Laboratory 7: RC Time Constant

1. Introduction

When a capacitor (C) is connected to a dc voltage source like a battery, charge builds up on its plates and the voltage across the plates increases until it equals the voltage (V) of the battery. At any time (t), the charge (Q) on the capacitor plates is given by Q = CV. The rate of voltage rise depends on the value of the capacitance and the resistance in the circuit. Similarly, when a capacitor is discharged, the rate of voltage decay depends on the same parameters.

Both charging and discharging times of a capacitor are characterized by a quantity called the **time constant** τ, which is the product of the capacitance (C) and the resistance (R), i.e. τ= RC.

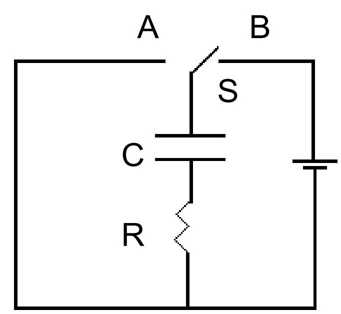


Figure 1

2. Theory

When a capacitor is charged through a resistor by a dc voltage source by putting the switch to position B in Figure 1, the charge in the capacitor and the voltage across the capacitor increase with time. The voltage V as a function of time t is given by:

$$V(t) = V_o(1 - e^{-t/RC}) = V_o(1 - e^{-t/\tau})$$

where the exponential e=2.718 is the base of natural logarithm and Vo is the voltage of the source. The quantity τ=RC is called the **time constant**. The curve of the exponential rise in voltage with time during the charging process is illustrated in Figure 2.

At time t=τ=RC (one time constant), the voltage across the capacitor has grown to a value:

$$V(t = RC) = V_o(1 - e^{-1}) = V_o(1 - \frac{1}{e}) = 0.63V_o$$

It will take an infinite amount of time for the capacitor to fully charge to its maximum value. For practical purposes we will assume the after five time constants the capacitor is fully charged.

When a fully charged capacitor is discharged through a resistor by putting the switch to position A in Figure 1, the voltage across the capacitor decreases with time. The voltage V as a function of time t is given by:

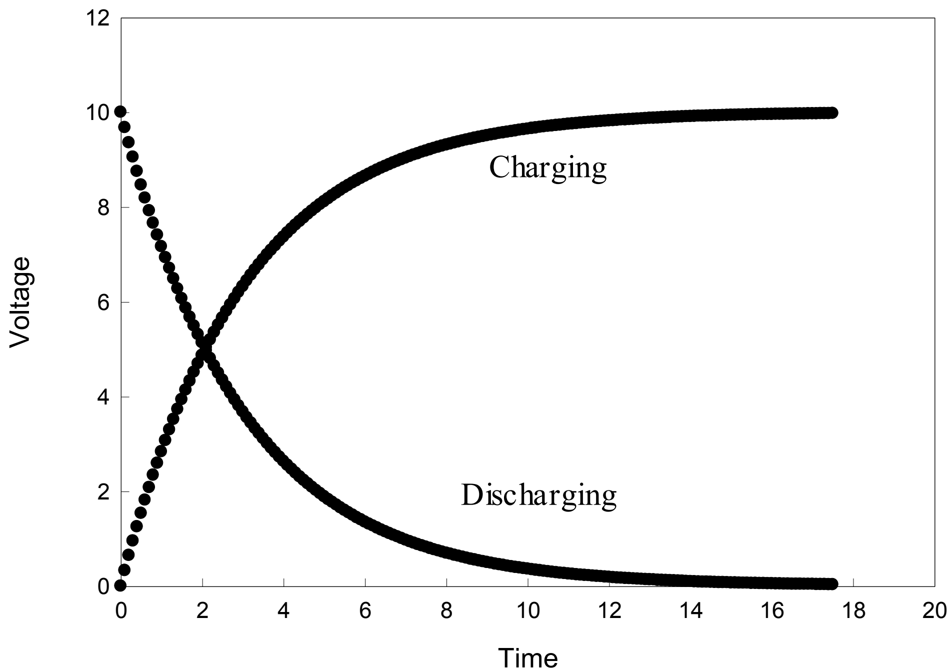


Figure 2

$$V(t) = V_o e^{-t/RC}$$

The exponential decay of the voltage with time is also illustrated in Figure 2. After a time t=τ=RC (one time constant), the voltage across the capacitor has decreased to a value:

$$V(t = RC) = V_o e^{-1} = V_o \frac{1}{e}) = 0.37 V_o$$

Similar equations for charging and discharging of the capacitor exist for the charge Q across the plates of the capacitor. These are:

Charging

$$Q(t) = Q_o (1 - e^{-t/RC}) = V_o (1 - e^{-t/\tau})$$

Discharging:

$$Q(t) = Q_o e^{-t/RC}$$

Examples:

1. Find the time constant of the circuit shown in Figure 1 if C = 1 μF and R = 10 MΩ.

T = RC = 1μF x 10 MΩ = 1 x 10^{-6} x 10 x 10^{6} = 10 s

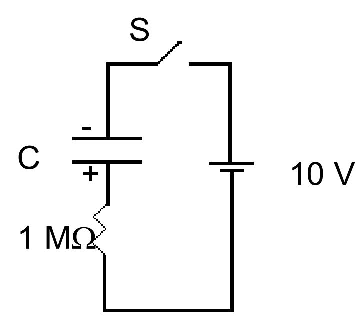
2. What is the voltage across the capacitor in Figure 1, 4 seconds after the switch is put in position B and the battery voltage = 10 V?

$$V(t=4s) = 10(1-e^{-4/10}) = 3.29V$$

3. Use the same values of the capacitor, resistor and battery in example 2. After the capacitor is fully charged in Figure 1, the switch is connected to position A. What is the voltage across the capacitor after 7 seconds.

$$V(t) = 10e^{-7/10} = 5.03V$$

4. A capacitor of 10 μF is connected to a 1 MΩ resistor as shown in the Figure below. The capacitor has an initial charge of 10 μC on its plates. At time t = 0 the switch is closed. What is the voltage across the capacitor after 5 seconds.



The time constant of this circuit is 10 μF x 1 MΩ = 10 s.
The initial voltage on the capacitor is V = Q/C = 10μC/10μF = 1 V

This voltage is negative with respective to the battery. Hence the voltage across the capacitor after 5 seconds is:

$$V = -1 + 10(1 - e^{5/10}) = 2.93V$$

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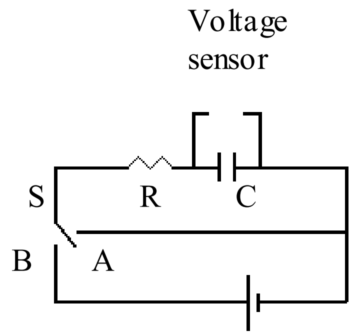
Laboratory Report: **Name:**

Part I: Charging a capacitor. Connect the circuit as shown in the Figure below with the switch S open:



Choose an RC combination that gives a time constant of about 1 second.

C = _____________ R = _______________

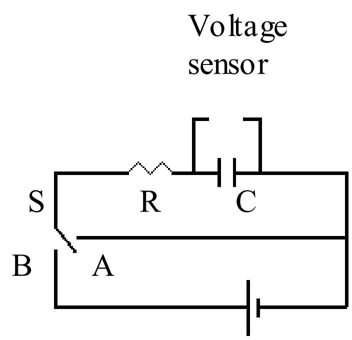
Time constant (calculated) = τ = RC = ____________

Procedure:

1) Set the output voltage to 5 V dc.
2) Connect the voltage sensor across the capacitor.
3) Make sure the capacitor is completely discharged by touching the two ends of the capacitor with a cable.
4) Select graphical representation for the data to plot the voltage across the capacitor as a function of time.
5) Put the switch to position B.
6) Run the program and stop the program once the capacitor is charged to its maximum value. Connect the switch to position A.
7) Using this graph calculate the time constant of the circuit. i.e. find the time taken for the capacitor to charge to 0.63 multiplied by the maximum voltage across the capacitor in the fully charged condition.
8) **Time constant from the graph =** _______________
9) Print out a copy of the graph.

% Error in τ = __________________

Part II: Discharging a capacitor. Connect the circuit as shown in the Figure below with the switch S open:



Choose an RC combination that gives a time constant of about 1 second.

C = ______________ R = ________________

Time constant (calculated) = τ = RC = _____________

Procedure:

1) Set the output voltage to 5 V dc.
2) Connect the voltage sensor across the capacitor.
3) Make sure the capacitor is completely discharged by touching the two ends of the capacitor with a cable.
4) Select graphical representation for the data to plot the voltage across the capacitor as a function of time.
5) Put the switch to position B and charge the capacitor to its maximum.
6) Run the program and stop the program once the capacitor is charged to its maximum value.
7) Erase the graph and start a fresh graph.
8) **Put the switch to position A and at the same time run the program.**
9) Stop the program when the voltage across the capacitor reaches zero.
10) Using this graph calculate the time constant of the circuit. i.e. find the time taken for the capacitor to discharge to 0.63 multiplied by the maximum voltage across the capacitor in the fully charged condition.
11) **Time constant from the graph =** ________________
12) Print out a copy of the graph.

% Error in τ = ___________________

Part III: Repeat parts I and II but with an RC combination that has a time constant greater than 10 seconds.

R = ___________________ C = ________________

Time constant (calculated) = τ = RC = _____________

Charging:

Time constant (from graph) = τ = RC = _____________

% Error in τ = ___________________

Attach copy of graph.

Discharging:

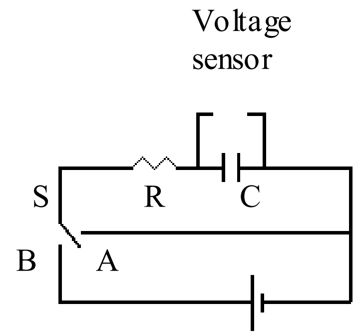
Time constant (from graph) = τ = RC = _____________

% Error in τ = ___________________

Attach copy of graph.

Part IV: Charging and Discharging in an RC circuit.

Choose the values of resistance and capacitance of Part I and connect the circuit shown below.



Procedure:

1) Set the output voltage to 5 V **square wave** 100 Hz or any appropriate frequency.
2) Connect the voltage sensor across the capacitor.
3) Make sure the capacitor is completely discharged by touching the two ends of the capacitor with a cable.
4) Select **two** graphical representations for the data to plot the voltage across the capacitor as a function of time and also the output voltage.
5) Put the switch to position B.
6) Run the program and stop the program once you have a few cycles plotted.
7) Change the frequency and repeat step 6. Try other frequencies also.
8) Print out a copy of the two graphs (at 100 Hz), one below the other on the same sheet.
9) **What can you conclude?**
10) Attach a copy of the graph.

Questions:

1) From your results of part I, what is the value of the voltage across the capacitor after 5 time constants? What can you conclude?

2) With $V(t) = V_o \exp(-t/RC)$, it mathematically takes an infinite time for the capacitor in an RC circuit to discharge completely. Practically, how many time constants does it take for the capacitor to discharge to less than 1% of its initial voltage?

3) What is the time constant of the circuit shown below: