DC techniques, dividers, and bridges

Purpose

You will gain a familiarity with the circuit board and work with a variety of DC techniques, including voltage dividers, the Wheatstone bridge, and 4-point measurement techniques. You also will gain a familiarity with the concept of input and output impedance.

Readings

D&H 1.9-1.13, 6.8-6.11

Theory

1. The Basic Wheatstone Bridge

Prior to very accurate DVM's, the Wheatsone bridge was widely used to determine an unknown resistance R_X by comparison with a precisely known standard R_S. It's operation illustrates many principles of DC circuits. In the basic bridge R_X, R_S, R_1, & R_2 are each >>0.1Ω so that all contact resistances can be ignored. R1 and R2 are the opposite legs of a potentiometer and can be continuously and accurately varied, until the null meter reads no current or voltage. Then, according to voltage divider rules, $Rs/(Rs+Rx) = R2/(R1+R2)$ and so Rx can be determined.

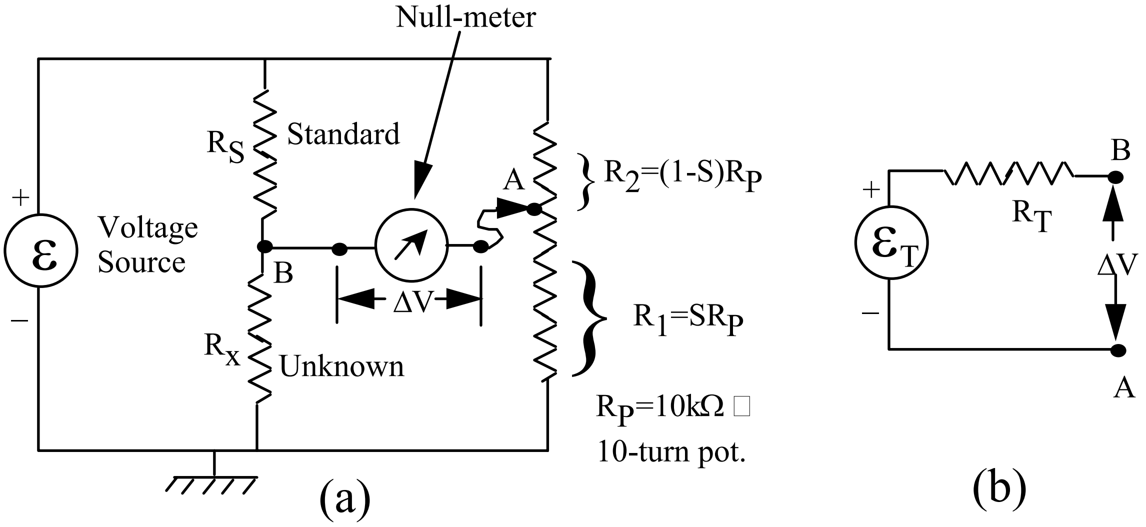

Fig. 2.1 a) Basic Wheatstone bridge. b) Thevinin equivalent circuit.

More generally, the operation of the bridge, both in the balanced and the unbalanced states, follows directly from its Thévenin equivalent and the equations:

$$\varepsilon_{\mathrm{T}} = \varepsilon\left(\frac{R_x}{R_x + R_s} - \frac{R_1}{R_1 + R_2}\right), \text{ and } R_T = \frac{R_s R_x}{R_s + R_x} + \frac{R_1 R_2}{R_1 + R_2}.$$

4 -Terminal Connections.

Typically, when we wish to measure the resistance of an element we can simply connect the element between the two leads of a DMM and read "Ohms". With the modern digital DMM this works quite well unless the resistance of the element to be measured is small - not very much higher than the resistances of the leads or contact resistances between the sample and the leads (probably less than an ohm for your leads). If the element's resistance is not much higher than the leads, then the lead resistance will make a sizeable impact on the measurement, skewing it badly. The way around this is to use the method of 4-terminal connections, in which the current leads are separate from the voltage leads. A schematic is shown in figure 2.2. A known large current passes into and out of the sample from CT1 to CT2. The voltage across the sample is measured at VT1 and VT2. Since there is essentially no current flowing through VT1 and VT2 there is also no voltage drop across them, even if they are of a sizeable resistance. Ohm's law is then used to determine the resistance of the sample.

The black dots CT. 1 and CT. 2 represent the current terminals through which heavy current flows into and out of the sample. The arrows VT. 1 and VT. 2 represent voltage terminals to which the voltmeter is lightly attached using spring loaded alligator clips. The sample length is the distance between the voltage terminals. Note that the contact resistance (about 0.1Ω) of the voltage terminals can be neglected since it appears in series with the 10 MΩ resistance of the DMM. Alternatively, the current terminals CT. 1 & CT. 2 are outside the voltmeter circuit. This is so that the IR drops in these contacts will not be measured by the voltmeter.

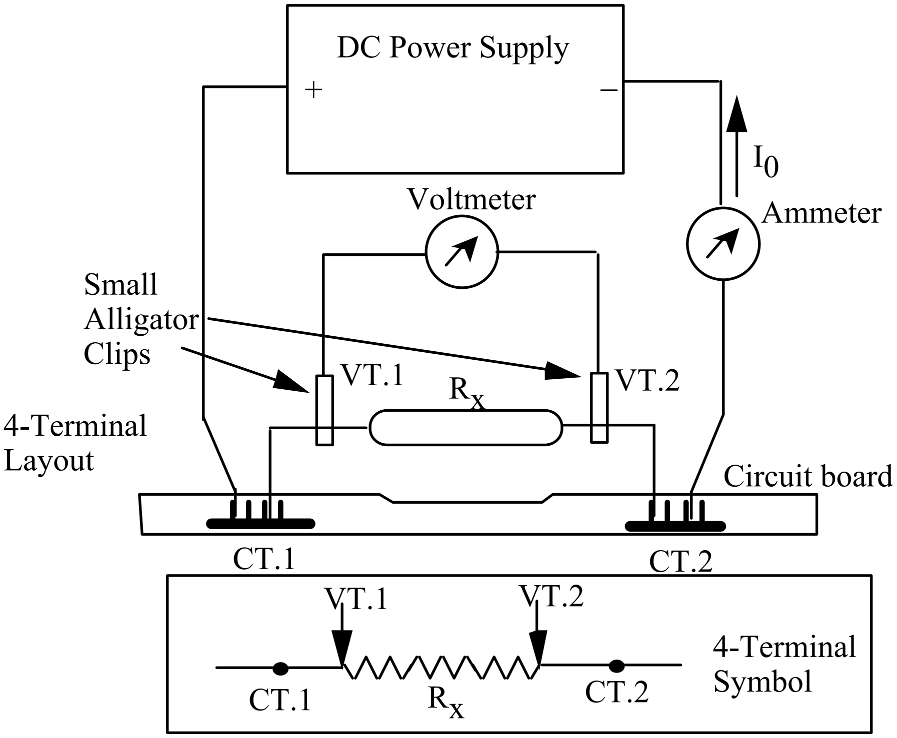

Fig 2.2 4 terminal connections: physical and symbolic

New Apparatus And Methods

CIRCUIT BOARDS (see figure 2.3)
Take a circuit board and write your name on it. You will use the same board all semester. An incomplete experiment can be left on the board and finished later. Store the board on the shelf labeled for your section. Components (resistors, capacitors, transistors, etc.) are available from the community stock. Take what components you need for the experiment. When it is over, stick them in a piece of foam or store them in a cardboard box for future use until the end of semester. Do not take a new component for an experiment unless you are sure you don't have it already. The complete circuit board contains a front panel, and a plug-in circuit board.

On the front panel, you will find:

- BNC cable sockets that carry electric signals between your circuit on the board and the function generator and oscilloscope.
- Colored banana jacks to bring in dc power for transistors or chips from an external power supply (+15 V red, -15 V blue, +5 V orange, and 0 V black).
- A precision 10 k? ten-turn potentiometer (linearity ± 1/4%), and several switches.

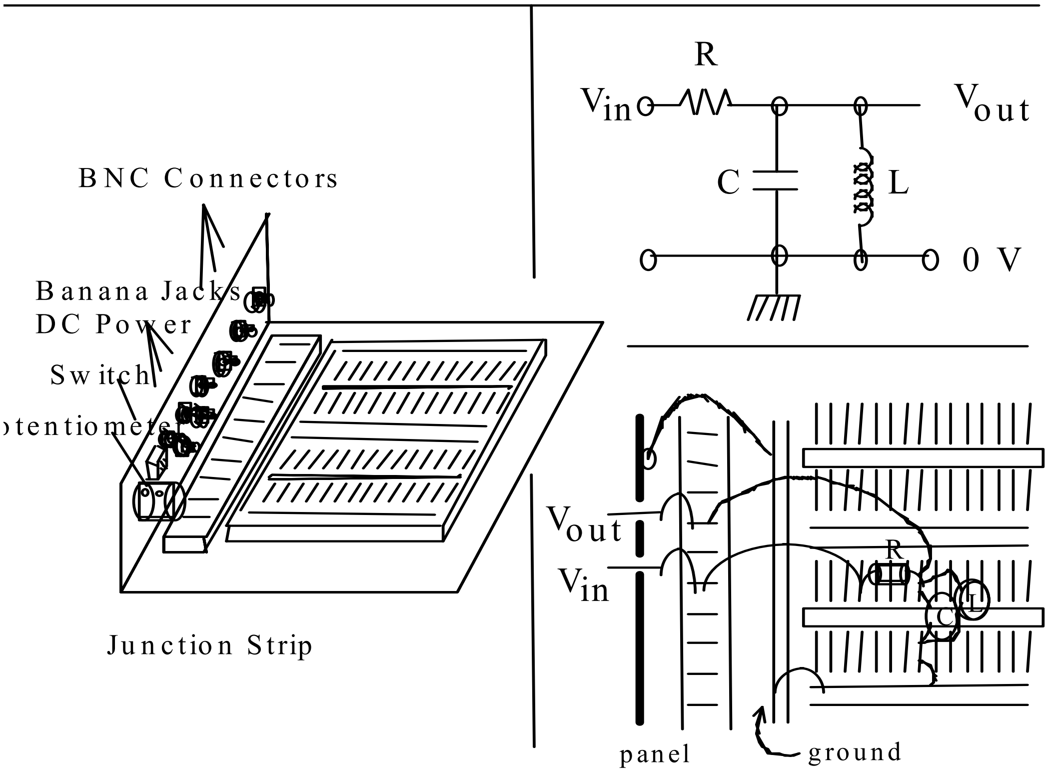

Figure 2.3

The circuit board contains arrays of holes, interconnected by buried conductors, into which components are plugged to build your circuit.

- The long lines of connected holes are used for power lines (+15 V red and -15 V blue) and ground lines (0 V black). They are never used for signals.
- The 5 holes in each short group at right angles to the long lines are interconnected, but separate from every other group of 5. A given short group is used to make the junction between two or more components of your circuit.
- There are four color coded binding posts on the circuit board. They are wired to the assiciated banana jacks on the front panel.
- Good electric contact is essential when you plug in components or wires. Use only 22 or 24 gauge solid wire, not stranded wire. Push in each wire with thin nosed pliers until you feel the contacts grip. A common fault is to plug in enamel insulated wire, used for winding inductors. First burn off the enamel with a lighter flame for 1/2" at the end and clean with emery paper. Contact resistances should measure less than one third of an ohm.
- If you get confused about which holes are connected together by buried conductors, explore the board with an ohm meter.

Reliable ground connections (0 V), readily accessible from any point on the board, are essential to the good functioning of most circuits. The front panel is the ground for your circuit board since the coax cable shields connect the front panel of your circuit board to the ground of other instruments in your experiment.

Problems - Turn these in to your instructor before the start of your lab!

1. Voltage dividers. An ideal voltage source drives current around the loop of resistors shown in Figure 2.4(a). Find a formula for the current I and the voltage V_{out}. What is V_{out} if V = 10 V, $R_1 = 1\ \mathrm{k\Omega}$, and $R_2 = 100\ \Omega$? For these component values, what is the Thévenin equivalent circuit? Also calculate the voltage V_{out} for the modified circuit shown in Figure 2.4(b) with $R_3 = 200\ \Omega$ and the other components unchanged. What is the Thévenin equivalent circuit now

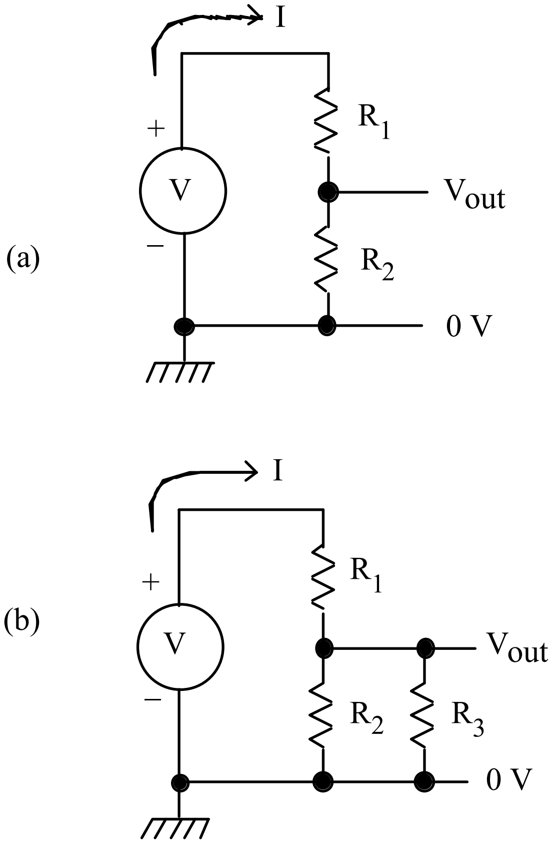

Figure 2.4 Voltage dividers

2. What is the diameter of a typical pure metal wire 6-ft long if its resistance is (at room temperature) is 0.1 Ω? Assume that the resistivity of typical pure metals at room temperature is about $2 \times 10^{-8}\ \Omega$-m. What is the maximum error in measuring the diameter that you can tolerate if the resistivity is to have a 1% uncertainty?

3. The American standard wire gauge gives a logarithmic measure of the diameter of a wire,

AWG gauge = $30 - 20\ \mathrm{Log}_{10}(d/0.01'')$.

Wires usually are available only in even gauges. What gauge should you use for the 6-ft, 0.1Ω wire?

4. Consider the basic Wheatstone bridge of Fig 2.2, Assume that the bridge has been balanced so that a resistance R_{xo} gives a potentiometer setting S_o. Now leave the potentiometer alone, but let the unknown resistance change by a small amount ΔR_x. Show that, as a consequence, the null-meter will measure an (approximate) unbalanced potential difference

$$\Delta V = \varepsilon\left(\Delta R_x / R_{x0}\right)\left(1 - S_0\right)S_0 \qquad (3)$$

What value of S_o gives you maximum sensitivity, $\Delta V/\Delta R_x$? What does your answer imply about choosing a standard resistance if you know the approximate value of the unknown resistance?

Experiment

1. Build three copies of the simple voltage divider circuit of figure 2.4a, with both resistors approximately equal to each other. In the first circuit use relatively low resistances of about 1 kΩ. In the second use resistances near 1 MΩ, and in the third use resistors of 10 MΩ or more. Measure each resistor with your DVM before inserting them into your circuit. Apply a voltage to the input and measure the output voltage using both your DVM and your oscilloscope. Record your findings. What does this tell you about the input impedance of your devices? What is the ideal input impedance?

2. In preparation for a Wheatstone bridge experiment, accurately measure the resistance of your potentiometer for a variety of settings. Determine the precision and linearity of this instrument.

3. Build the basic Wheatstone bridge on your circuit board. Validate the null procedure by using the balanced bridge to compare a cheap (10%) 1 kΩ resistor as R_X with a 1 kΩ, 1% resistor as the standard R_S. Compare your result with a measurement of the cheap resistor with a DMM.

4. Test the unbalanced bridge equation (Eq. 3) by first using a cheap 820 Ω as R_X, then repeat with $R_X = 1.2$ kΩ.

5. Cut a 6-ft length of copper wire of a diameter such that the resistance is likely to be about 0.1 Ω. Define a "sample" length with two pieces of masking tape. See if you can measure the resistance of the sample with a digital multimeter (DMM).

6. Determine the resistivity of copper by measuring the resistance of the sample and its dimensions. Use 4-terminal connections (Fig 2.2) and measure the voltage drop along the sample to find the resistance. When measuring the diameter of the wire remember that the thickness of the enamel coating is not completely negligible.

7. Optional: Measure the temperature variation of the resistivity of copper $\rho(T)$ between water ice and boiling points and a couple of points between.