Frequency Response of Transistor Amplifiers

for
Electronic Circuits

http://cktse.eie.polyu.edu.hk/eie304

by
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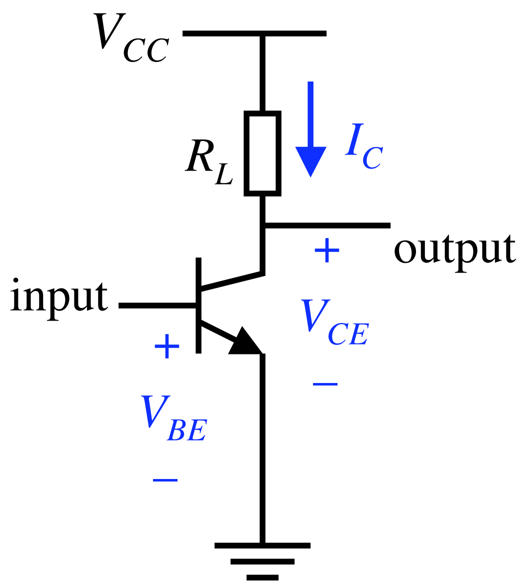
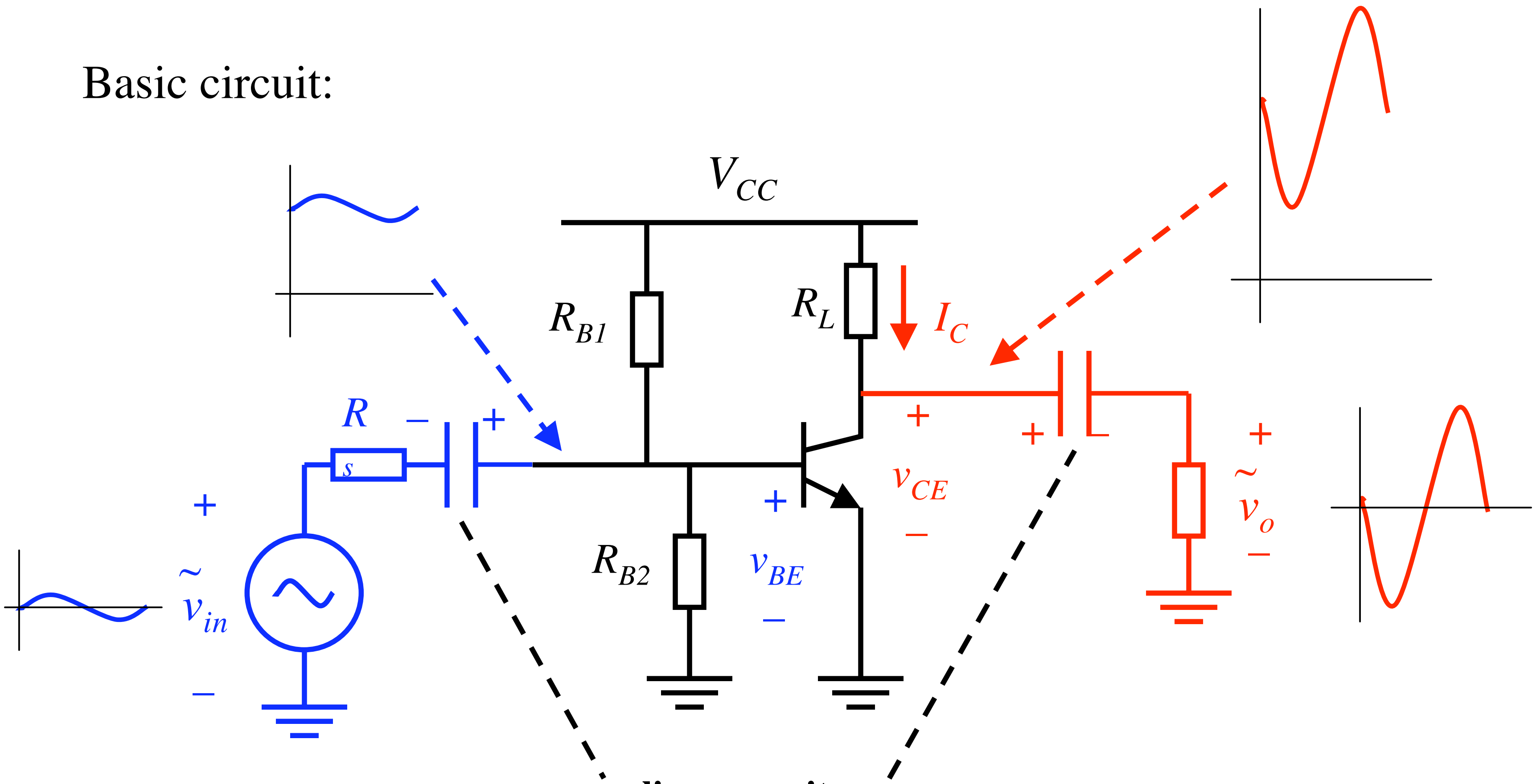
Introduction

A convenient starting point is the common-emitter amplifier. Our understanding of this amplifier (up to EC1) is that it can provide a fair amount of voltage gain. We apply a small input at the base and we expect to get a pretty wide swing at the collector side. That's amplification. But, so far, we have assumed that the signals are not of very high frequency such that the parasitic capacitances in the transistor do not have any significant effects. Remember that the impedance of a capacitor is inversely proportional to $1/\omega C$. So, if C is very small, it is practically an open-circuit at moderately low frequency. But as soon as we increase the signal frequency, the capacitor becomes less and less like an open-circuit, and in fact its impedance begins to drop. Therefore, at high frequencies, we must take into account the presence of parasitic capacitances in order to get a fuller picture of the voltage gain.



Review of common-emitter configuration

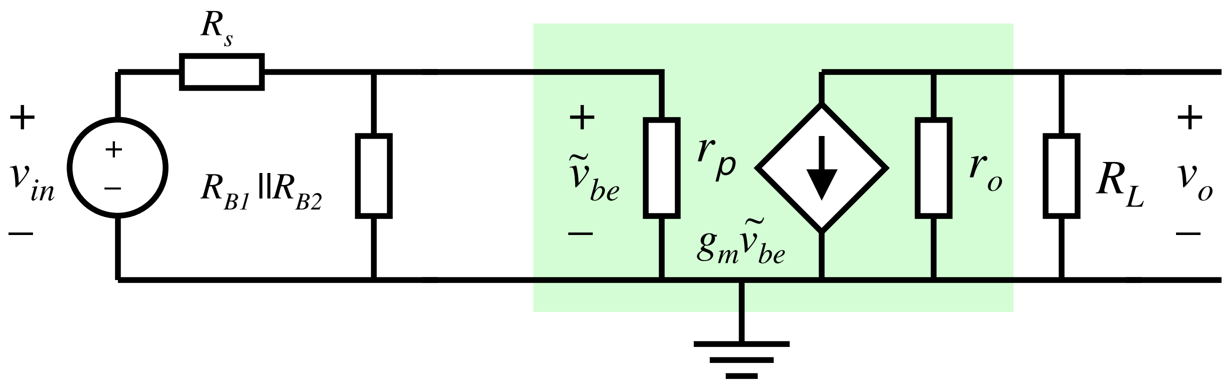
Basic circuit:



coupling capacitors
(large enough so that they become short-circuit at signal frequencies)

Review of common-emitter configuration

Small-signal model for low-frequency operation:

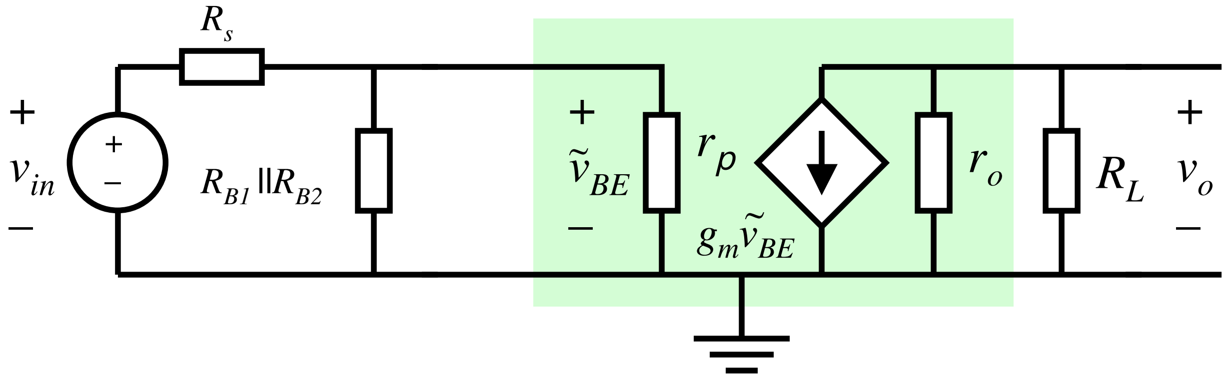


Small-signal parameters of BJT:

transconductance i_c/v_{be}	$g_m = \frac{q}{kT} I_c = \frac{I_c}{V_T}$	$\approx \frac{I_c}{25\text{mV}}$ at room temperature
current gain i_c/i_b	β	
input resistance of BJT v_{be}/i_b	$r_\pi = \frac{\beta}{g_m}$	**This is not an ohmic resistor!**
output resistance of BJT	$r_o = \frac{V_A}{I_c}$	where V_A is Early voltage of the BJT

Review of common-emitter configuration

Small-signal model for low-frequency operation:



Parameters of CE amplifier:

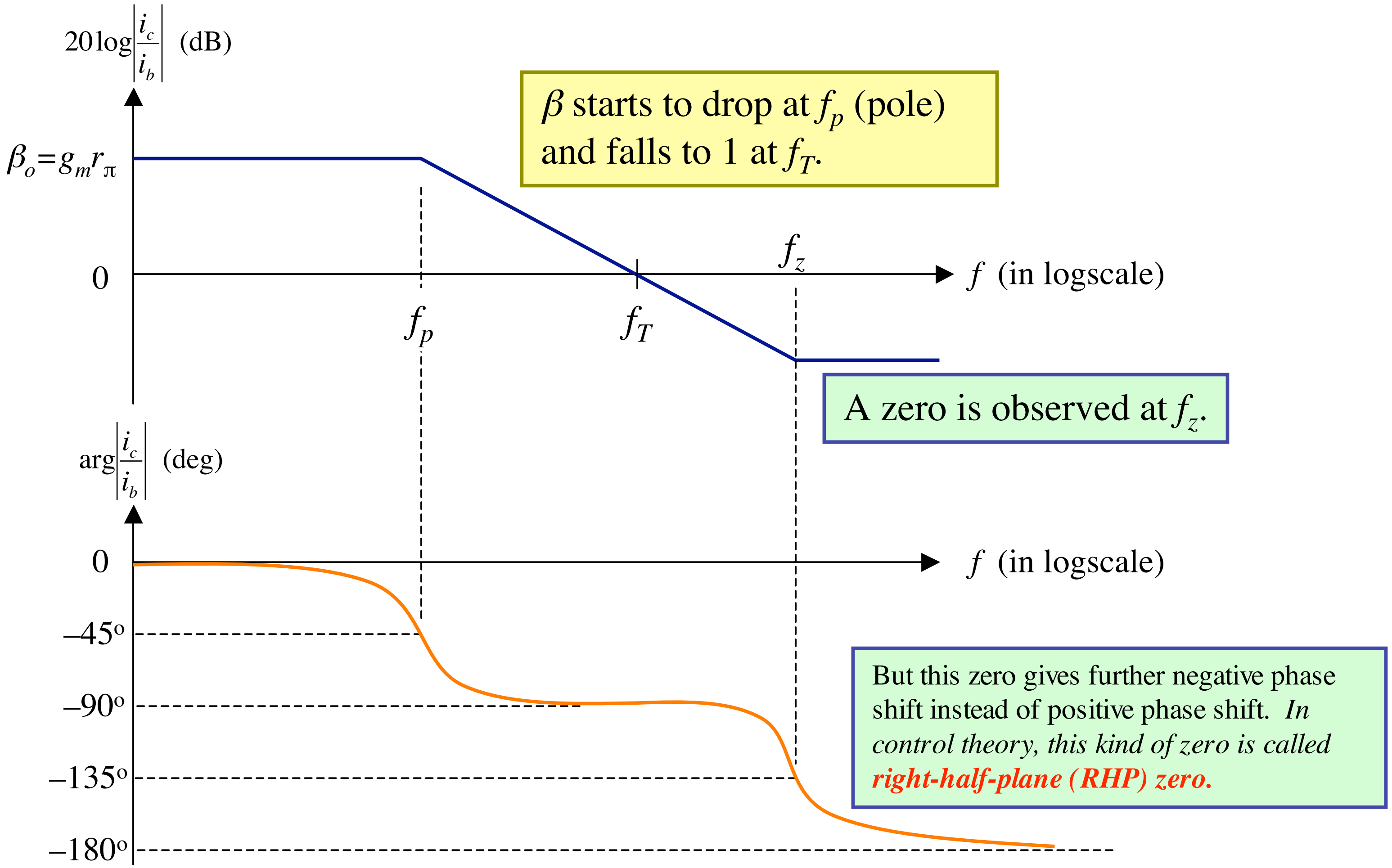
$$\text{Gain} = \frac{v_o}{v_{in}} = \left[\frac{R_{B1} \,||\, R_{B2}}{(R_{B1} \,||\, R_{B2}) + R_s}\right]\left[g_m (R_L \,||\, r_o)\right] \approx g_m R_L \quad \text{if } R_s \approx 0 \text{ and } r_o \to \infty$$

Input resistance = $R_{in} = R_{B1} || R_{B2} || r_\pi$

Output resistance = $R_o = R_L || r_o$

What happens when frequency increases?

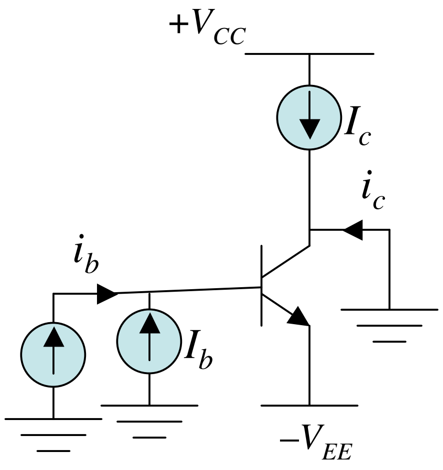
Let's measure the **current gain** β which is just $|i_c/i_b|$.



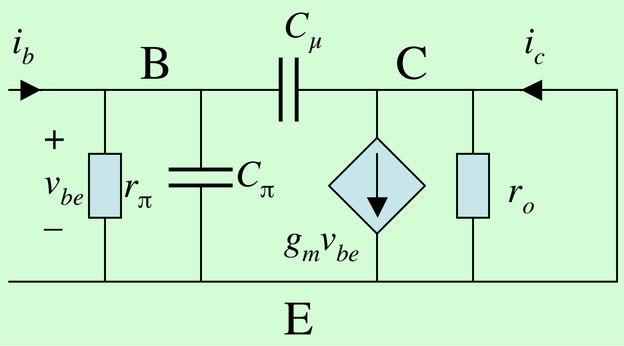
Why?

Let's look at the circuit.

We omit the biasing circuit, and focus on the short-circuit current gain.



Small-signal model:



We insert a parasitic capacitor across each pair of terminals of the BJT. But since the output is shorted, we can omit the one across C and E in this case.

Nodal equation of the B node:

$$\frac{v_{be}}{r_\pi} + \frac{v_{be}}{(1/j\omega C_\pi)} + \frac{v_{be}}{(1/j\omega C_\mu)} - i_b = 0$$

$$\Rightarrow \quad v_{be} = \frac{i_b}{\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)}$$

The current i_c, from KCL, is

$$i_c = g_m v_{be} - v_{be} j\omega C_\mu = v_{be}\left(g_m - j\omega C_\mu\right)$$

Hence, we get

$$\frac{i_c}{i_b} = g_m r_\pi \left[\frac{1 - j\omega \frac{C_\mu}{g_m}}{1 + j\omega\left(C_\mu + C_\pi\right) r_\pi}\right]$$

Current gain at high frequencies

From the above analysis, we see that the current gain β has a pole and a RHP zero.

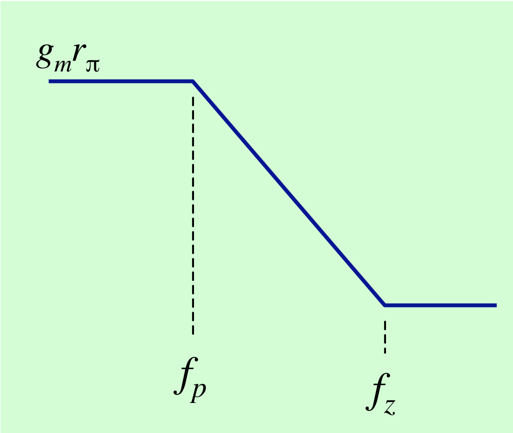
$$\beta(j\omega) = \frac{i_c}{i_b} = g_m r_\pi \left[\frac{1 - j\omega \frac{C_\mu}{g_m}}{1 + j\omega\left(C_\mu + C_\pi\right) r_\pi} \right]$$

This is consistent with the frequency response of $|i_c/i_b|$ we saw earlier.

The pole frequency is $f_p = \dfrac{1}{2\pi\left(C_\pi + C_\mu\right) r_\pi}$ (in Hz)

The zero frequency is $f_z = \dfrac{g_m}{2\pi C_\mu}$ (in Hz)

The low-frequency current gain is $\beta_o = g_m r_\pi$



Transition frequency (unity current gain frequency)

We can find f_T by setting $|\beta(j\omega_T)| = 1$.

$$1 = g_m r_\pi \left| \frac{1 - j\omega_T \frac{C_\mu}{g_m}}{1 + j\omega_T (C_\mu + C_\pi) r_\pi} \right| \quad \Rightarrow \quad 1 = (g_m r_\pi)^2 \left(\frac{1 + \omega_T^2 \left(\frac{C_\mu}{g_m}\right)^2}{1 + \omega_T^2 (C_\mu + C_\pi)^2 r_\pi^2} \right)$$

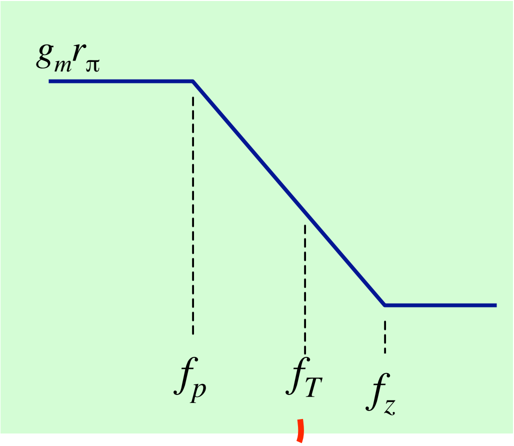
$$1 + \omega_T^2 (C_\mu + C_\pi)^2 r_\pi^2 = (g_m r_\pi)^2 + \omega_T^2 \left(\frac{C_\mu}{g_m}\right)^2$$

$$\omega_T^2 = \frac{(g_m r_\pi)^2 - 1}{(C_\mu + C_\pi)^2 r_\pi^2 - \left(\frac{C_\mu}{g_m}\right)^2}$$

$$\approx \frac{(g_m r_\pi)^2}{(C_\mu + C_\pi)^2 r_\pi^2}$$

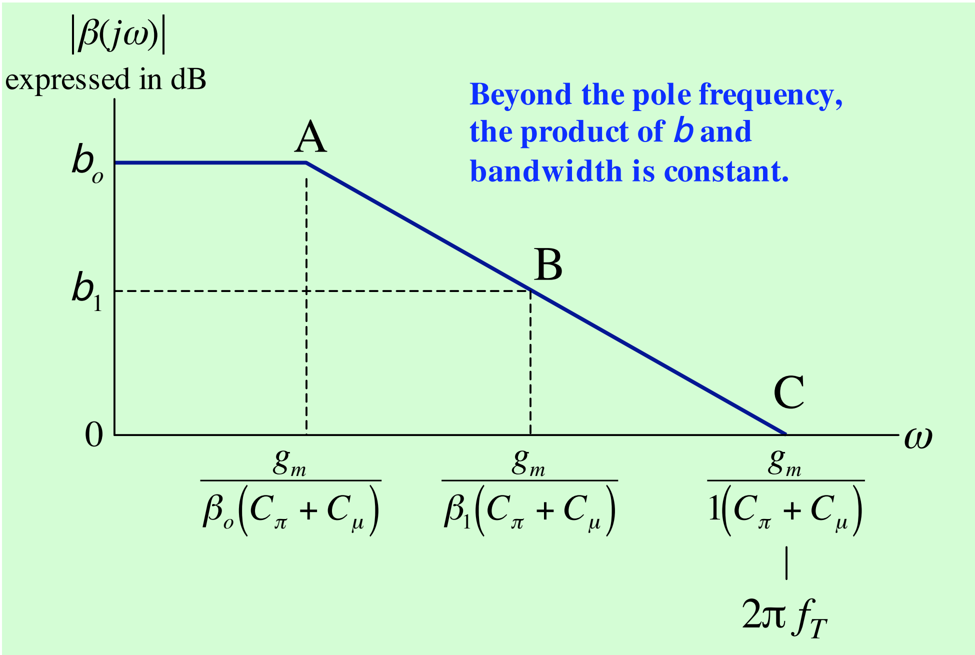
$$\omega_T \approx \frac{g_m}{(C_\mu + C_\pi)} \quad \text{or} \quad f_T = \frac{g_m}{2\pi (C_\mu + C_\pi)}$$

transition frequency



Principle of gain-bandwidth product

There is an interesting property about $\beta(j\omega)$.



A: $\beta_o \times \frac{g_m}{\beta_o(C_\pi + C_\mu)} = \frac{g_m}{(C_\pi + C_\mu)}$

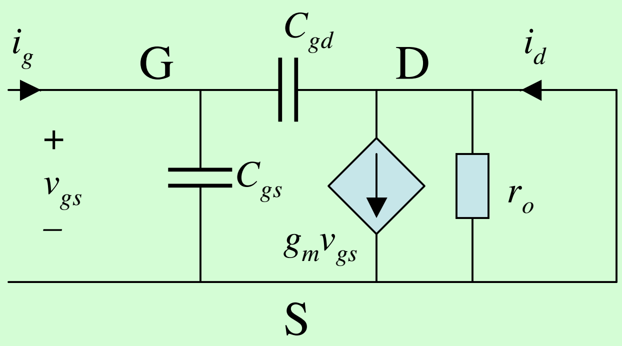
B: $\beta_1 \times \frac{g_m}{\beta_1(C_\pi + C_\mu)} = \frac{g_m}{(C_\pi + C_\mu)}$

C: $1 \times \frac{g_m}{1(C_\pi + C_\mu)} = \frac{g_m}{(C_\pi + C_\mu)} = 2\pi f_T$

The transition frequency f_T is a very important parameter for studying the frequency response of a BJT amplifier. It tells us how β (and r_π) changes as frequency increases.

How about MOSFET ?

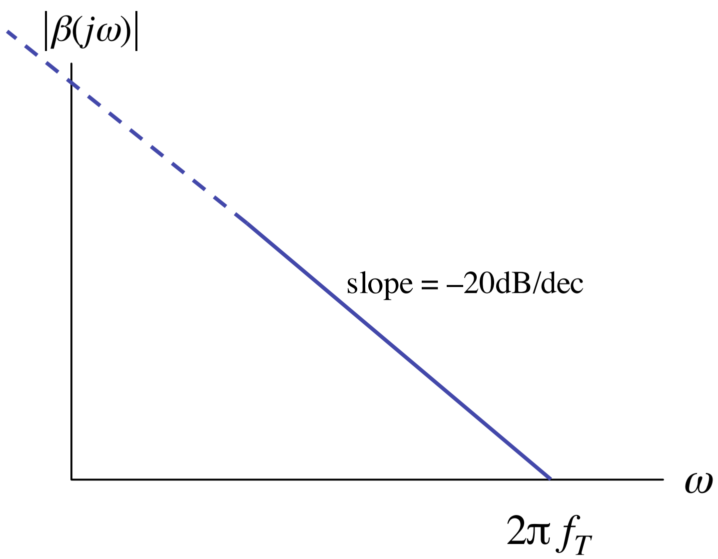
Small-signal model:



The main difference between MOSFET and BJT is that there is no r_π in the model of MOSFET or $r_\pi = \infty$.

We can easily find the transfer function of the current gain as:

$$\beta(j\omega) = \frac{i_d}{i_g} = g_m \left[\frac{1 - j\omega \frac{C_{gd}}{g_m}}{j\omega\left(C_{gs} + C_{gd}\right)} \right]$$



The transition frequency is given by

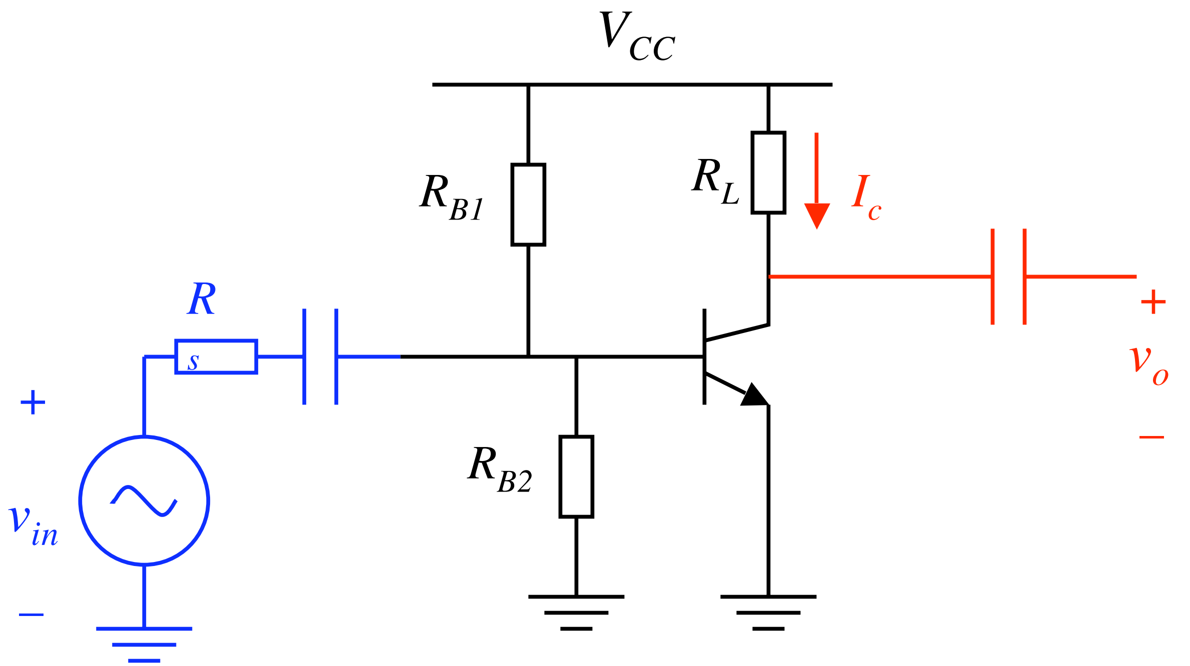
$$\omega_T = \frac{g_m}{\left(C_{gs} + C_{gd}\right)} \approx \frac{g_m}{C_{gs}}$$

since C_{gd} is much smaller than C_{gs}.

The gain-bandwidth product principle is valid at any frequency.

Frequency response of CE amplifier

We consider the CE amplifier again, but this time, as a voltage amplifier.

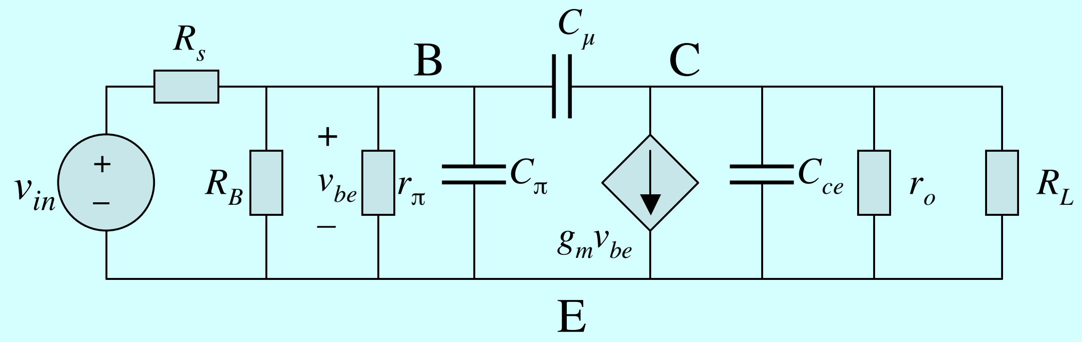


Recall that the voltage gain ay low frequencies is

$$\frac{v_o}{v_{in}} = \left[\frac{R_{B1} \parallel R_{B2}}{(R_{B1} \parallel R_{B2}) + R_s}\right]\left[g_m(R_L \parallel r_o)\right] \approx g_m R_L \quad \text{if } R_s \approx 0 \text{ and } r_o \to \infty$$

Frequency response of CE amplifier

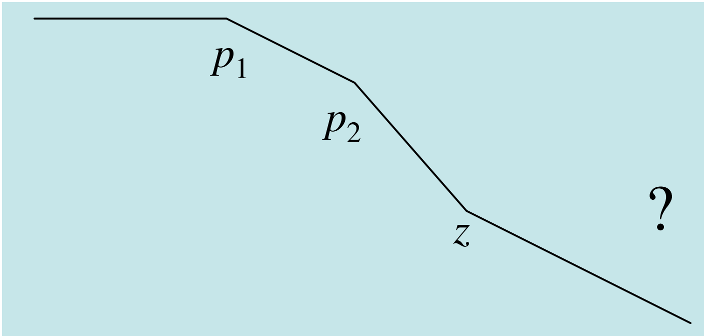
Complete small-signal model:

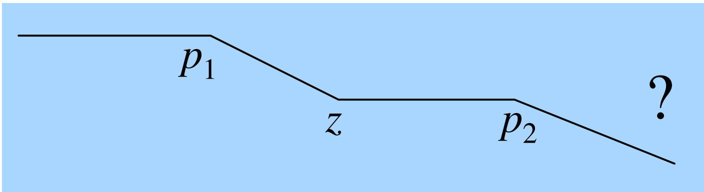


Questions

How many poles?
How many zeros?
Where are the poles and zeros?

Most important is the first dominant pole p_1 which limits the gain!





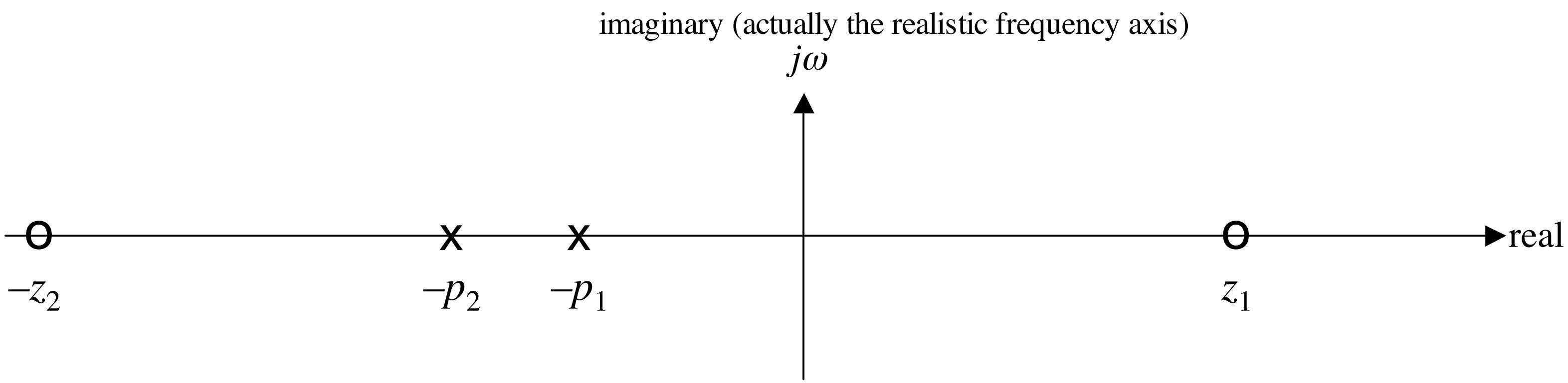
What are poles and zeros? (Year 1 material)

Theory (if you still remember!)

On the complex number plane, a pole is the point where the output response is ∞ even when the input is finite. Also, a zero is the point where the output response is 0 even when the input is finite.

All poles must be on LHS of the complex plane. Otherwise, the circuit is unstable.
But zeros can be anywhere.

For circuits with ONLY ONE type of reactive elements (capacitors only), all poles and zeros are real numbers. Thus, **all poles are LHS real numbers. [Some textbooks use the term *negative real poles* instead of LHS real poles!]**



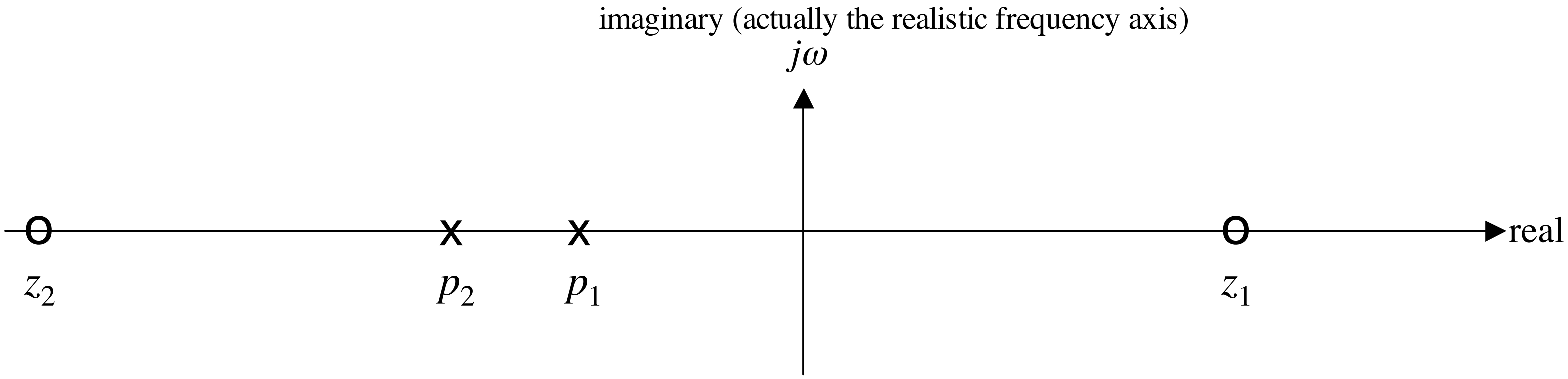
What are poles and zeros? (Year 1 material)

In transfer function forms, we write

$$F(j\omega) = A_o \frac{\left(1+\frac{j\omega}{z_2}\right)\left(1-\frac{j\omega}{z_1}\right)}{\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)}$$

where A_o is the low-frequency gain.

Careful! Watch the sign for the RHP zero!

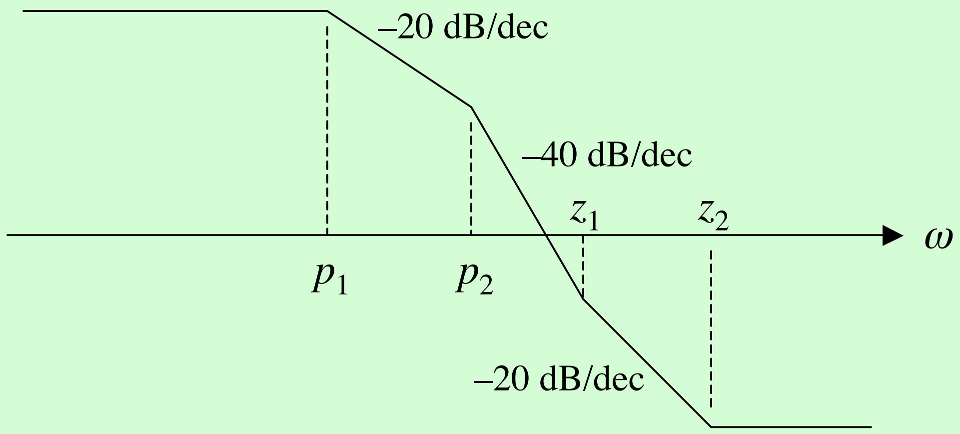


What are poles and zeros? (Year 1 material)

In practice

We look at the imaginary axis, which is actually the realistic frequency axis.

A pole [or zero] becomes just a corner point on the realistic frequency axis where the response starts to fall [rise] at a rate of 20 dB/dec.



We know all poles are LHS poles, but zeros can be LHS or RHS.

What is the difference between a RHS zero and a LHS zero? To see the difference, we have to look at the phase.

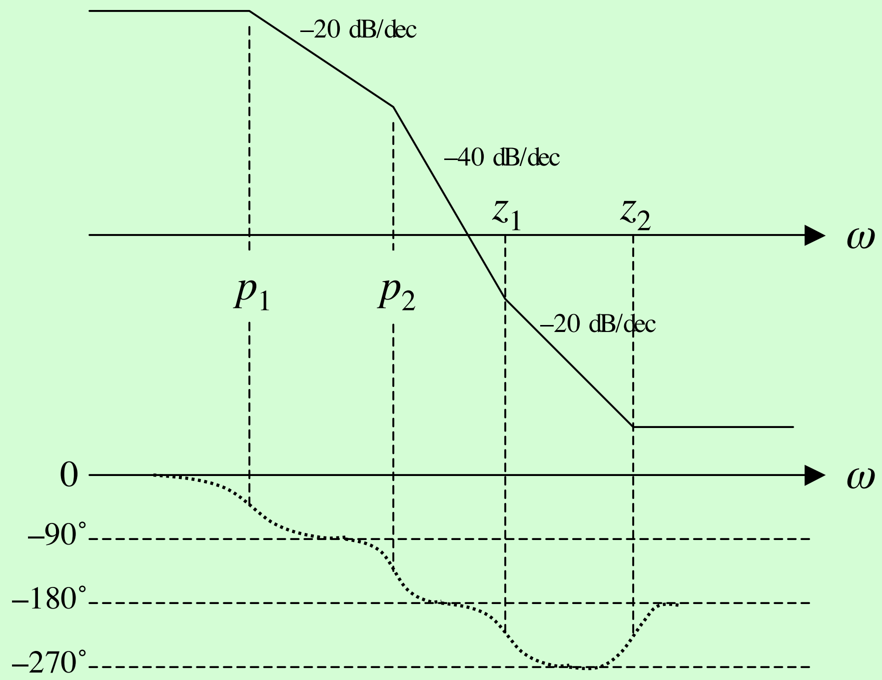
What are poles and zeros? (Year 1 material)

Phase shift

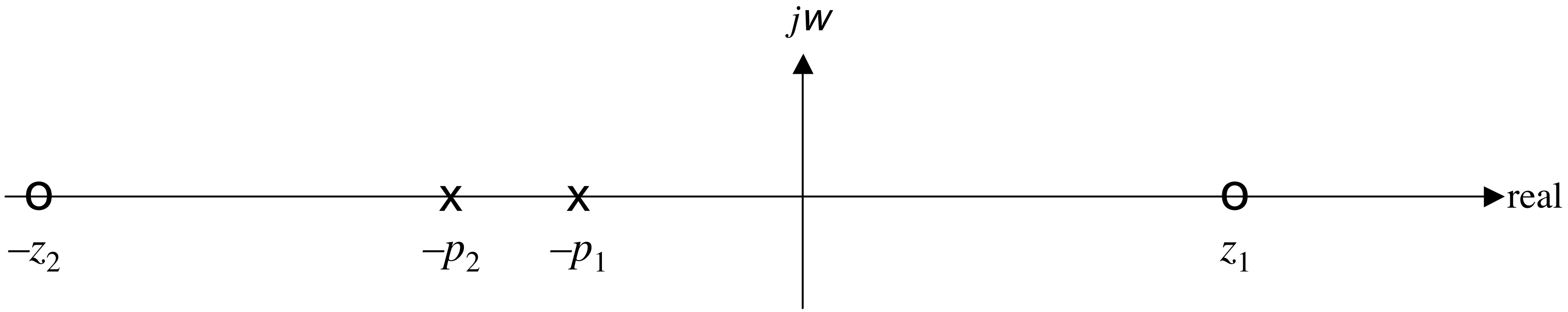
A LHS pole comes with negative phase shift.
One LHS pole gives –90˚.

A LHS zero comes with positive phase shift.
One LHS zero gives +90˚.

A RHS zero comes with **negative** phase shift.
One RHS zero gives –90˚.

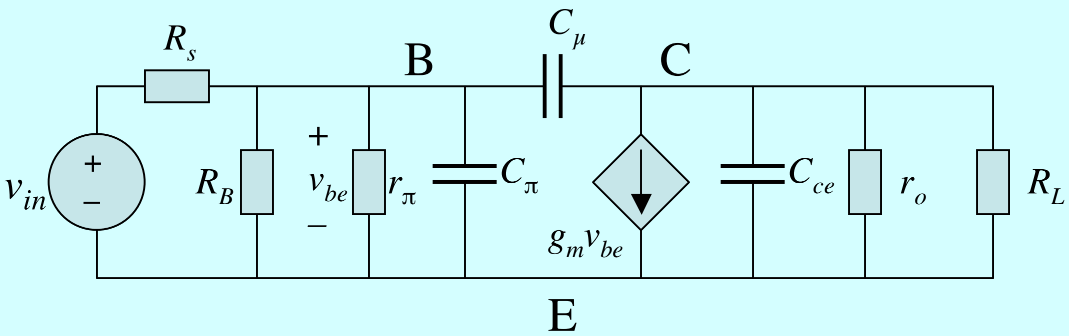


NOTE: z_1 is a RHP zero, and z_2 is a LHP zero.



Frequency response of CE amplifier

Complete small-signal model:



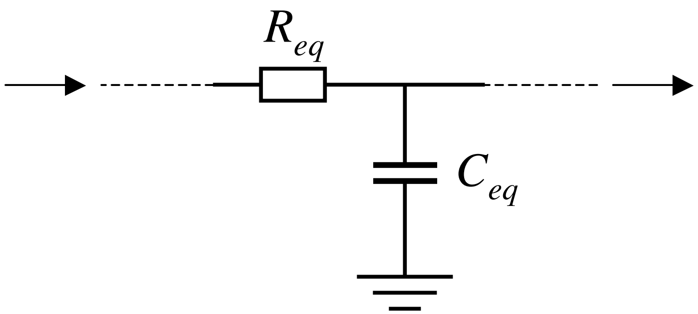
For this circuit, there are two independent capacitors, hence the circuit is second order. Hence, there must be TWO LHS poles. Moreover, there is at least ONE zero due to C_μ because $g_m v_{be}$ can pull all its current through C_μ resulting in zero current flowing to R_L. Let's find the frequency where this happens!

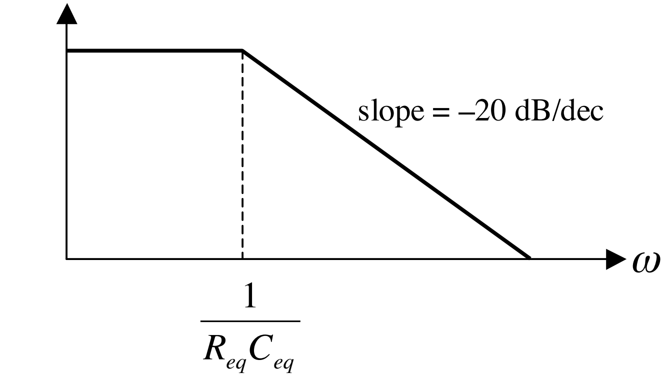
The low-frequency gain:

$$\frac{v_o}{v_{in}} = \left[\frac{R_{B1} \| R_{B2}}{(R_{B1} \| R_{B2}) + R_s}\right]\left[g_m(R_L \| r_o)\right]\frac{\left(1 \pm \frac{j\omega}{z}\right)}{\left(1 + \frac{j\omega}{p_1}\right)\left(1 + \frac{j\omega}{p_2}\right)}$$

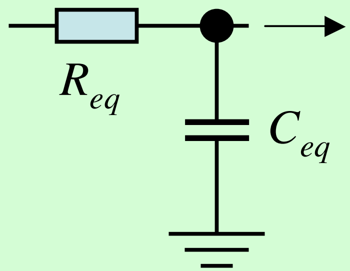
What is a pole?   A very simple approach!

To find the poles, we first recall that the roll-off is typically a result of an equivalent low-pass RC filter.



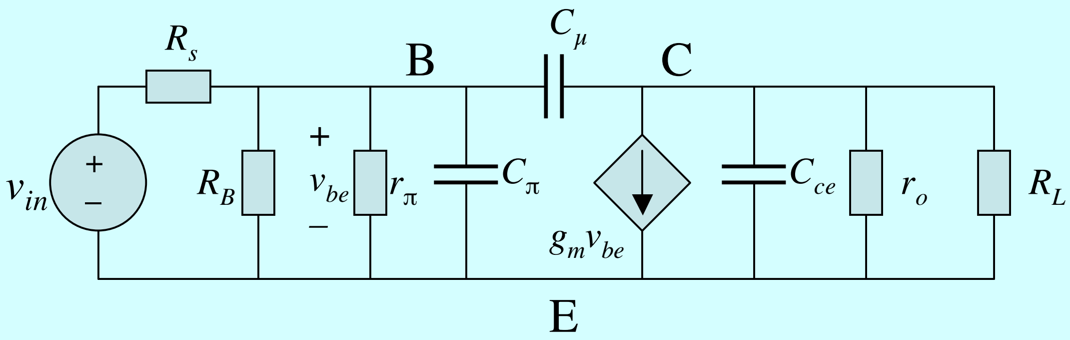


Basically, if we see a node along the signal path which has
(i)   a substantial resistance toward the input side; and
(ii)   a substantial capacitance to ground,
then there will be a pole!



Finding poles of CE amplifier

Complete small-signal model:



To find the poles, we examine the two nodes B and C. Our aim is to find the equivalent RC filters at B and C.

First consider node B. We can assume that (before roll-off) the current source $g_m v_{be}$ flows in the output load $R_L // r_o$ such that

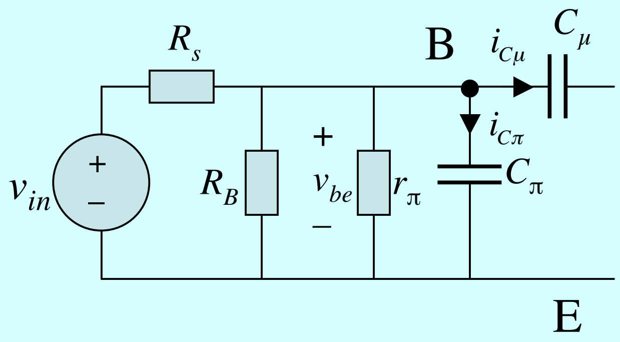
$$v_{CE} = -g_m(R_L \parallel r_o)v_{be}$$

So, the voltage across the capacitor C_μ is

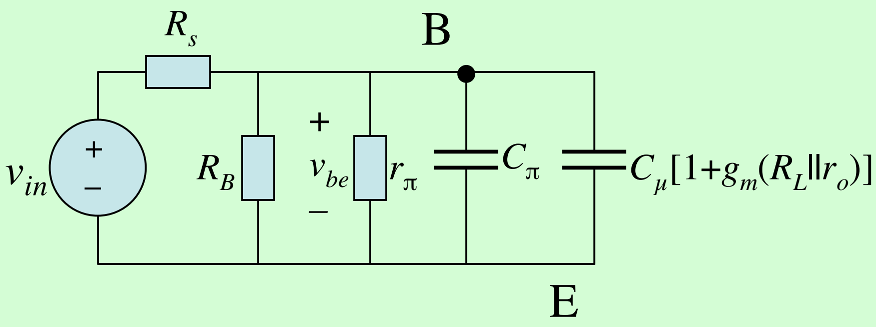
$$v_{BC} = v_{be}[1 + g_m(R_L \parallel r_o)]$$

Finding the first pole of CE amplifier

Consider node B:



Equivalent model at node B:



Clearly, at node B, we can see that

(i) capacitor C_μ is pumping current equal to $i_{C\mu} = j\omega C_\mu v_{be}[1 + g_m(R_L \| r_o)]$

(ii) capacitor C_π is pumping current equal to $i_{C\pi} = j\omega C_\pi v_{be}$

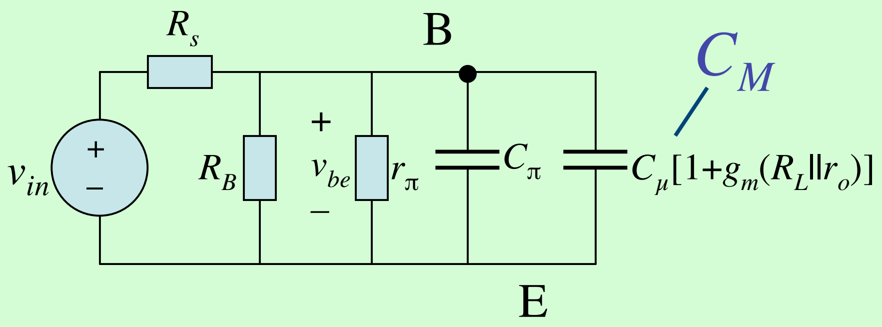
They combine to pump current equal totally to

$$i_{C\mu} + i_{C\pi} = j\omega C_\mu v_{be}[1 + g_m(R_L \| r_o)] + j\omega C_\pi v_{be}$$
$$= j\omega\left\{C_\mu\left[1 + g_m(R_L \| r_o)\right] + C_\pi\right\}v_{be}$$

So, we can put one equivalent capacitor to node B: $C_{eq} = C_\mu\left[1 + g_m(R_L \| r_o)\right] + C_\pi$

First pole is due to Miller

Equivalent model for forward signal flow:



Observe that C_μ has been EXPANDED by a factor of $[1+g_m(R_L||r_o)]$ which is just the dc gain.

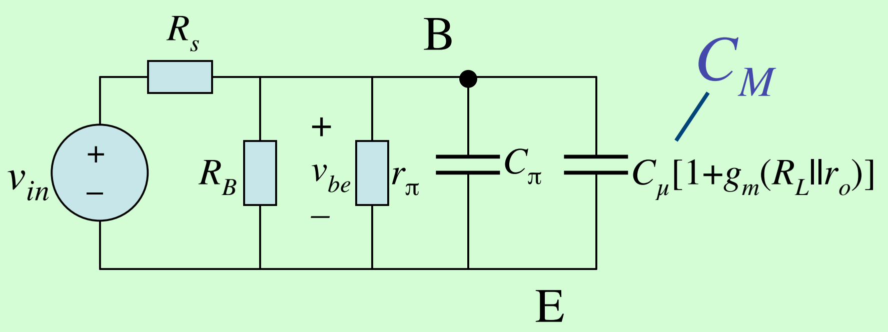
This is called Miller effect.

The expanded capacitor at node B to ground is called Miller capacitor:

$$C_M = C_\mu[1+g_m(R_L||r_o)]$$

First pole of CE amplifier

Equivalent model for forward signal flow:



A pole can be found from the equivalent low-pass filter at node B. **This is the Miller effect pole.** Clearly, the equivalent R and C are

$$C_{eq} = C_\pi + C_M = C_\pi + C_\mu[1+g_m(R_L||r_o)]$$
$$R_{eq} = R_s \,||\, R_B \,||\, r_\pi$$

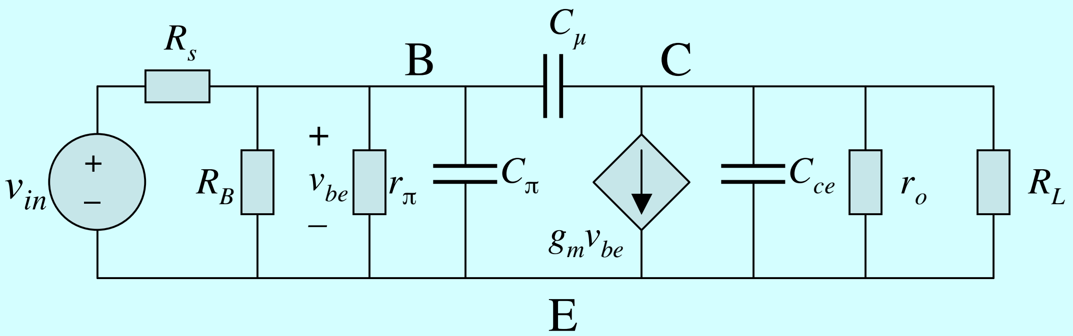
This pole is

$$p_1 = \frac{1}{C_{eq}R_{eq}} = \frac{1}{\left\{C_\pi + C_\mu\left[1+g_m\left(R_L \,||\, r_o\right)\right]\right\}\left(R_s \,||\, R_B \,||\, r_\pi\right)}$$

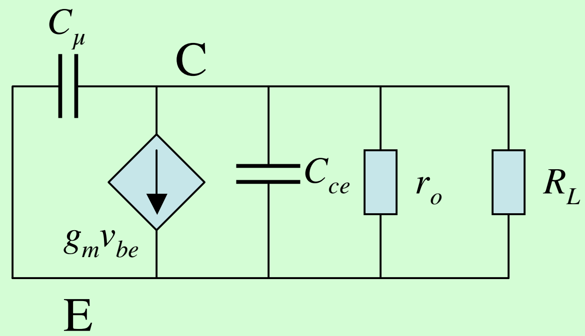
This is a normal LHP pole.

Finding the second pole of CE amplifier

Complete small-signal model:



Equivalent model at node C:



Next consider node C. We can assume that the signal at B looking from C is *comparatively* small and that node B is essentially grounded. Therefore, we can approximate that C_μ goes to ground at C.

Clearly, the combined capacitance to ground is $C_\mu + C_{ce}$.

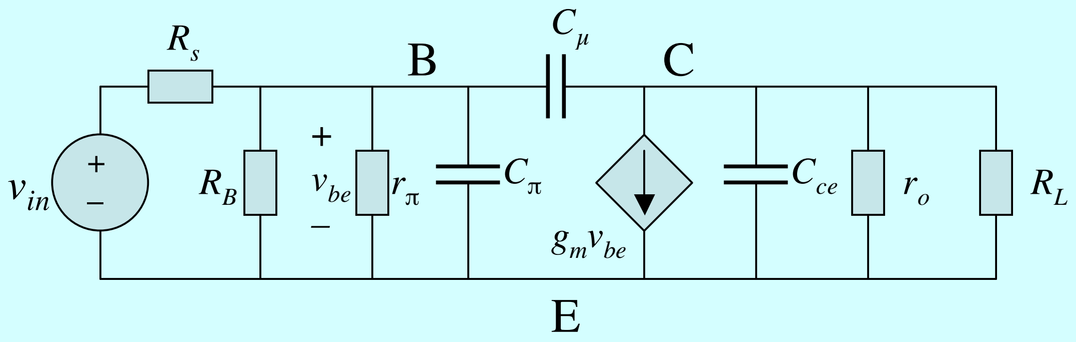
Also, since the equivalent resistance is $R_L \parallel r_o$, the pole is

$$p_2 = \frac{1}{(C_\mu + C_{ce})(R_L \parallel r_o)}$$

This is a normal LHP pole.

Finding the zero of CE amplifier

Complete small-signal model:



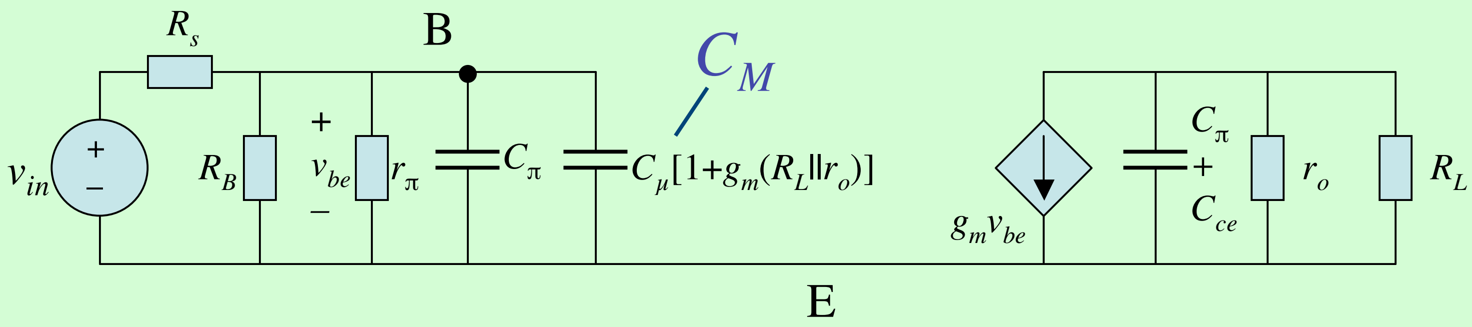
To find the zero, we can simply try to find the frequency (on complex plane) where the response is 0. This is quite easy! Almost by inspection, we have, when output is 0,

$$v_{bc} = v_{be} \quad \Rightarrow \quad \frac{g_m v_{be}}{sC} = v_{be} \quad \Rightarrow \quad s = \frac{g_m}{C}$$

So, the zero is $+g_m/C$, **which is a RHP zero.**

Complete transfer function of CE amplifier

Equivalent model for forward signal flow:



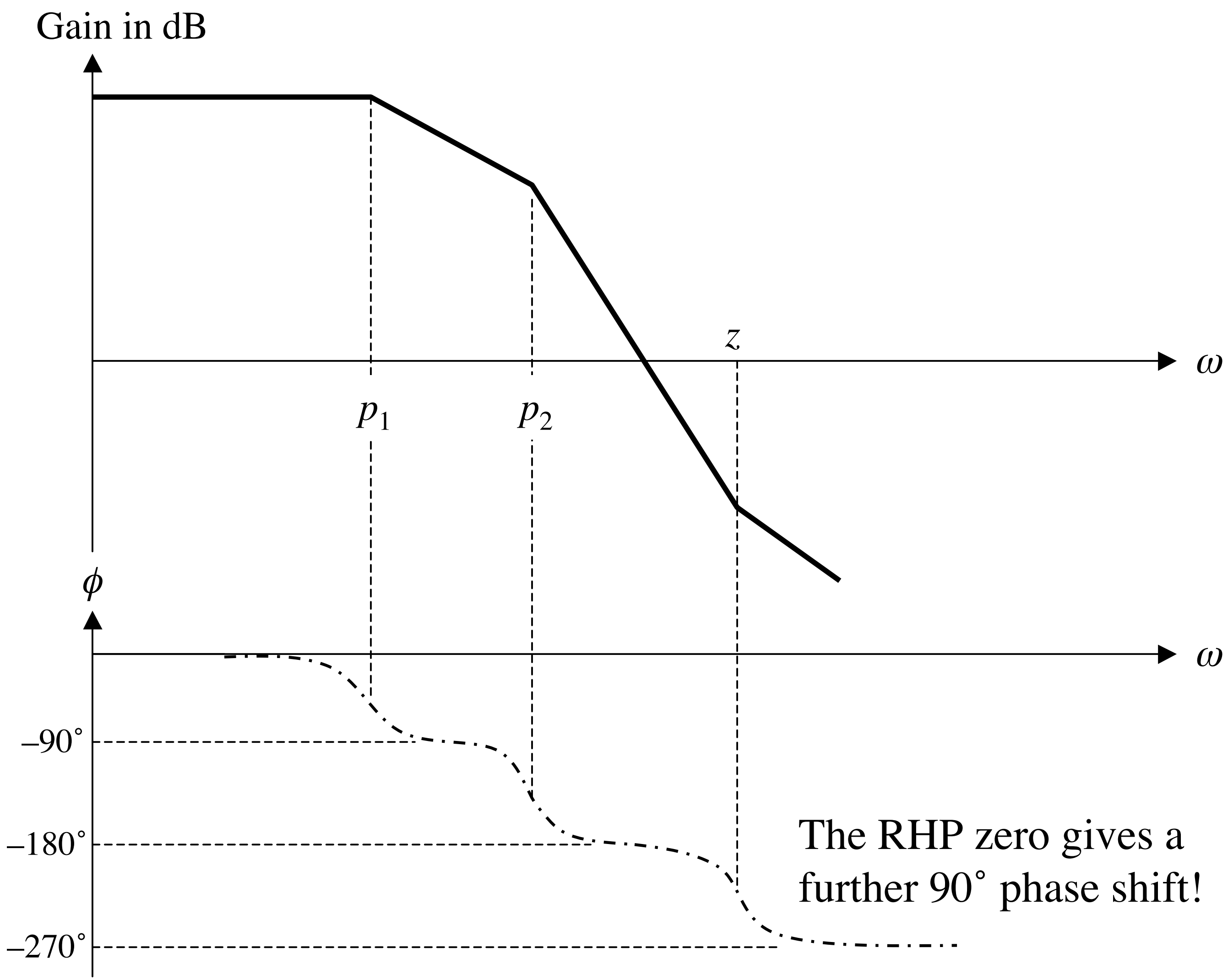
The complete transfer function for the gain is

note the -ve sign (it's RHP)

$$\frac{v_o}{v_{in}} = \left[\frac{R_{B1} \parallel R_{B2}}{(R_{B1} \parallel R_{B2}) + R_s}\right]\left[g_m (R_L \parallel r_o)\right] \frac{\left(1 - \frac{j\omega}{z}\right)}{\left(1 + \frac{j\omega}{p_1}\right)\left(1 + \frac{j\omega}{p_2}\right)}$$

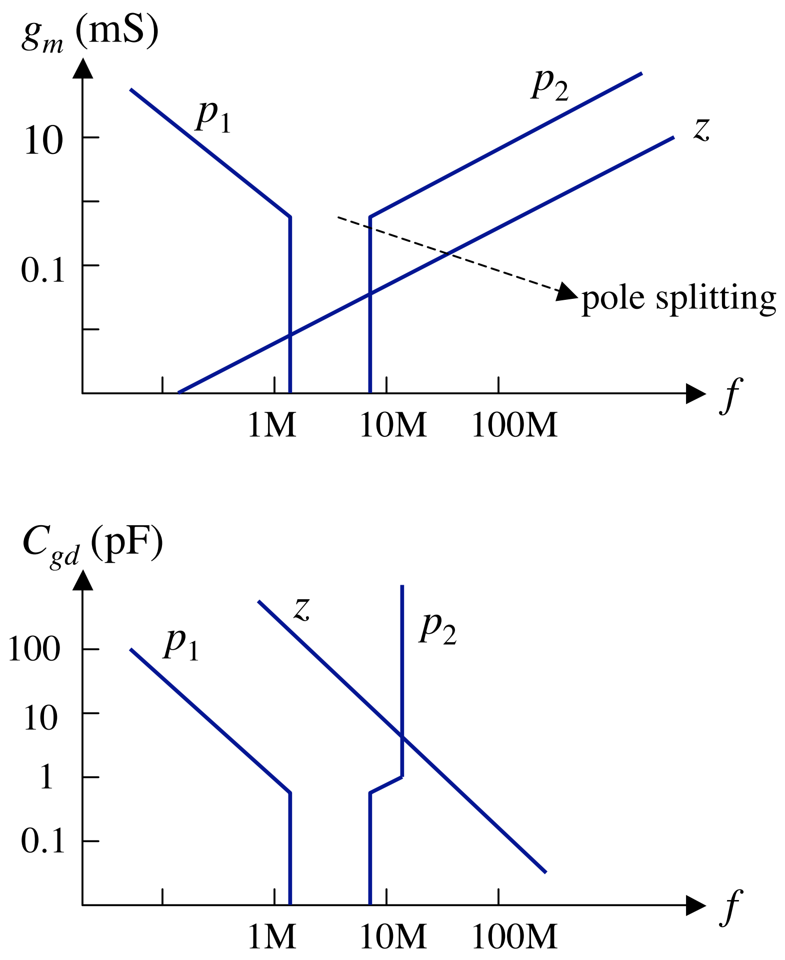
$$p_1 = \frac{1}{C_{eq} R_{eq}} = \frac{1}{\{C_\pi + C_\mu [1 + g_m (R_L \parallel r_o)]\}(R_s \parallel R_B \parallel r_\pi)} \qquad p_2 = \frac{1}{(C_\mu + C_{ce})(R_L \parallel r_o)} \qquad z = \frac{g_m}{C}$$

Complete frequency response of CE amplifier



The RHP zero gives a further 90˚ phase shift!

Locations of poles and the RHP zero



These are not Bode plots! They actually show the positions of the poles and the RHP zero for different values of g_m and C_{gd}.

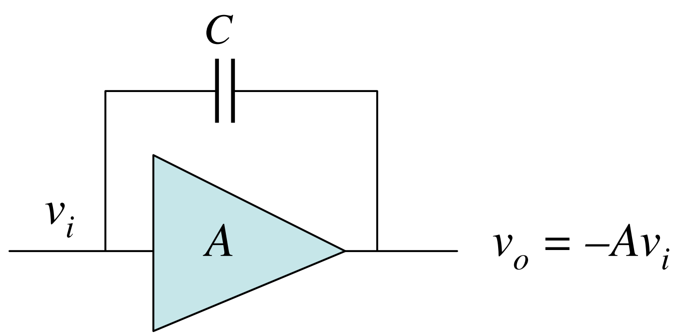
Note the **pole splitting** when g_m is greater than a certain value. This actually improves the stability, as we will see when we discuss feedback later.

Miller effect gives the dominant pole

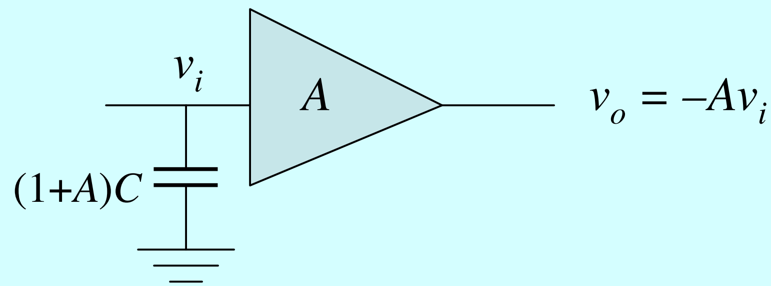
The first pole (dominant pole) defines the ***roll-off frequency*** and limits the bandwidth of the amplifier.

The main cause, as we have seen, is Miller effect!

In general, we can extend Miller effect to any situation where a capacitor appears across the input and output nodes of an amplifier.



Forward signal model:



Discussion

Is there any way to beat Miller?

Direction 1: kill C

A capacitor flying over a signal input and a swinging amplified output. We can kill Miller if we can either stop the input or the output from moving! The question is how to do it, with the signal amplification still maintained.

Direction 2: kill R

How about R_s, which is clearly an evil that causes the roll off? Can we make it as small as possible? What kind of amplifier should be used to buffer the input so that R_s can be smaller?

The process for making Miller disappear is called broadbanding an amplifier. I will tell you more about beating Miller in the final-year elective High Frequency Circuit Design.