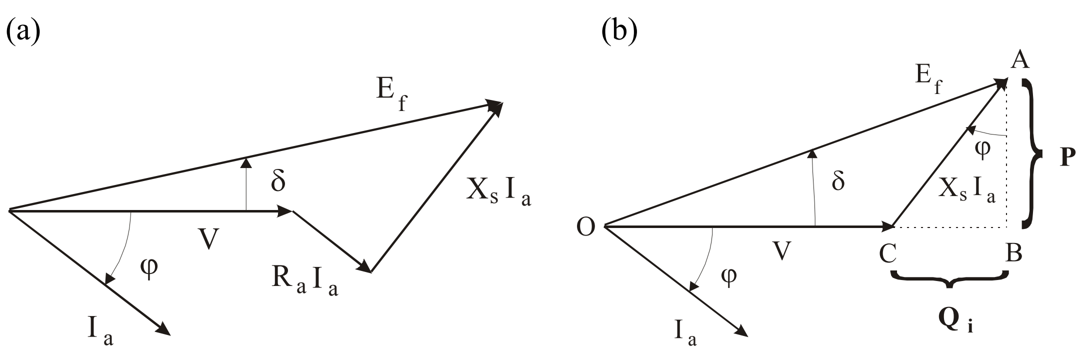
$$\underline{V} = \underline{E}_f - R_a\underline{I}_a - jX_s\underline{I}_a \tag{2.8}$$

or (Fig.2.5.c)

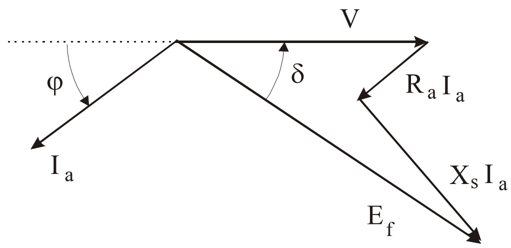
$$\underline{V} = \underline{E}_f - Z_s\underline{I}_a \tag{2.9}$$

where:
$X_s = X_{ar} + X_{as}$ - synchronous reactance
$Z_s = R_a + jX_s$ - synchronous impedance

The phasor diagram for generator and motor operation is shown in Fig.2.6. The terminal voltage is taken as the reference phasor. The angle $\boldsymbol{\delta}$ between $\boldsymbol{V}$ and $\boldsymbol{E_f}$, known as the power angle, is positive for generating action and negative for the motoring action. This angle referred to the mutual position of field winding flux and armature winding flux is shown in Fig.2.3. In generating operation the rotor flux $\boldsymbol{\Phi_f}$ (the rotor is driven by the external machine-turbine) is pulling the stator flux and the angle $\boldsymbol{\delta}$ is positive, while in motoring operation the stator flux $\boldsymbol{\Phi_a}$ is pulling the rotor.



(c)



(d)

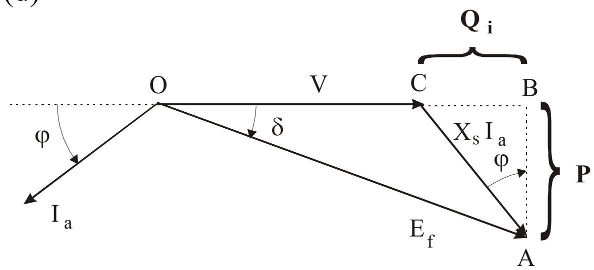


Fig.2.6 Phasor diagrams for: (a) synchronous generator with the armature resistance R_a and (b) without R_a; (c) synchronous motor with the armature resistance R_a and (d) without R_a

2.3 Power and torque characteristics

The complex power at the terminals is

$$S = mVI_a^* \tag{2.10}$$

The stator current from the equivalent circuit (Fig.2.5.c):

$$\begin{aligned} I_a^* &= \left(\frac{E_f - V}{Z_s}\right)^* = \frac{E_f^*}{Z_s^*} - \frac{V^*}{Z_s^*} \\ &= \frac{|E_f|\angle -\delta}{|Z_s|\angle -\varphi_s} - \frac{|V|\angle 0}{|Z_s|\angle -\varphi_s} \\ &= \frac{|E_f|}{|Z_s|}\angle(\varphi_s - \delta) - \frac{|V|}{|Z_s|}\angle\varphi_s \end{aligned} \tag{2.11}$$

$$S = m\left(\frac{|V||E_f|}{|Z_s|}\angle(\varphi_s - \delta) - \frac{|V|^2}{|Z_s|}\angle\varphi_s\right) \tag{2.12}$$

$$S = m\left(\frac{|V||E_f|}{|Z_s|}\cos(\varphi_s - \delta) - \frac{|V|^2}{|Z_s|}\cos\varphi_s\right) \tag{2.13}$$

$$S = m\left(\frac{|V||E_f|}{|Z_s|}\sin(\varphi_s - \delta) - \frac{|V|^2}{|Z_s|}\sin\varphi_s\right) \tag{2.14}$$

In large synchronous machines $R_a << X_s$, thus $Z_s = X_s$ and $\Theta_s = 90^o$. From the above equations:

$$P = m\frac{|V||E_f|}{|Z_s|}\sin(\delta) \qquad (2.15)$$

$$Q = m\left(\frac{|V||E_f|}{|Z_s|}\cos(\delta) - \frac{|V|^2}{|Z_s|}\right) \qquad (2.16)$$

The Eq.2.15 can be directly derived from the phasor diagram drawn for $R_a = 0$ (Fig.2.6.b and d).

In general, the active power

$$P = mVI_a\cos\varphi \qquad (2.17)$$

and reactive power

$$Q = mVI_a\sin\varphi \qquad (2.18)$$

From the phasor diagram the section:

- $\overline{AB} = X_s I_a \cos\varphi$, or (2.19)
- $\overline{AB} = E_f \sin\varphi$ (2.20)

From these two formulae:

$$X_s I_a \cos\varphi = E_f \sin\delta\text{, and} \qquad (2.21)$$

$$I_a\cos\varphi = \frac{E_f}{X_s}\sin\delta \qquad (2.22)$$

Combining Eq.2.22 and 2.17 we get:

$$P = m\frac{VE_f}{X_s}\sin\delta \qquad (2.23)$$

Because the stator losses are neglected in this analysis, the power developed at the terminals is also the air gap power. The torque developed by the machine is:

$$T = \frac{P}{\omega_s} \qquad (2.24)$$

Inserting Eq.2.23 to 2.24:

$$T = \frac{m}{\omega_s}\frac{VE_f}{X_s}\sin\delta \qquad (2.25)$$

The torque-power angle characteristic drawn at $\boldsymbol{V = const}$, and $\boldsymbol{I_f = const}$ is shown in Fig.2.7. The maximum torque, known also as the pull-out torque is at $\boldsymbol{\delta = 90^o}$. The machine will lose synchronism if $\boldsymbol{\delta > 90^o}$. The pull-out torque can be increased by increasing the excitation current $\boldsymbol{I_f}$.

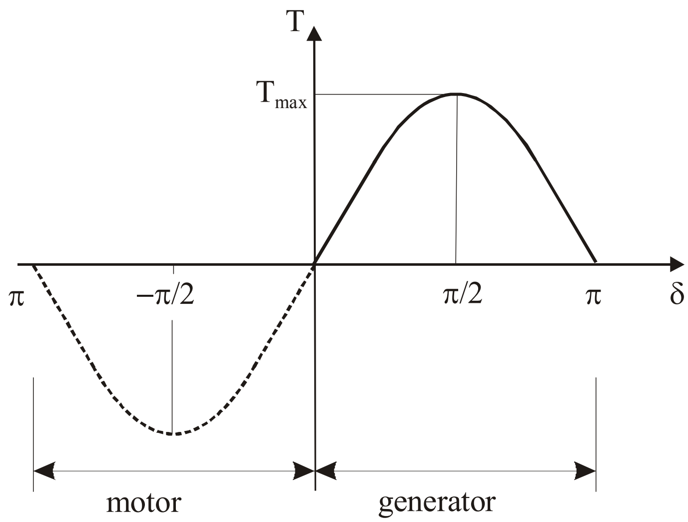


Fig.2.7 Torque-power angle characteristic

2.4 Power factor control

Looking at phasor diagram we see that:

$$\overline{AB} = X_s I_a \cos\varphi \approx P\text{, and} \qquad (2.26)$$

$$\overline{CB} = X_s I_a \sin\varphi \approx Q \qquad (2.27)$$

If the synchronous machine is connected to the system with $\boldsymbol{V = const}$. and $\boldsymbol{f = const}$, and the power on the rotor shaft $\boldsymbol{P = const}$, then changing the field current $\boldsymbol{I_f}$ the induced voltage $\boldsymbol{E_f}$ changes too and the locus of the voltage $\boldsymbol{E_f}$ (Fig.2.8) is the horizontal line $\boldsymbol{p}$ (point $\boldsymbol{A}$ slides on $\boldsymbol{p}$ line). At the same time the current $\boldsymbol{I_a}$ changes too and its locus is also straight line $\boldsymbol{i}$, perpendicular to phasor $\boldsymbol{V}$. When the field current changes the section $\overline{CB}$, which symbolizes the reactive power $\boldsymbol{Q}$ changes too. For the low field current $\boldsymbol{I_{f1}}$ (point $\boldsymbol{A_1}$) the reactive power is capacitive, the stator current $\boldsymbol{I_{a1}}$ is large and leading. This state is called under-excitation.

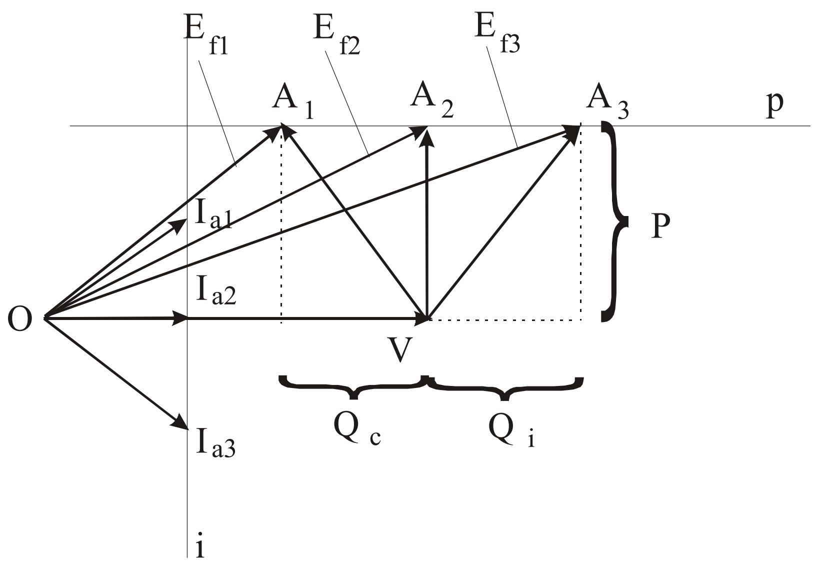


Fig.2.8 Operation of synchronous generator at constant active power ***P*** and constant voltage and frequency

At point A_2 the field current is equal I_{f2}, the armature current is minimum I_{a2} and is in phase with voltage ***V (PF = 1)*** and the reactive power ***Q*** (section $\overline{CB}$) is zero. This state is called normal excitation. For larger field current $I_f = I_{f3}$ (point A_3) the stator current I_{a3} is large again and is lagging and the reactive power ***Q*** is inductive.

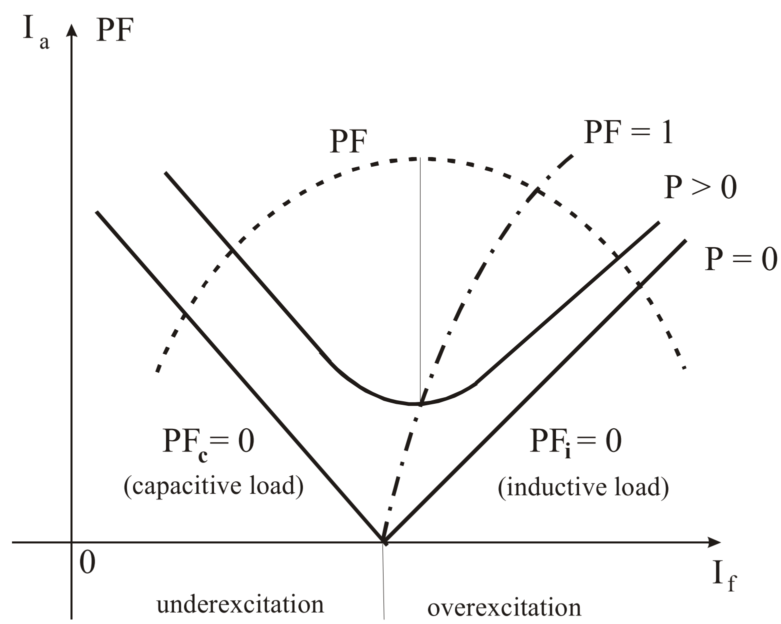


Fig.2.9 ***V-curves*** characteristics of machine operating at constant voltage ***V*** and frequency

The variation of the stator current with the field current for constant-power operation is shown in Fig.2.9. The set of characteristics drawn for different power are known as ***V-curves*** because of their shape. The variation of the power factor with the field current (dashed curves) are inverted ***V-curves***.

This feature of the power factor control by the field current is utilized to improve the power factor of a plant. If the synchronous machine is not transferring any power but is simply floating on the infinite bus, the power factor is zero. The stator current either leads or lags the stator voltage by 90^o (Fig.2.10). The magnitude of the stator current changes as the field current is changed, but the stator current is always reactive. The machine behaves like a variable inductor or capacitor as the field current is changed. Therefore, the unloaded synchronous machine is called a ***synchronous condenser*** and may be used to regulate the receiving-end voltage of a long power transmission line.

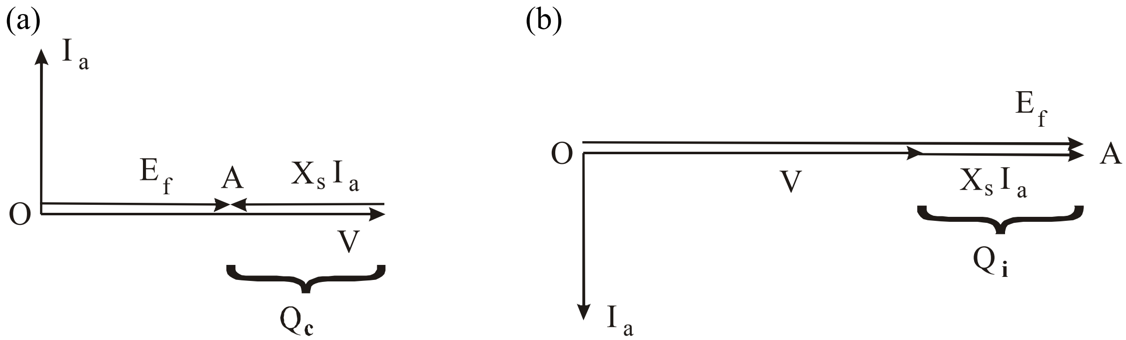


Fig.2.10 Operation of the synchronous machine as a ***synchronous condenser:*** machine deliver to the load (***infinite bus***) (a) capacitive reactive power, (b) inductive power

2.5 Operation of synchronous machine as an independent generator

Synchronous machines are normally connected to an infinite bus. However, small synchronous generators may be required to supply independent electric loads. As an example, a gasoline engine can drive the synchronous generator at constant frequency. In such a system, the terminal voltage tends to change with varying load.

To determine the terminal characteristics of an independent synchronous generator, consider the equivalent circuit in Fig.2.11. At open circuit $V = E_f$, $I_a = 0$, and at short circuit $V = 0$, $I_a = I_{sc} = E_f/X_s$. If the load current is changed from 0 to I_{sc}, the terminal voltage V will change from E_f to zero. Fig.2.12 illustrates these changes in form of ***V-I characteristics*** drawn for various ***PF*** of the load. For the purely reactive load:

$$\underline{V} = \underline{E}_f - jX_s \underline{I}_a \tag{28}$$

and from the phasor diagram the straight line ***V-I*** characteristic is concluded for pure inductive and pure capacitive load.

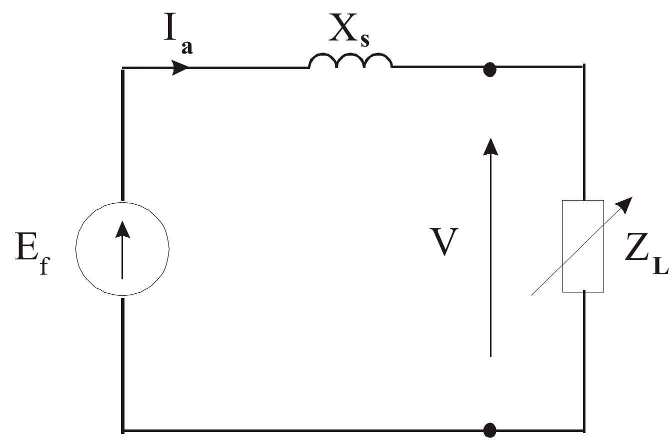


Fig.2.11 Equivalent circuit of synchronous generator operating on the load $\boldsymbol{Z_L}$

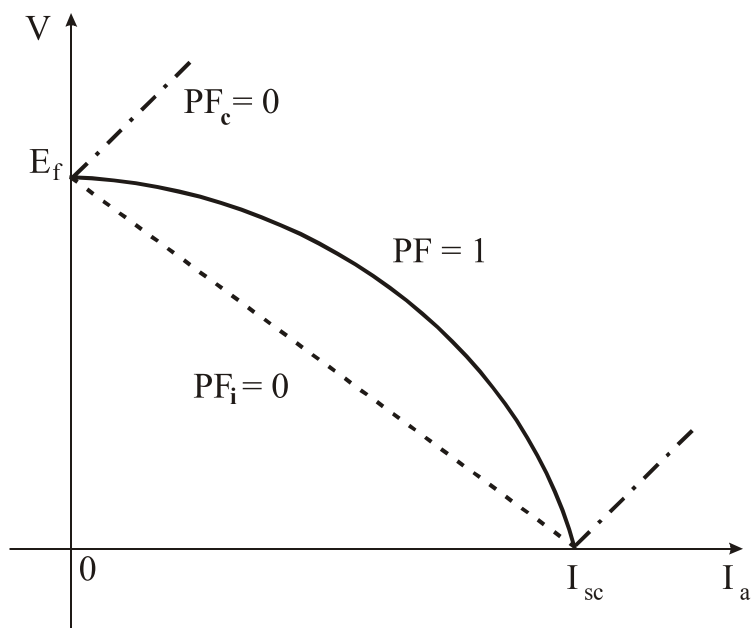


Fig.2.12 V-I characteristics of synchronous machine operating as an independent generator

2.6 Salient pole synchronous machine

Low speed multipolar synchronous machine has salient poles and nonuniform air gap (Fig.2.13). The magnetic reluctance is low along the poles (***d-axis***) and high between poles (***q-axis***). If the flux Φ_a produced by the armature (stator) current is aligned with d-axis the current I_a experiences reactance X_{ad}. Such a reactance experiences the stator current for cylindrical rotor, independently on the rotor position with respect to stator flux Φ_a. When the stator flux takes the position parallel to the ***q-axis*** the armature reaction reactance for the stator current is X_{aq}, smaller than X_{ad}. Thus, the armature reaction reactance changes vs. power angle δ as is shown in Fig.2.14.

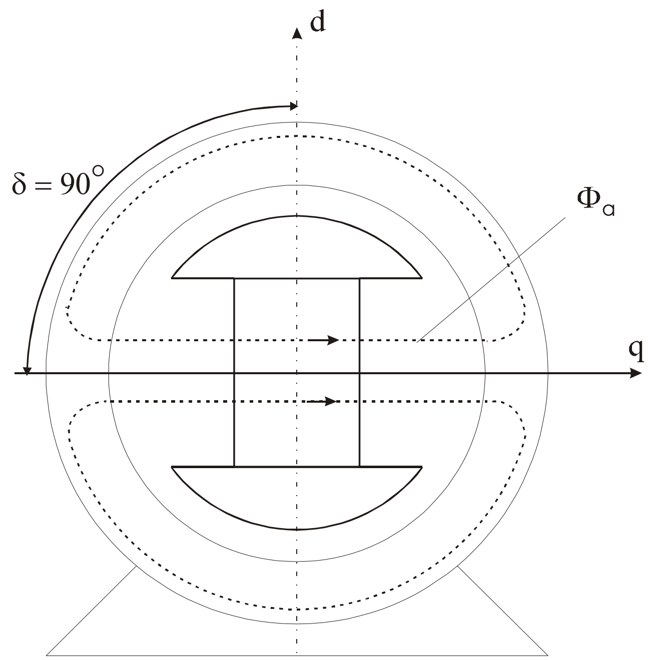


Fig.2.13 Armature reaction magnetic flux Φ_a of synchronous machine with salient poles, directed along ***q-axis***

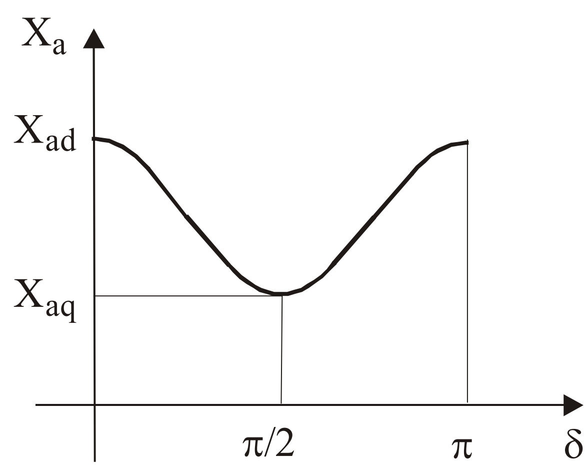


Fig.2.14 Variation of armature reaction reactance X_a along the stator circumference of the salient pole synchronous machine

When the power angle varies (the mutual position of stator flux and the rotor) the unexcited rotor experiences the torque called reluctance torque, what illustrates Fig.2.15. This torque changes according to equation:

$$T_r = \frac{m}{\omega_s}\frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta \tag{2.29}$$

The excited salient synchronous machine develops the resultant torque that is the sum of electromagnetic torque and reluctance torque:

$$T = \frac{m}{\omega_s}\frac{VE_f}{X_s}\sin\delta + \frac{m}{\omega_s}\frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta = T_{em} + T_r \tag{2.30}$$

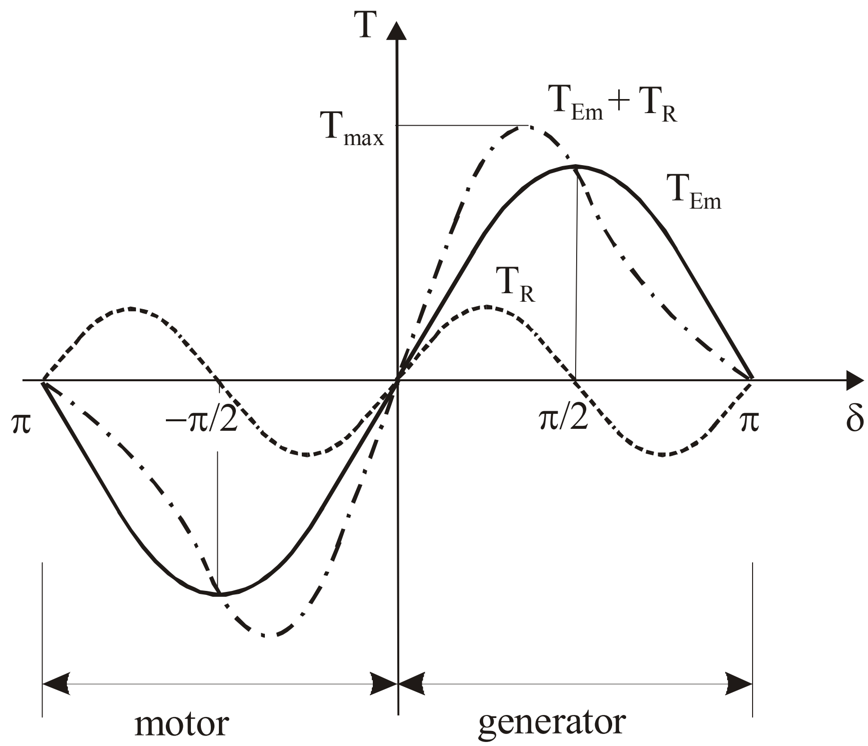


Fig.2.15 Torque-power angle characteristics of synchronous machine with salient-pole rotor

The torque-power angle characteristics drawn at different excitation currents and constant terminal voltage ***V*** are shown in Fig.2.16. For salient pole machine the field current may be reduced to zero and the reluctance torque can keep machine still in synchronism.

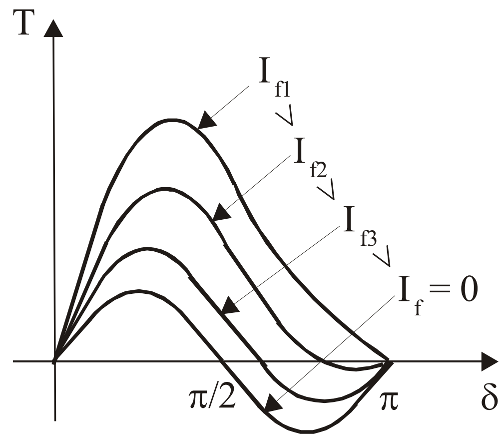


Fig.2.16 Torque-power angle characteristics of synchronous machine with salient-pole rotor at various field currents

2.7 Connection of a synchronous generator to the infinite bus

Synchronous generators are rarely used to supply the individual loads. These generators, in general, are connected to a power system known as *an infinite bus* or *grid*. The voltage and frequency of the infinite bus hardly change.

The operation of connecting a synchronous generator to the infinite bus is known as paralleling with the infinite bus. Before the generator can be connected to the infinite bus, the incoming generator and the infinite bus must have the same:

- Voltage,
- Frequency,
- Phase sequence,
- Phase.

In the power plant the satisfaction of these conditions is checked by an instrument known as a synchronoscope. Fig.2.17 illustrates the situation when one of the above mentions quantities are not equal:

1) Rms voltages are not the same, but frequency and phase sequence are the same: - Fig.2.17.a
2) Frequencies are not the same, but voltages and phase sequences are the same: - Fig.2.17.b
3) Phase sequences are not the same, but voltages and frequencies are are the same: - Fig.2.17.c
4) Phase is not the same, but voltage, frequency and phase sequence are the same: - Fig.2.17.d.

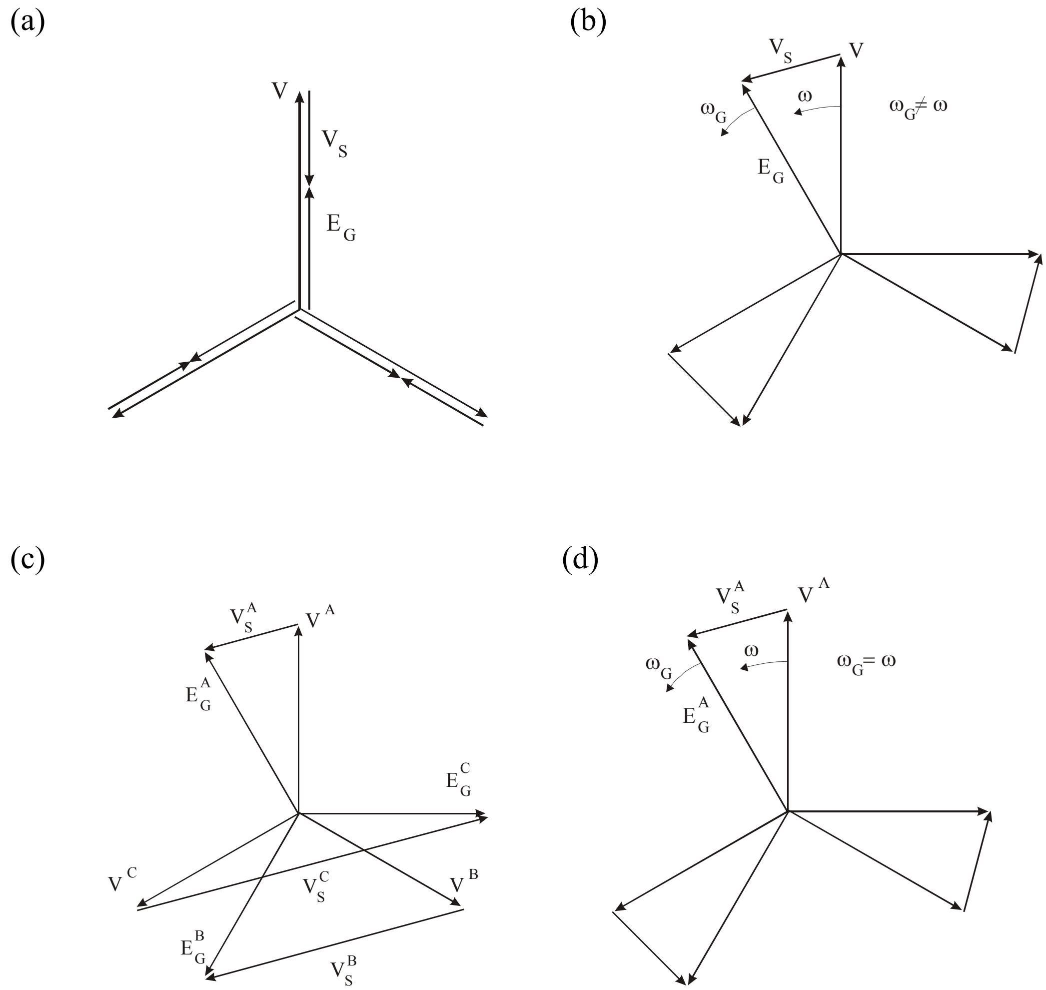


Fig.2.17 Phasor diagrams illustrating the situation when one of the conditions for equal instantaneous generator and infinite base voltages are not met

Each case mentioned above causes the voltage difference across the switch (between generator and infinite bus terminals) and if are connected it will produce a disastrous situation for the generator.