HYDRO POWER

Hydro power has played an important historical role in the industrialization of society from grinding flour to powering industry. Hydro energy originates from the sun, and hence, is renewable and its fuel is free.

The analysis of hydroelectric generation begins with the potential energy of the water. The gravitational potential energy (*PE*) is defined based on a material's mass (*m*) and height (*H*) from a reference point.

$$PE = m\,g\,H \tag{H.1}$$

where *g* is gravitational constant. The power generation (*P*) depends upon the period (*T*) over which the water is discharged through that height, oftentimes referred to as the *head*.

$$P = \frac{PE}{T} = \frac{m\,g\,H}{T} \tag{H.2}$$

The water mass may be expressed in terms of its density (ρ) and volume (*V*), *i.e.*, $m = \rho V$. Often, the volume of water is measured in acre-feet which is the volume occupied by a foot of water covering an acre of area; one acre-foot is equivalent to 43,560 ft^3. The standard density of water is 1,000 kg/m^3 or 62.4 lbm/ft^3. The power can then be represented in terms of the mass flow rate ($\dot{m}$) or volumetric flow rate ($\dot{V}$).

$$P = \dot{m}\ g\ H = \rho\ g\dot{V}\ H \tag{H.3}$$

The electric power output is reduced by the hydraulic turbine-generator efficiency.

<u>Example:</u>

Given: Bonneville Dam has eight large generators rated at 54 MWe each under a 59 ft head and two smaller generators rated at 43.2 MWe each under a 49 ft head. Each turbine discharges water at a rate of 13,300 cfs. Determine the overall efficiency of the hydro plant.

Solution: First, determine the theoretical power for the large and small generator types

$$P_L = \frac{\rho\ g\dot{V}\ H_L}{g_c} = \frac{\left(62.4\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{sec}^2}\right)\left(13{,}300\,\frac{\text{ft}^3}{\text{sec}}\right)(59\,\text{ft})}{\left(32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{sec}^2}\right)\left(550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{sec}}\right)\left(1.341\,\frac{\text{hp}}{\text{kW}}\right)} = 66.39\text{ MW}$$

$$P_S = \frac{\rho\ g\dot{V}\ H_S}{g_c} = \frac{\left(62.4\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{sec}^2}\right)\left(13{,}300\,\frac{\text{ft}^3}{\text{sec}}\right)(49\,\text{ft})}{\left(32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{sec}^2}\right)\left(550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{sec}}\right)\left(1.341\,\frac{\text{hp}}{\text{kW}}\right)} = 55.14\text{ MW}$$

Finally, the overall efficiency is the ratio of energy output to input:

$$\eta = \frac{W_{out}}{W_{in}} = \frac{(8)(54\text{ MWe}) + (2)(43.3\text{ MWe})}{(8)(66.39\text{ MW}) + (2)(55.14\text{ MW})} = 0.809$$

TIDAL POWER

Tidal power results from changing gravitational effects due to the relative positions of the earth, sun and moon. Extraction of potential energy from the movement of the tides is akin to hydroelectric facilities. In fact, the simplest implementation of tidal power would be to construct a dam across the mouth of a bay. As the tide comes into the bay, the dam gates are opened allowing water to flow into the bay. At high tide, the gates are closed. As the tide recedes, water is released as in a hydroelectric plant to produce electricity.

The potential energy created by the tides is may be calculated using Eq. (H.1); however, further clarification of the water mass and head is needed. The water mass stored in the "bay" is $m = R\,S\,\rho$ where R is the tidal range (height difference between high and low tides) and S is the bay surface area. The head (H) is not the tidal range (R) since as water exits the bay, the level of the reservoir decreases. The actual head is obtained as half the range, *i.e.*, $H = R/2$. The potential energy from the movement of water in one-half tidal cycle is

$$PE = m\,g\,H = (R\,S\,\rho)\,g\left(\frac{R}{2}\right) = \frac{R^2\,S\,\rho\,g}{2} \qquad \text{(T.1)}$$

In addition, the time (T) period of Eq. (H.2) requires some elaboration since the tidal cycle is 12 hrs and 24.6 minutes (*i.e.*, 4.46×10^4 sec). For systems constructed to utilize both the incoming and outgoing tides, the potential energy of Eq. (T.1) is doubled over the 12-hr tidal cycle. The maximum average power for a plant using both tidal directions is

$$P = \frac{PE}{T} = \frac{R^2\,S\,\rho\,g}{T} \qquad \text{(T.2)}$$

Example

Given: The Passamaquoddy Bay site on the U.S.-Canadian border has an average tidal range of 18.1 ft and a basin area of 101 square miles. Determine both the maximum potential energy stored in the bay and the maximum average power that could be produced at the site.

Solution: Seawater density is approximately 64 lbm/ft^3. The maximum potential energy storage for one tidal cycle would be a scheme that relies on both incoming and outgoing tides, so using the doubled form of Eq. (T.1)

$$E_{max} = \frac{R^2\,S\,\rho\,g}{g_c} = \frac{(18.1\,\text{ft})^2(101\,\text{mi}^2)\left(64\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{sec}^2}\right)\left(5280\,\frac{\text{ft}}{\text{mi}}\right)^2}{\left(32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{sec}^2}\right)\left(550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{sec}}\right)\left(1.341\,\frac{\text{hp}}{\text{kW}}\right)\left(3600\,\frac{\text{sec}}{\text{hr}}\right)} = 22.23\times10^6\ \frac{\text{kW}\cdot\text{hr}}{\text{cycle}}$$

The average power per cycle is then

$$P = \frac{PE}{T} = \frac{22.23\times10^6\ \text{kW}\cdot\text{hr}}{12.41\ \text{hrs}} = 1{,}790\,\text{MWe}$$

Annually, this would be (1,790 MWe)(8760 hrs) = 15.7×10^9 kWe·hr.