EE 560
MOS TRANSISTOR THEORY

PART 1

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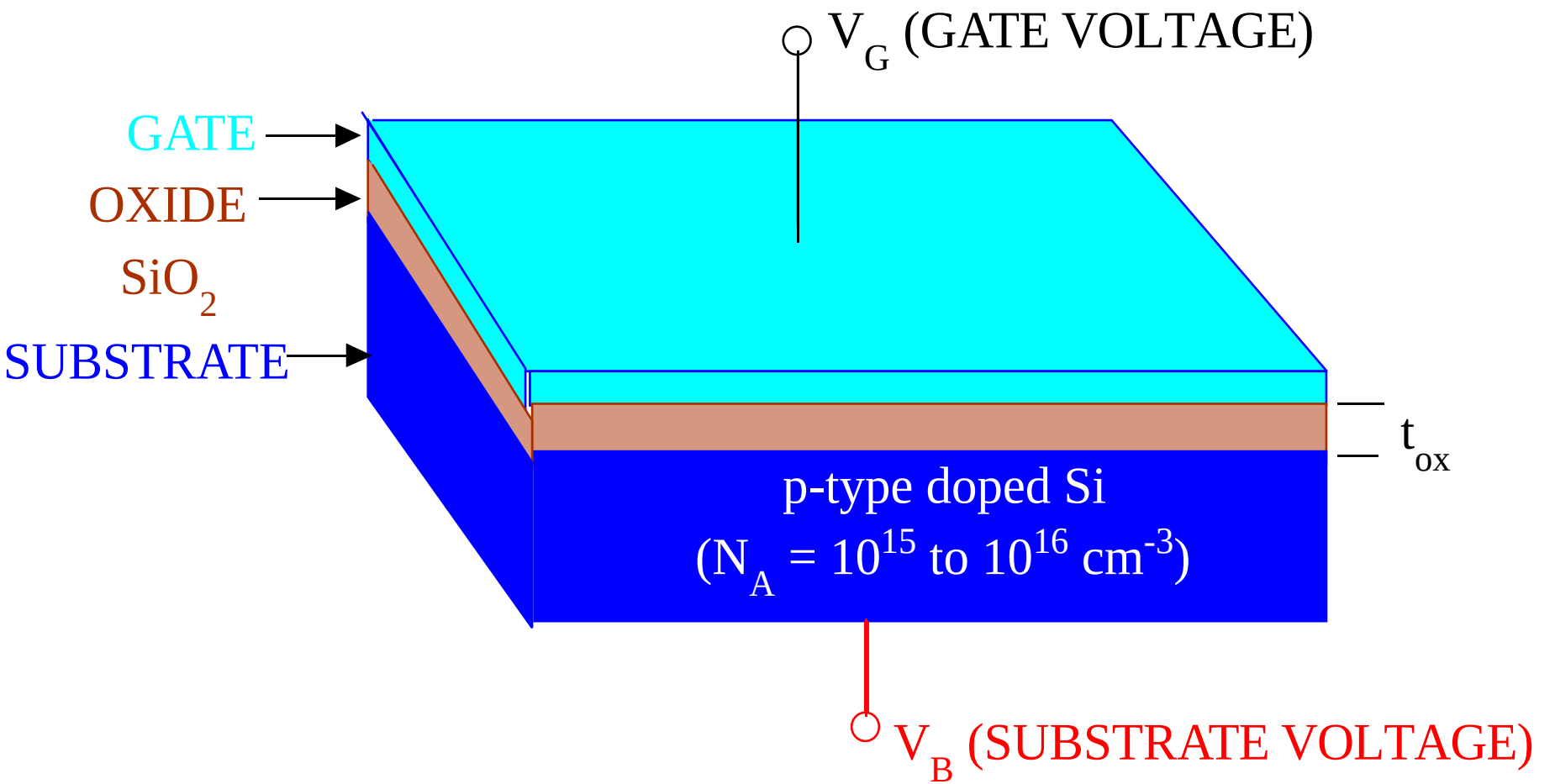
TWO TERMINAL MOS STRUCTURE



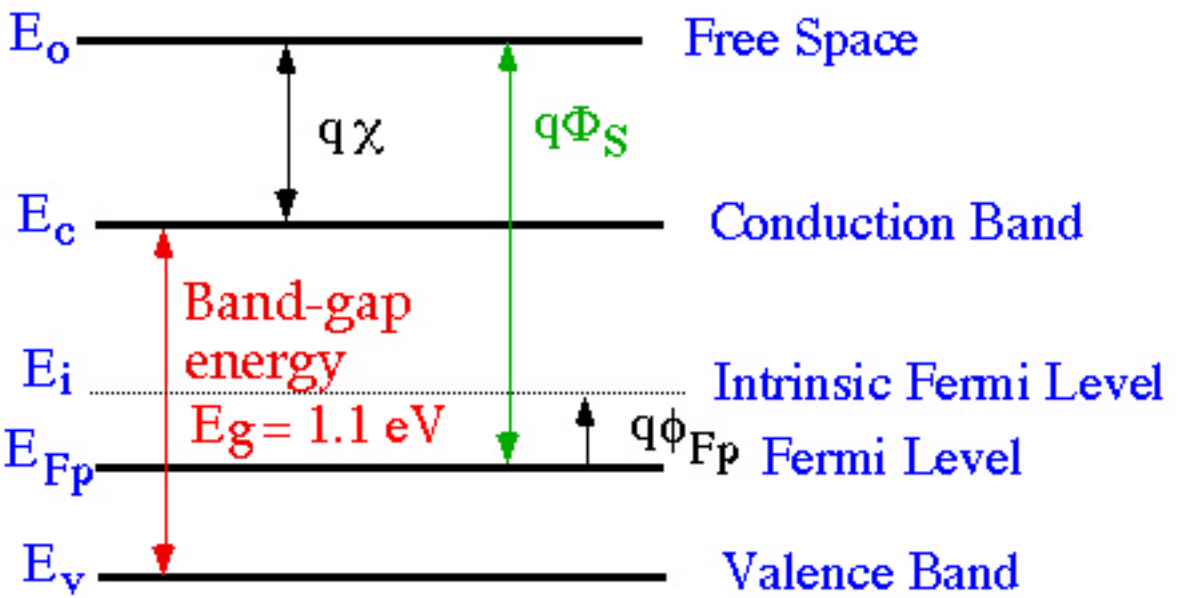
EQUILIBRIUM: $n\,p = n_i^2$ ($n_i \approx 1.45 \times 10^{10}$ cm^{-3})

Let SUBSTRATE be uniformy doped @ N_A

$$n_{p\,0} \approx \frac{n_i^2}{N_A} \quad \text{and} \quad p_{p0} = N_A$$

(BULK concentrations)

ENERGY BAND DIAGRAM FOR p - TYPE SUBSTRATE



$q\chi$ = electron affinity of Si

$$\varphi_F = \frac{E_F - E_i}{q}$$

Work Function

$$q\Phi_S = q\chi + (E_c - E_F)$$

$$\varphi_{Fp} = \frac{kT}{q} \ln \frac{n_i}{N_A} \quad (N_A >> n_i)$$

$$\varphi_{Fn} = \frac{kT}{q} \ln \frac{N_D}{n_i} \quad (N_D >> n_i)$$

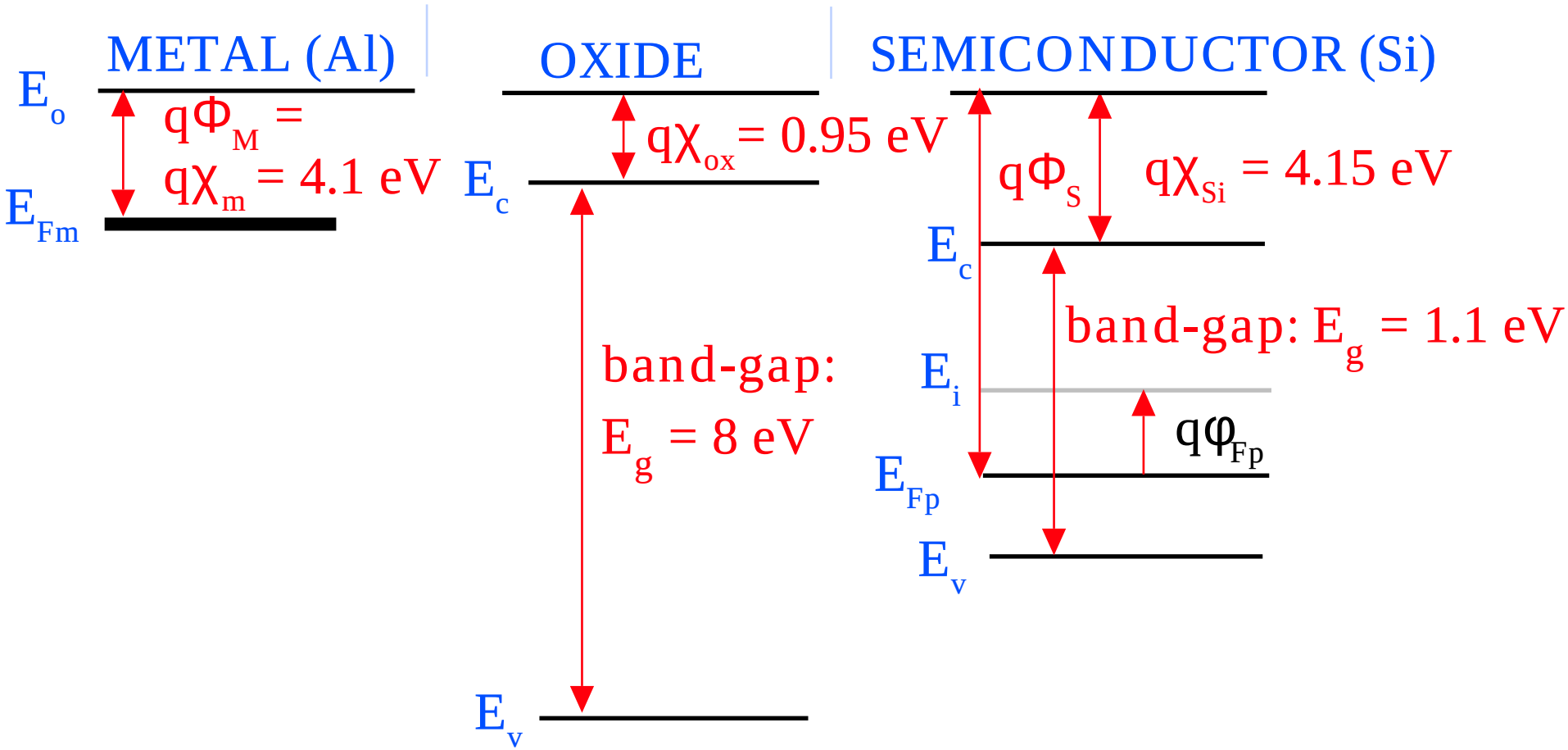
$n_i = 1.45 \times 10^{-10}$ cm^{-3} @ room temp,

$k = 1.38 \times 10^{-23}$ J/^oK,

$q = 1.6 \times 10^{-19}$ C

=> $kT/q = 26$ mV @ room temp

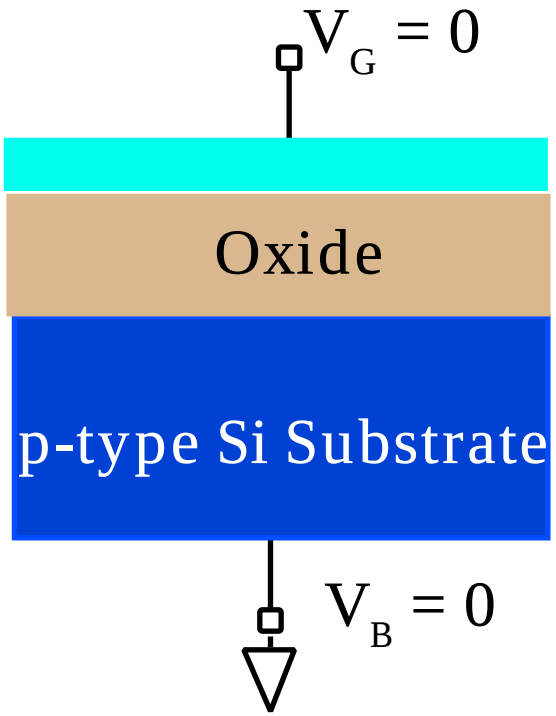
ENERGY BAND DIAGRAMS FOR COMPONENTS OF MOS STRUCTURE



$\Phi_S > \Phi_M$ => p-type Si

$\Phi_M > \Phi_S$ => n-type Si

Equilibrium



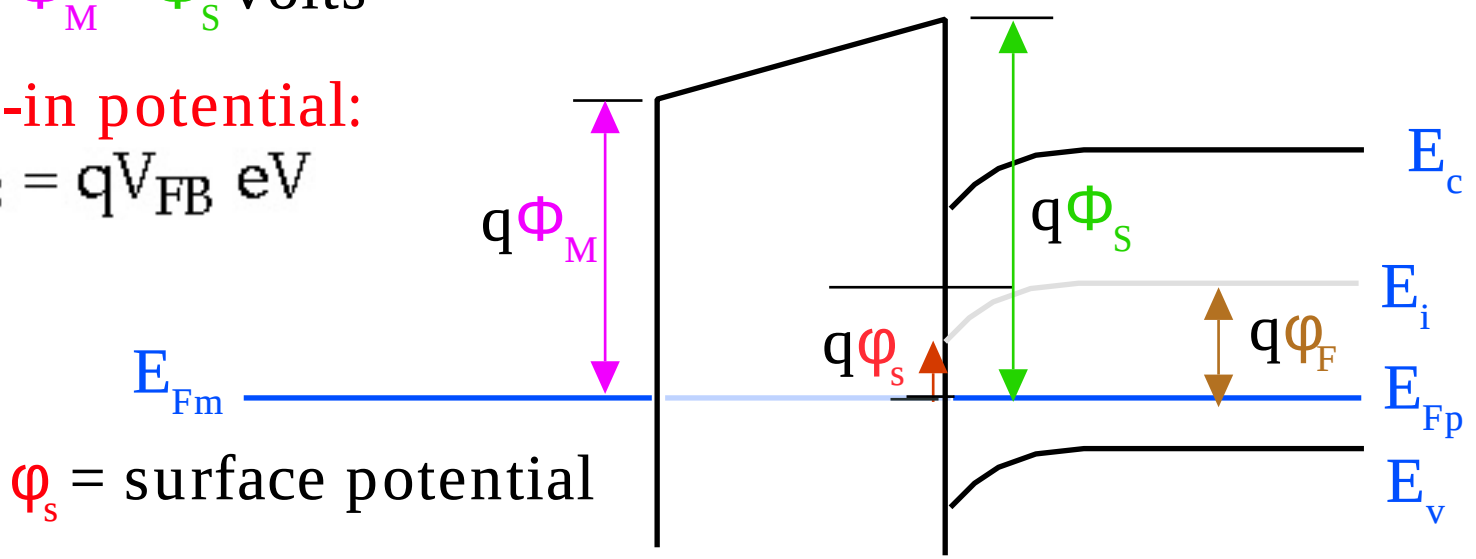
Flat-band voltage:

$V_{FB} = \Phi_M - \Phi_S$ volts

Built-in potential:

$\Phi_{MS} = qV_{FB}$ eV

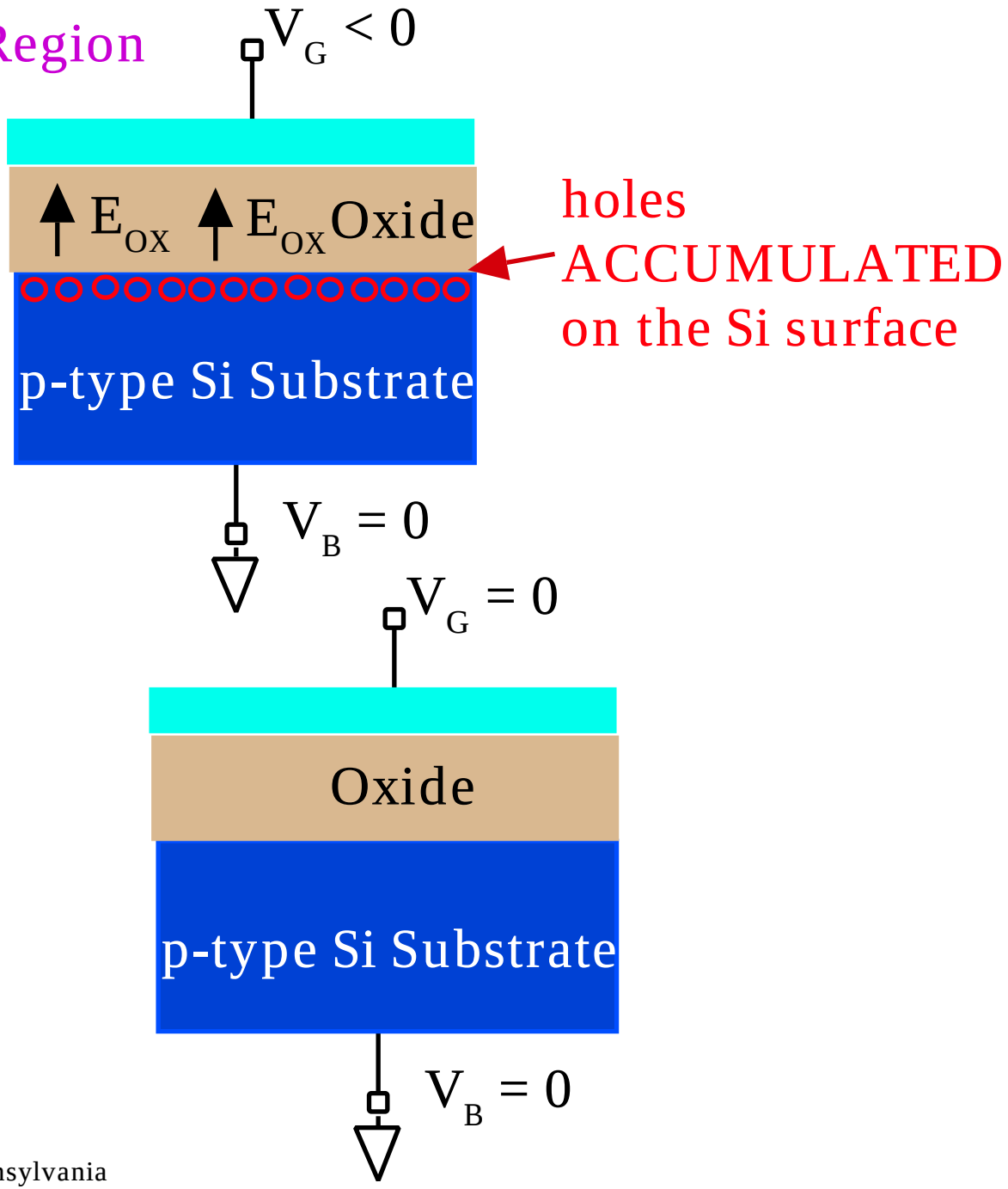
Equilibrium



φ_s = surface potential

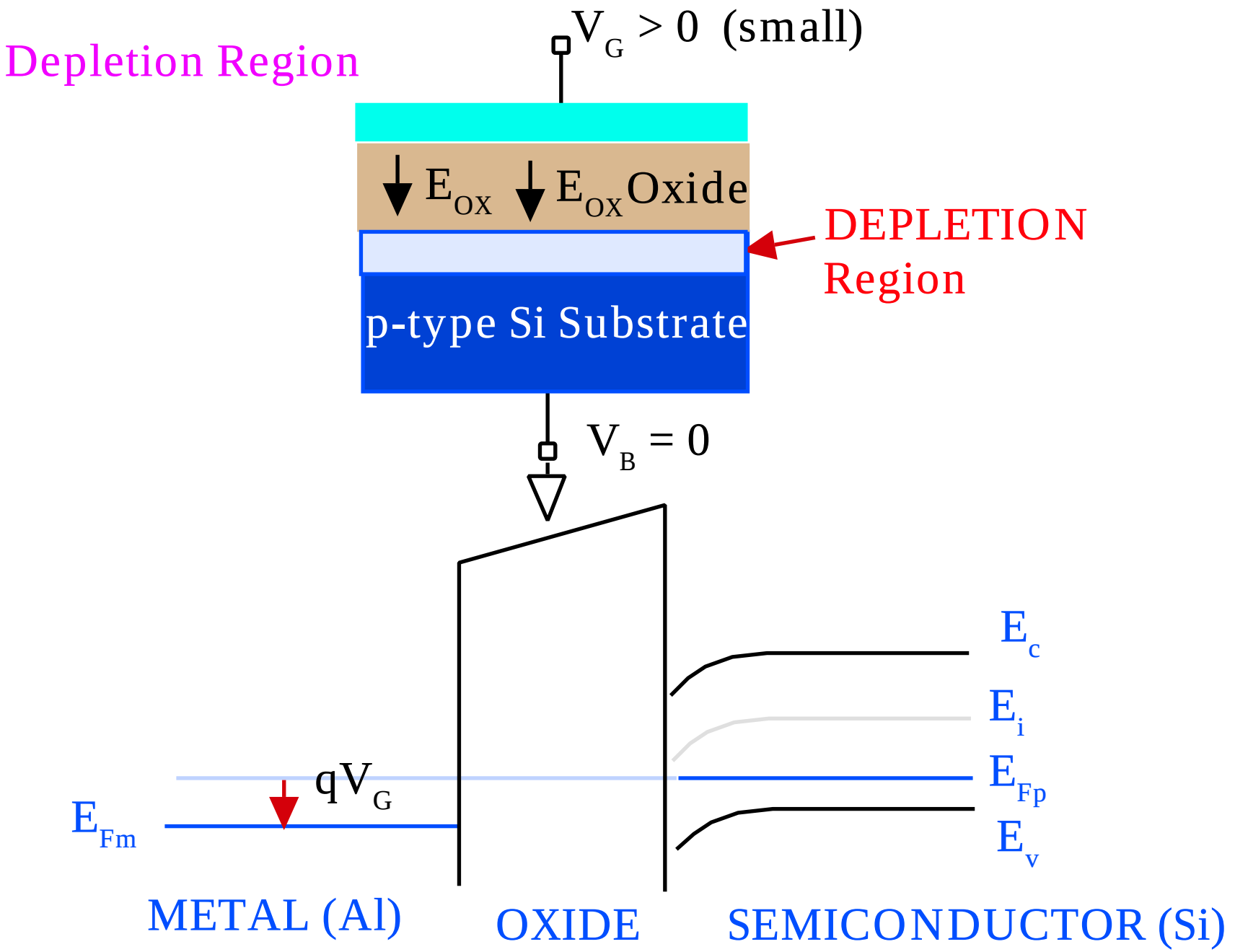
MOS SYSTEM WITH EXTERNAL BIAS

Accumulation Region

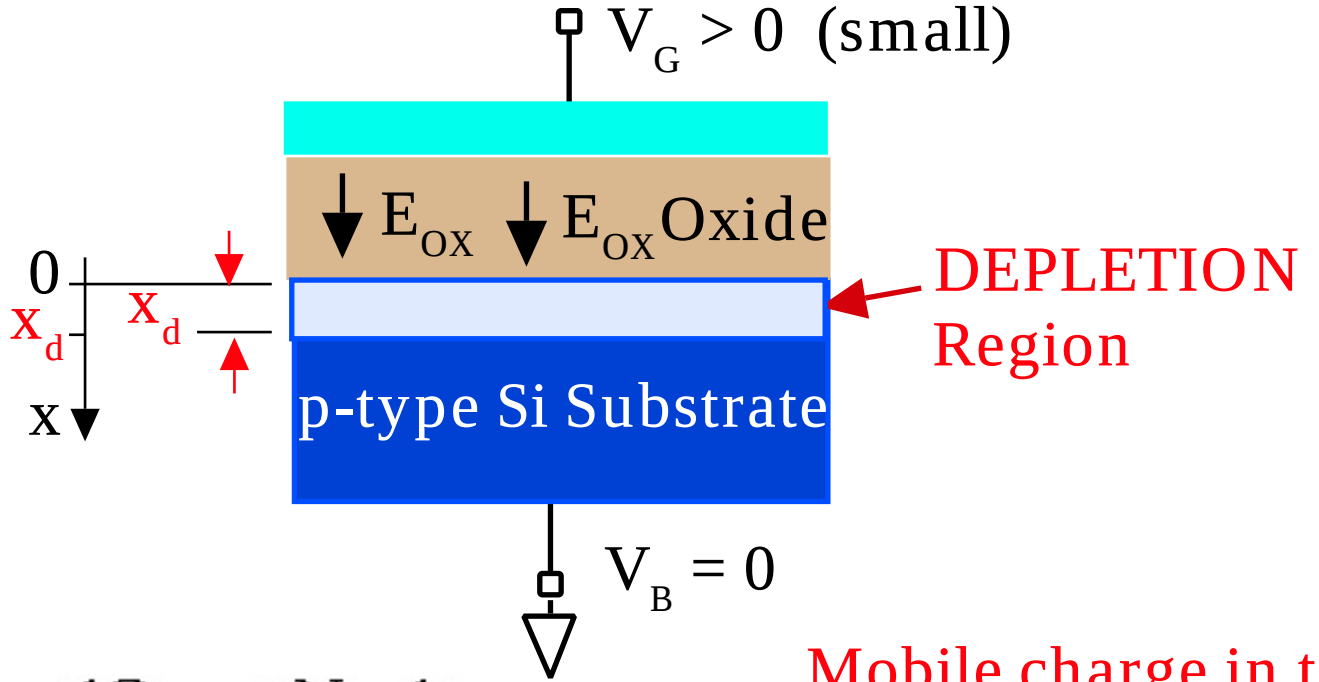


Equilibrium

MOS SYSTEM WITH EXTERNAL BIAS



MOS SYSTEM BIASED IN DEPLETION REGION



$$dQ = -qN_A dx$$

Mobile charge in thin layer parallel to Si surface

$$d\phi = -\frac{x}{\varepsilon_{Si}} dQ = x\frac{qN_A}{\varepsilon_{Si}} dx$$

Change in surface potential to displace dQ

$$\int_{\phi_s}^{\phi_F} d\phi = \int_0^{x_d} x\frac{qN_A}{\varepsilon_{Si}} dx \rightarrow \phi_F - \phi_s = \frac{qN_A}{2\varepsilon_{Si}} x_d^2$$

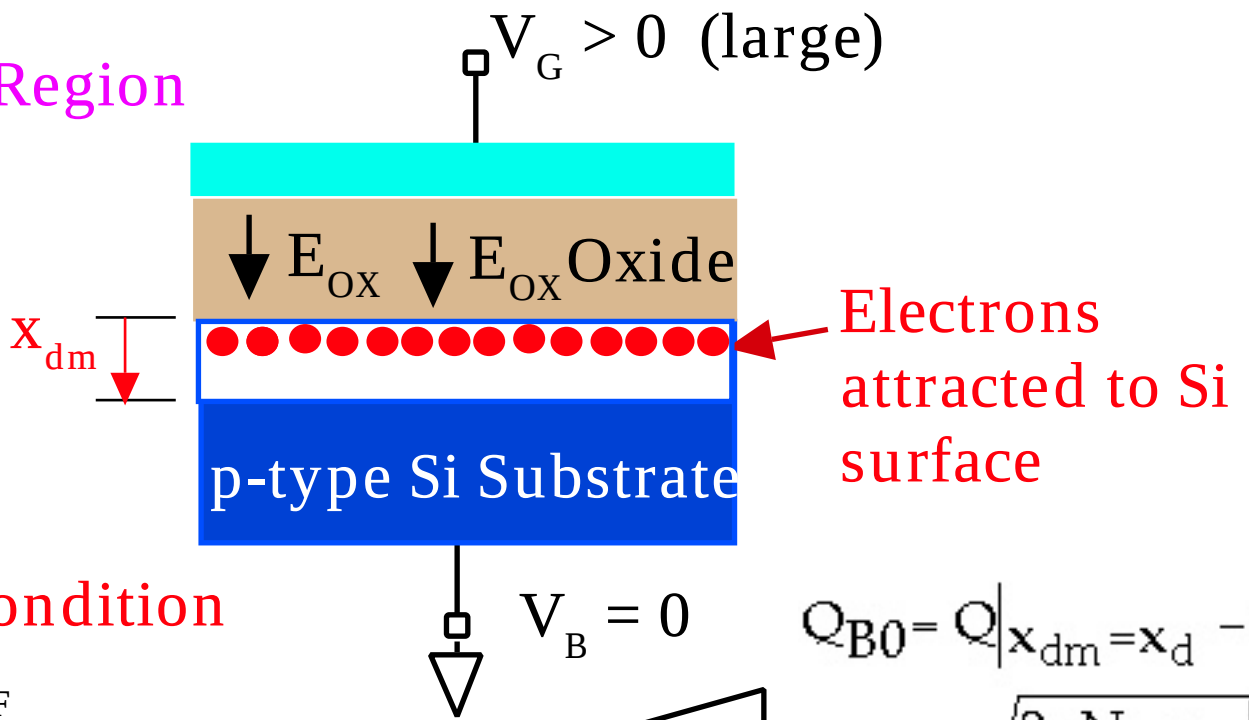
$$x_d = \sqrt{\frac{2\varepsilon_{Si}|\phi_F - \phi_s|}{qN_A}}$$

Depletion Region Charge

$$Q = -qN_A x_d = -\sqrt{2qN_A\varepsilon_{Si}|\phi_F - \phi_s|}$$

MOS SYSTEM WITH EXTERNAL BIAS

Inversion Region



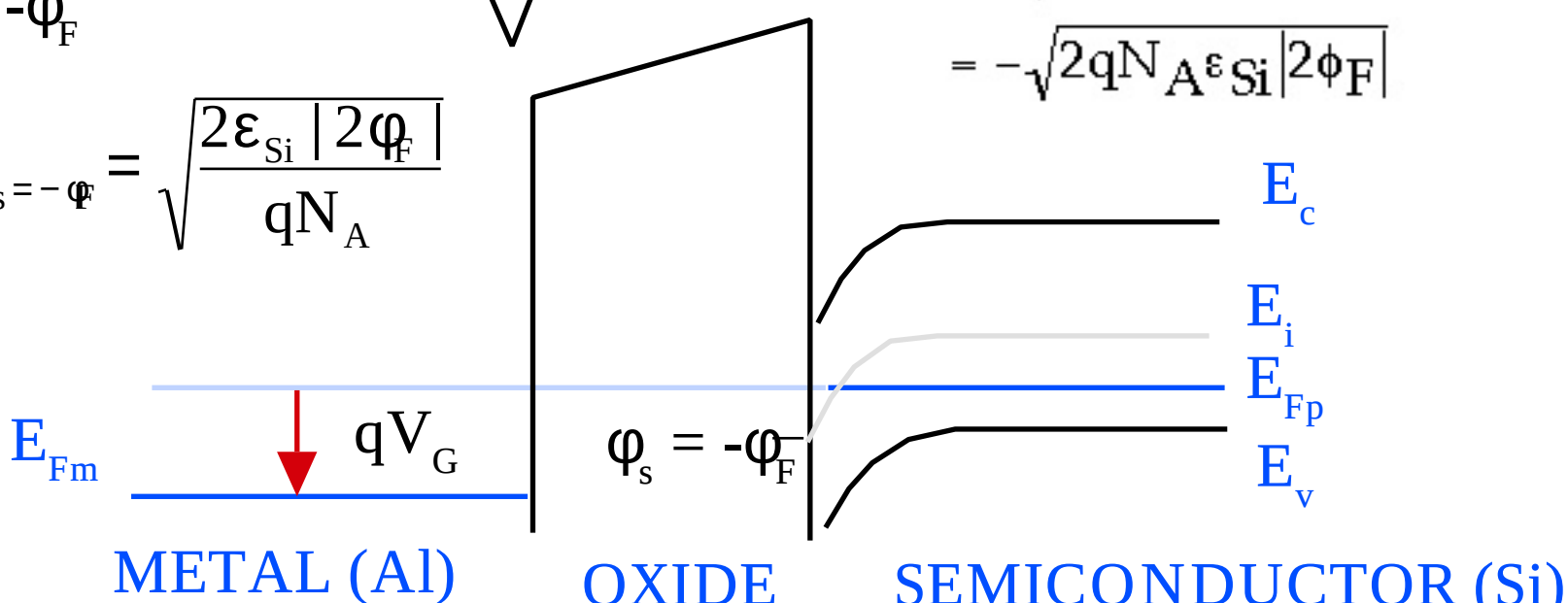
Inversion Condition

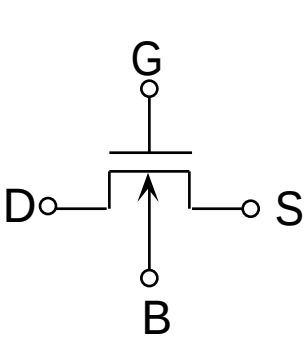
$$\varphi_s = -\varphi_F$$

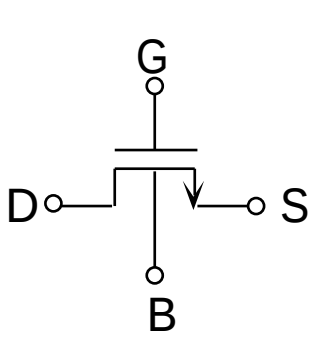
$$x_{dm} = x_d\Big|_{\varphi_s = -\varphi_F} = \sqrt{\frac{2\varepsilon_{Si}\,|2\varphi_F|}{qN_A}}$$

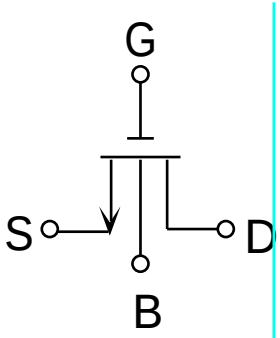
$$Q_{B0} = Q\Big|_{x_{dm}=x_d} - qN_A x_{dm}$$

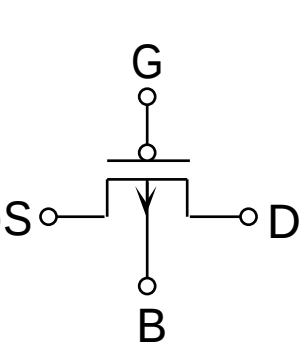
$$= -\sqrt{2qN_A\varepsilon_{Si}\,|2\phi_F|}$$

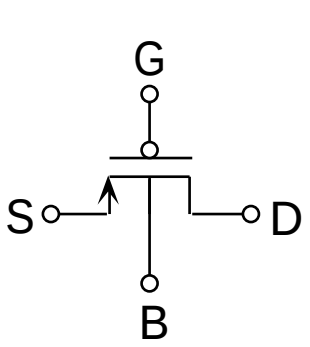


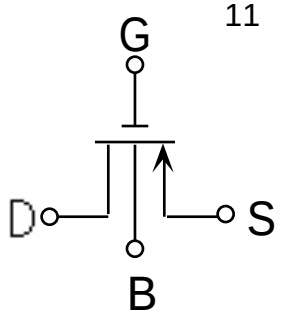




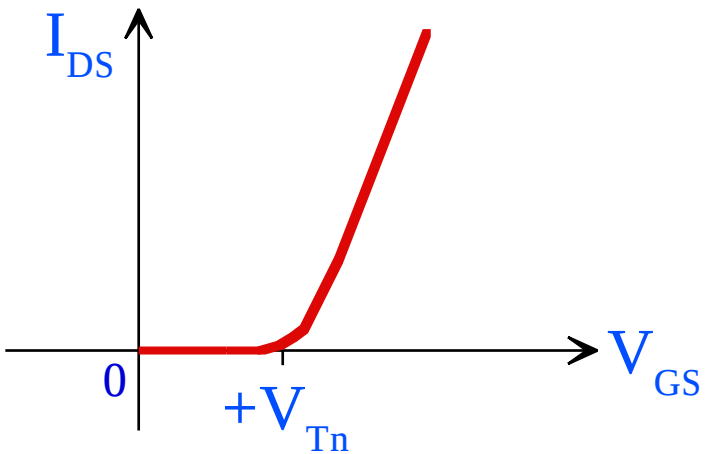




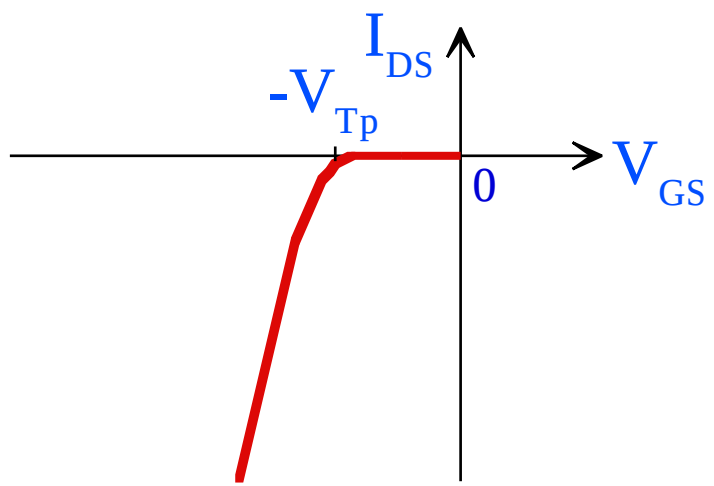




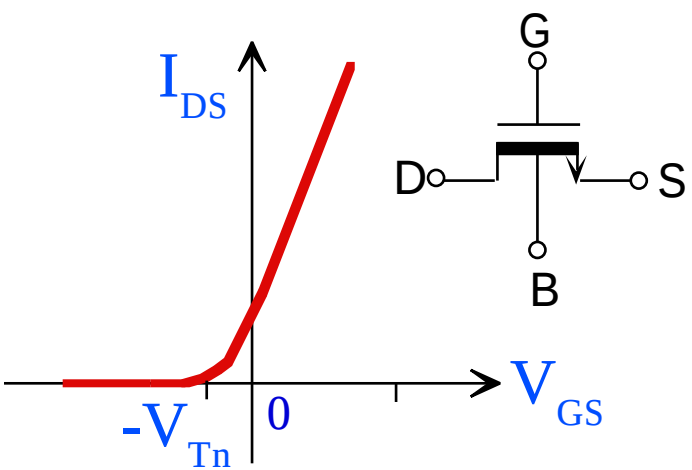
n-channel enhancement



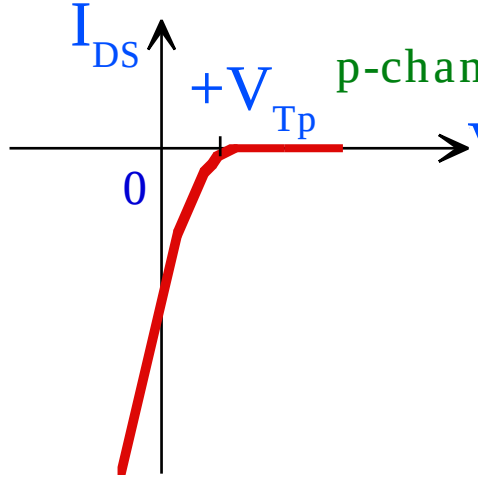
p-channel enhancement

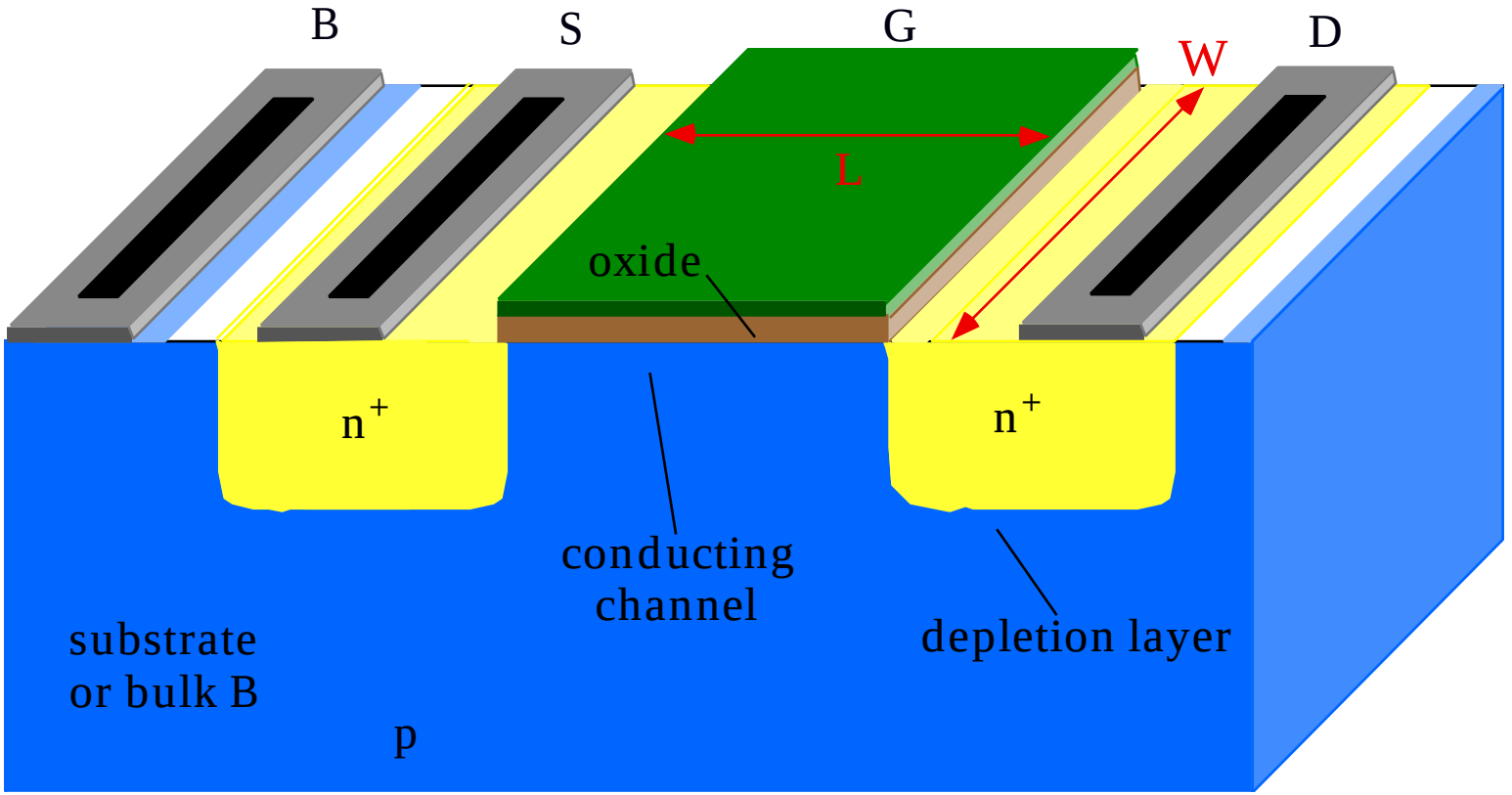


n-channel depletion

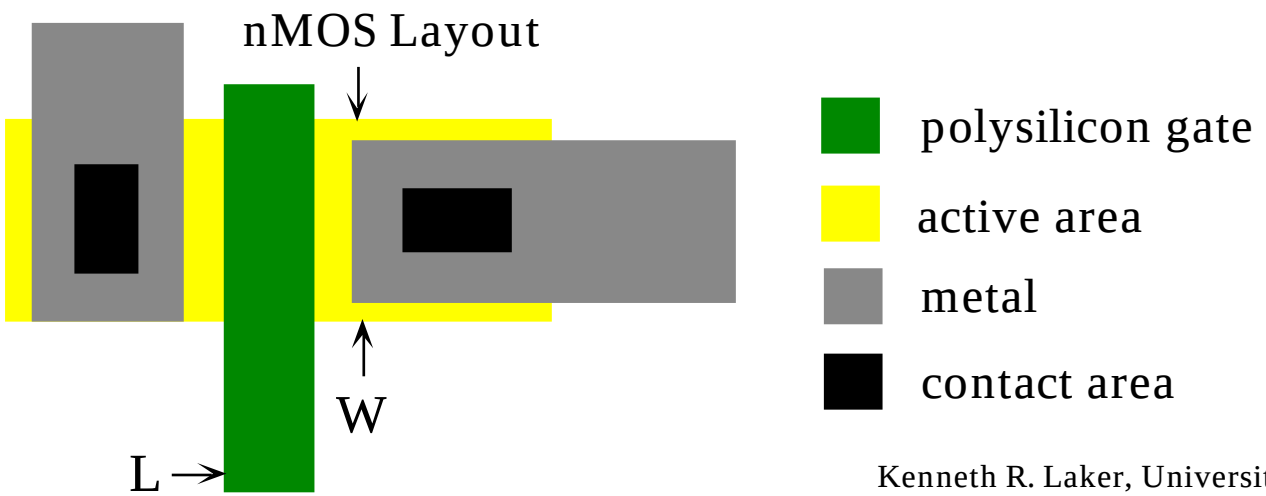


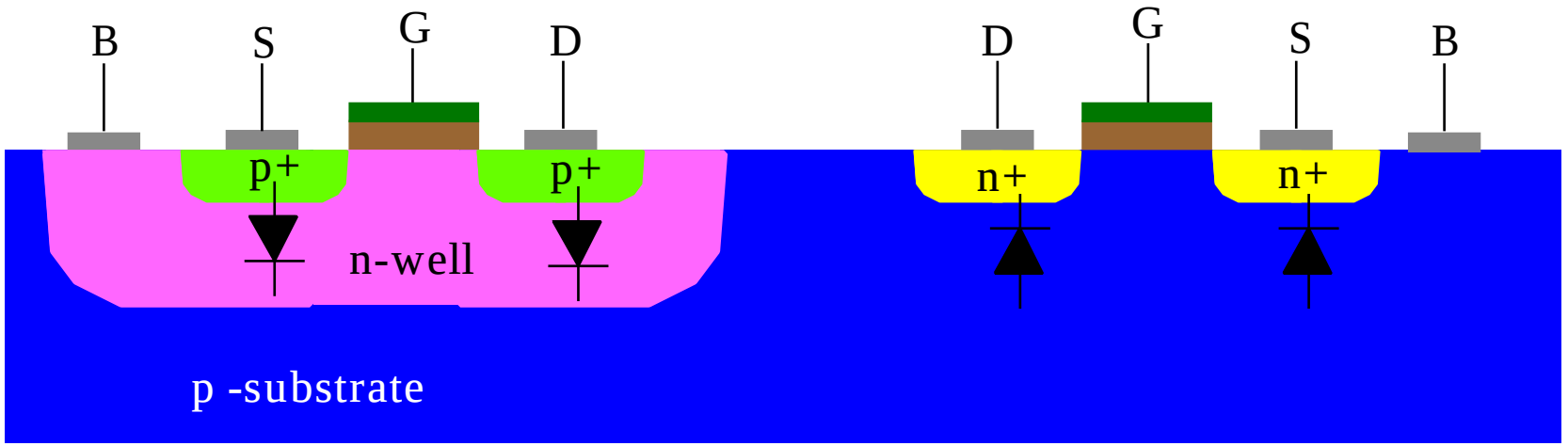
p-channel depletion





N-CHANNEL ENHANCEMENT-TYPE MOSFET

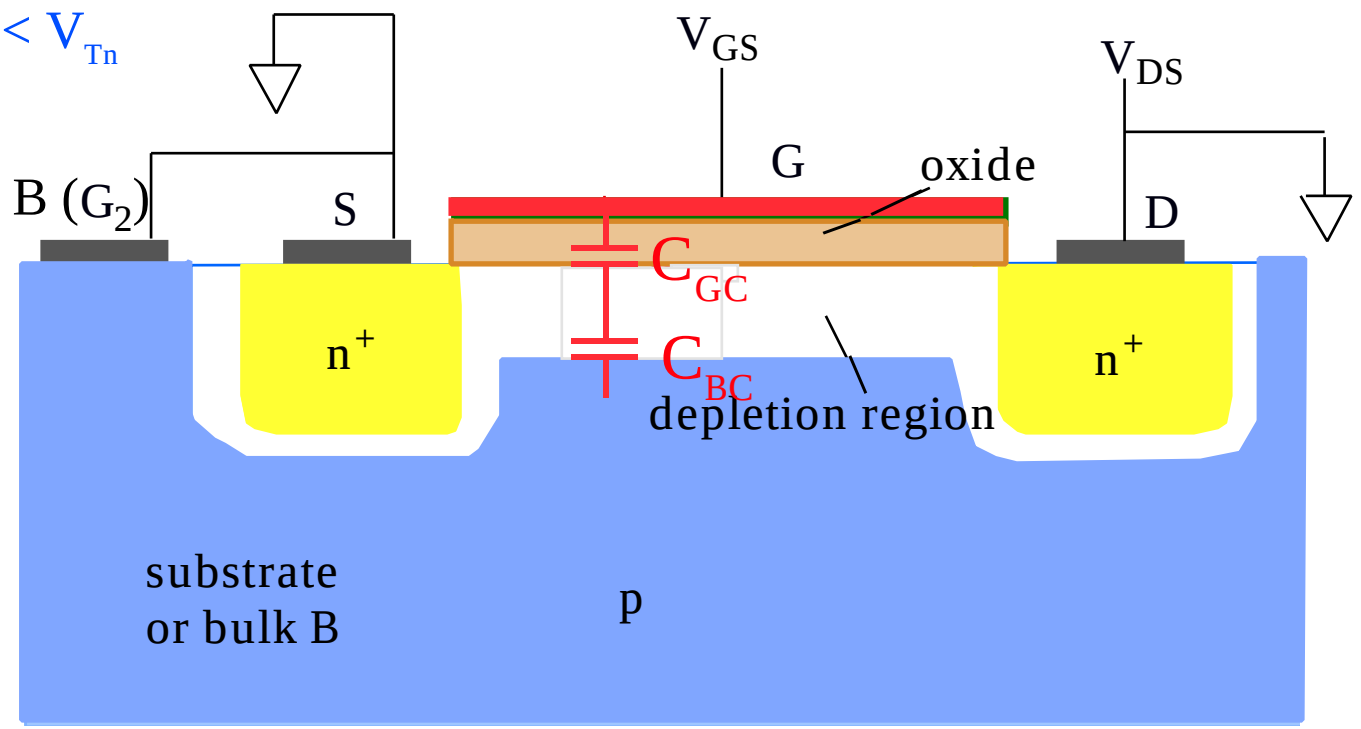




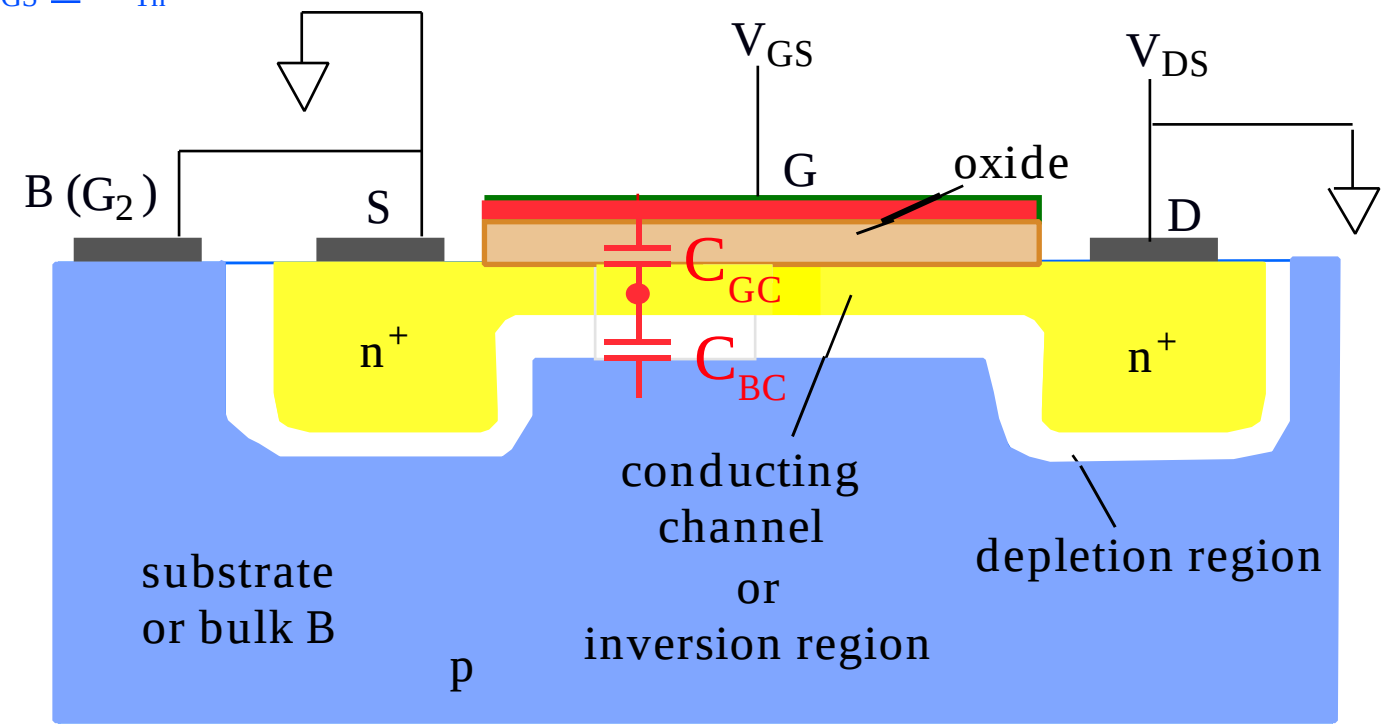
$V_{GS} < V_{Tn}$ → CUT-OFF REGION

DEPLETION REGION

$0 < V_{GS} < V_{Tn}$

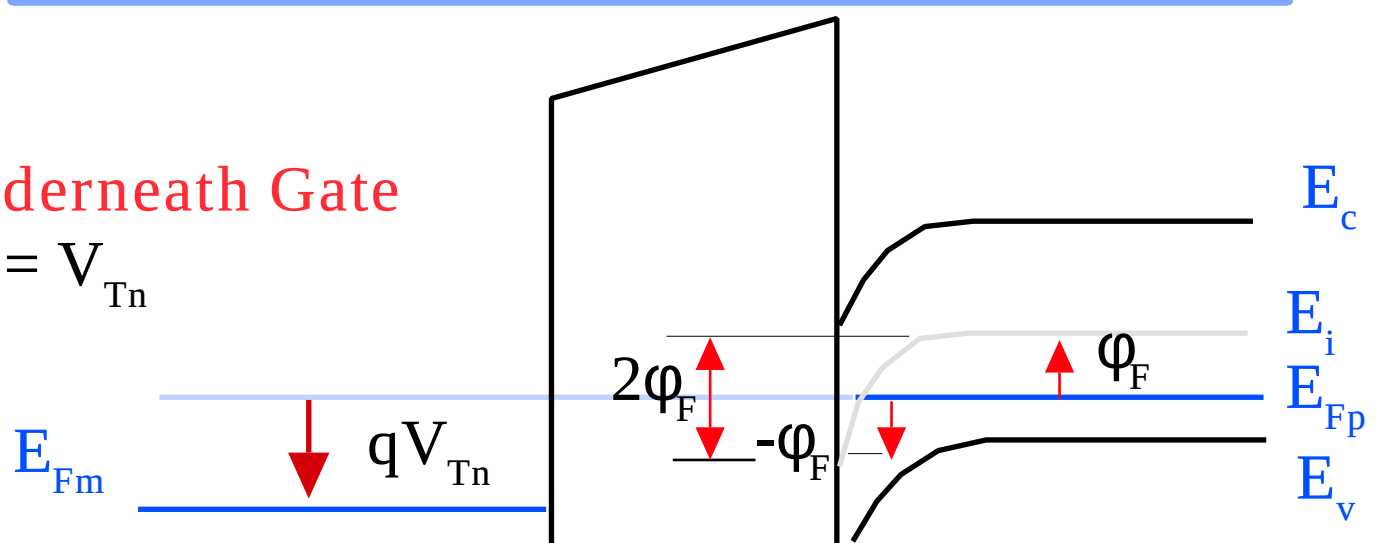


$V_{GS} \geq V_{Tn}$ INVERSION REGION



Underneath Gate

$V_G = V_{Tn}$



METAL (Al) OXIDE SEMICONDUCTOR (Si)

MOS Capacitance $C_{GC} = WLC_{ox}$, $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$ [t_{ox} -> TOX in SPICE] [C_{ox} -> COX in SPICE]

t_{ox} = 50 nm, ε_{ox} = 0.34 pF/cm => C_{ox} = 6.8 x 10^{-8} F/cm^2

W x L = 50 µm x 50 µm => C_{GC} = 170 fF

Depletion Capacitance $C_{BC} = WLC_j$, $C_j = \frac{\varepsilon_{Si}}{x_d}$

$$x_d = \sqrt{\frac{2\varepsilon_{Si}|\phi_F - V_{SB}|}{qN_A}}$$

[N_{SUB} -> NSUB in SPICE]

N_{SUB} (p - substrate)

$$\varphi_{Fp} = \frac{kT}{q} \ln \frac{n_i}{N_A}$$

N_A = 3 x 10^{17} cm^{-3}, n_i = 1.45 x 10^{10} cm^{-3} => φ_F = -0.438 V

(recall that at room temp or 27°C kT/q = 26 mV)

V_{SB} = 0 V, ε_{Si} = 1.06 pF/cm, q = 1.6 x 10^{-19} C, N_A, φ_F => x_d = 6.22 µm

ε_{Si}, x_d => C_j = 0.17 x 10^{-8} F/cm^2

W x L = 50 µm x 50 µm => C_{BC} = 42.5 fF

Threshold Voltage for MOS Transistors

n-channel enhancement

For $V_{SB} = 0$, the threshold voltage is denoted as V_{T0} or $V_{T0n,p}$ [V_{T0} -> VT0 in SPICE]

$$V_{T0} = \Phi_{GC} - 2\varphi_F - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

(+ for nMOS and - for pMOS)

[$2\varphi_F$ = PHI in SPICE]

$$Q_{B0} = -\sqrt{2qN_A\varepsilon_{Si}|2\phi_F|}$$

[N_A = NSUB in SPICE]

Threshold Voltage factors:

-> Gate conductor material;

-> Gate oxide material & thickness;

-> Channel doping;

-> Impurities in Si-oxide interface;

-> Source-bulk voltage V_{sb};

-> Temperature.

$\Phi_{GC} = \varphi_F(\text{substrate}) - \varphi_M$ metal gate

$\Phi_{GC} = \varphi_F(\text{substrate}) - \varphi_F(\text{gate})$ polysilicon gate

[Q_{ox} = qNSS in SPICE]

Threshold Voltage for MOS Transistors

n-channel enhancement

For $V_{SB} \neq 0$: the threshold voltage is denoted as V_T or $V_{Tn,p}$

$$Q_B = -\sqrt{2qN_A\varepsilon_{Si}|2\phi_F - V_{SB}|}$$

$$V_T = \Phi_{GC} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

$$= \Phi_{GC} - 2\phi_F - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_B - Q_{B0}}{C_{ox}}$$

where V_{T0}

$$\frac{Q_B - Q_{B0}}{C_{ox}} = \sqrt{\frac{2qN_A\varepsilon_{Si}}{C_{ox}}}\left(\sqrt{|2\phi_F - V_{SB}|} - \sqrt{|2\phi_F|}\right)$$

(γ = Body-effect coefficient) [γ = GAMMA in SPICE]

$$V_{Tn} = V_{T0n} + \gamma\left(\sqrt{|2\phi_F - V_{SB}|} - \sqrt{|2\phi_F|}\right)$$

Threshold Voltage for MOS Transistors

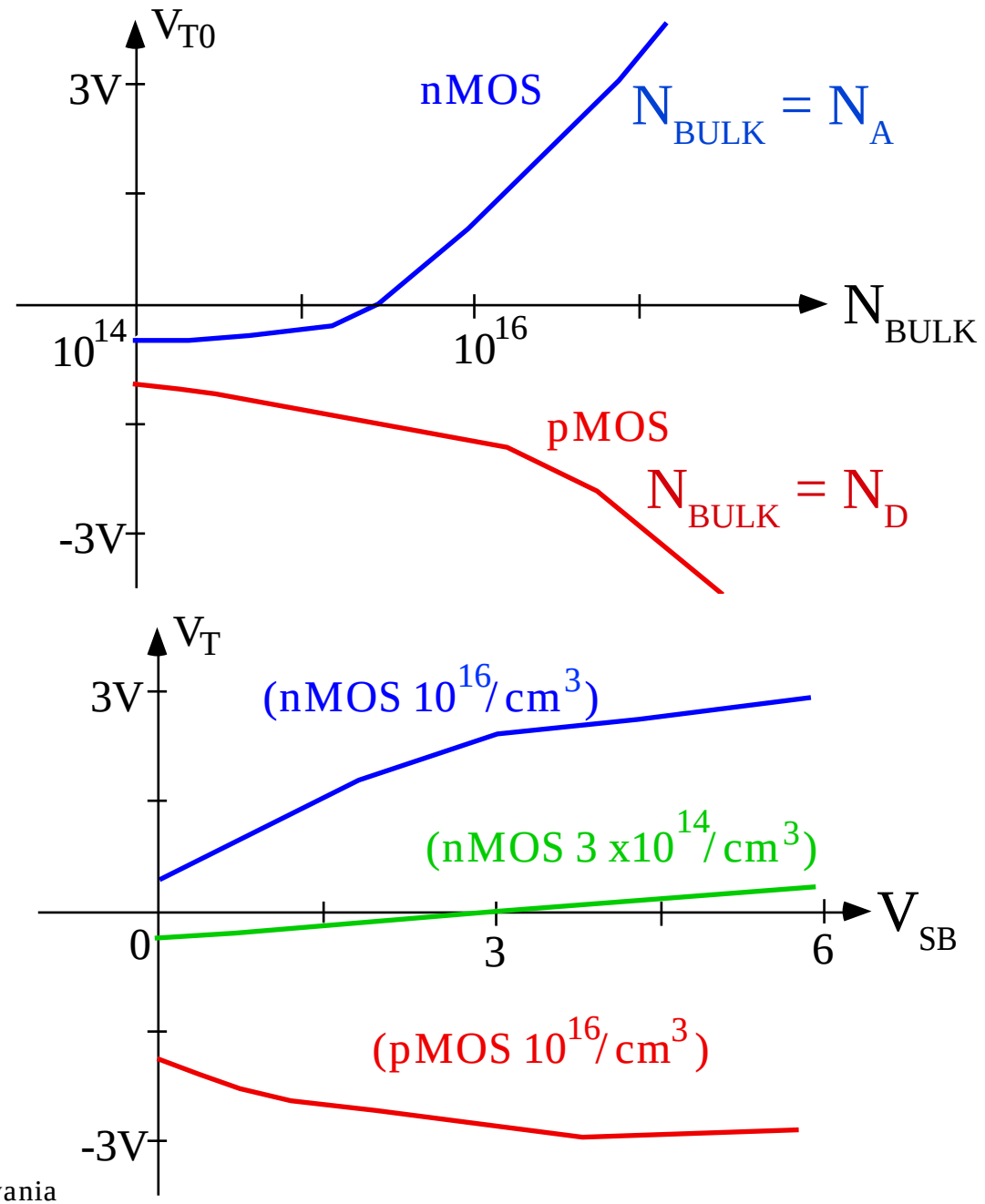
n-channel -> p-channel

******BE CAREFULL*** WITH SIGNS**

- φ_F is negative in nMOS, positive in pMOS
- Q_{B0}, Q_B are negtive in nMOS, positive in pMOS
- γ is positive in nMOS, negative in pMOS
- V_{SB} is negative in nMOS, positive in pMOS

NOTE: $\gamma \quad \frac{C_{BC}}{C_{GC}}$

Threshold Voltage for MOS Transistors



EXAMPLE 3.2 Calculate the threshold voltage V_{T0} at $V_{BS} = 0$, for a polysilicon gate n-channel MOS transistor with the following parameters:

substrate doping density $N_A = 10^{16}\ cm^{-3}$,
polysilicon doping density $N_D = 2 \times 10^{20}\ cm^{-3}$,
gate oxide thickness $t_{ox} = 500$ Angstroms,
oxide-interface fixed charge density $N_{ox} = 4 \times 10^{10}\ cm^{-2}$.

$$V_{T0} = \Phi_{GC} - 2\varphi_{F(sub)} - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

$\varphi_{F(sub)}$, Φ_{GC}: $\Phi_{GC} = \varphi_{F(sub)} - \varphi_{F(gate)}$

$$\varphi_{F(sub)} = \frac{kT}{q}\ln\frac{n_i}{N_A} = 0.026\,V \ln\left(\frac{1.45\times10^{10}}{10^{16}}\right) = -0.35\,V$$

$$\varphi_{F(gate)} = \frac{kT}{q}\ln\frac{N_D}{n_i} = 0.026\,V \ln\left(\frac{2\times10^{20}}{1.45\times10^{10}}\right) = 0.60\,V$$

$$\Phi_{GC} = \varphi_{F(sub)} - \varphi_{F(gate)} = -0.35\ V - 0.60\ V = -0.95\ V$$

EXAMPLE 3-2 CONT.

$$V_{T0} = \Phi_{GC} - 2\varphi_{F(sub)} - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

Q_{B0}:

$$Q_{B0} = -\sqrt{2qN_A\varepsilon_{Si}|2\phi_{F(sub)}|}$$

$$= -\sqrt{2(1.6\text{x}10^{-19}\,\text{C})(10^{16}\,\text{cm}^{-3})(1.06\,\text{x}10^{-12}\,\text{Fcm}^{-1})|2\,\text{x}\,0.35\,\text{V}|}$$

F = C/V

$$= -4.87 \times 10^{-8}\ \text{C/cm}^2$$

C_{ox}:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{0.34\,\text{x}10^{-12}\,\text{Fcm}^{-1}}{500\,\text{x}10^{-8}\,\text{cm}} = 6.8\,\text{x}10^{-8}\,\text{F/cm}^2$$

Q_{ox}:

$$Q_{ox} = qN_{ox} = (1.6\,\text{x}10^{-19}\,\text{C})(4\,\text{x}10^{10}\,\text{cm}^{-2}) = 6.4\,\text{x}10^{-9}\,\text{C/cm}^2$$

$$\frac{Q_{B0}}{C_{ox}} = \frac{-4.87\,\text{x}10^{-8}\,\text{C/cm}^2}{6.8\,\text{x}10^{-8}\,\text{F/cm}^2} = -0.716\,\text{V} \qquad \frac{Q_{ox}}{C_{ox}} = \frac{6.4\,\text{x}10^{-9}\,\text{C/cm}^2}{6.8\,\text{x}10^{-8}\,\text{F/cm}^2} = 0.094\,\text{V}$$

$$V_{T0} = -0.95\,\text{V} - (-0.70\,\text{V}) - (-0.72\,\text{V}) - (0.09\,\text{V}) = 0.38\,\text{V}$$

EXAMPLE 3.3 Consider the n-channel MOS transistor with the following process parameters:

substrate doping density $N_A = 10^{16}\ cm^{-3}$,
polysilicon doping density $N_D = 2 \times 10^{20}\ cm^{-3}$,
gate oxide thickness $t_{ox} = 500$ Angstroms,
oxide-interface fixed charge density $N_{ox} = 4 \times 10^{10}\ cm^{-2}$.

In digital circuit design, the condition $V_{SB} = 0$ can not always be quaranteed for all transistors. Plot the threshold voltage V_T as a function of V_{SB}.

$$V_T = V_{T0} + \gamma\left(\sqrt{|2\phi_F - V_{SB}|} - \sqrt{|2\phi_F|}\right)$$

γ - Body-effect coefficient:

F = C / V

$$\gamma = \frac{\sqrt{2qN_A e_{Si}}}{C_{ox}} = \frac{\sqrt{2(1.6\times10^{-19}\,C)(10^{16}\,cm^{-3})(1.06\times10^{-12}\,F cm^{-1})}}{6.8\times10^{-8}\,F/cm^2}$$

$$= \frac{5.824\times10^{-8}\,C/V^{-1/2}cm^2}{6.8\times10^{-8}\,C/Vcm^2} = 0.85\,V^{1/2}$$

EXAMPLE 3-3 CONT.

$$V_T = V_{T0} + \gamma\left(\sqrt{|2\phi_F - V_{SB}|} - \sqrt{|2\phi_F|}\right)$$

where

$V_{T0} = 0.38V$ (from EX 3-2)

$\gamma = 0.85V^{1/2}$