

Lab 10: RC, RL, and RLC Circuits

In this experiment, we will investigate the behavior of circuits containing combinations of resistors, capacitors, and inductors. We will study the way voltages and currents change in these circuits when voltages are suddenly applied or removed. To change the voltage suddenly, a function generator will be used. In order to observe these rapid changes we will use an oscilloscope.

1. The Square Wave Generator

Introduction

We can quickly charge and discharge a capacitor by using a function generator set to generate a square wave. The output of this voltage source is shown in Figure 1.

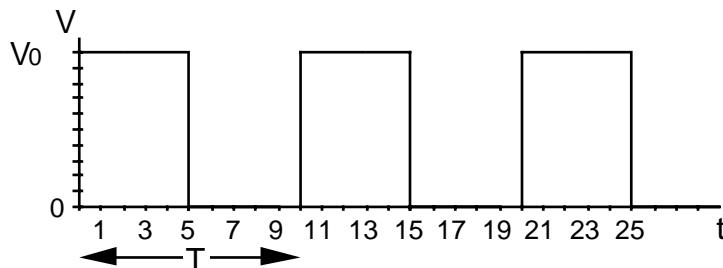


Figure 1. Output of square-wave generator

One control on the generator lets you vary the amplitude, V_0 . You can change the time period over which the cycle repeats itself, T , adjusting the repetition frequency $f = 1/T$.

The generator is not an ideal voltage source because it has an internal resistance of 50Ω . Thus, for purpose of analysis, the square-wave generator may be replaced by the two circuits shown in Figure 2. When the voltage is “on,” the circuit is a battery with an EMF of V_0 volts in series with a $50\text{-}\Omega$ resistor. When the voltage is “off,” the circuit is simply a $50\text{-}\Omega$ resistor.

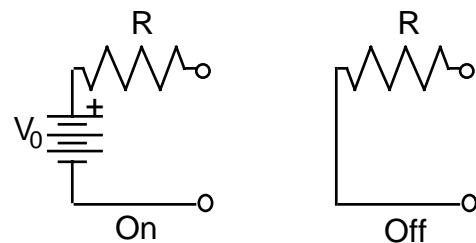


Figure 2. Square-wave generator equivalent circuit

Procedure

- To learn how to operate the oscilloscope and function generator, set the function generator for square wave output and connect the generator to the vertical input of the oscilloscope.
- Adjust the oscilloscope to obtain each of the patterns shown in Figure 3.
- Try changing the amplitude and repetition frequency of the generator and observe what corresponding changes are needed in the oscilloscope controls to keep the trace on the screen the same.
- Now set the function generator to a frequency of about 100 Hz. Observe pattern and adjust the frequency until the period $T = 10.0$ ms.

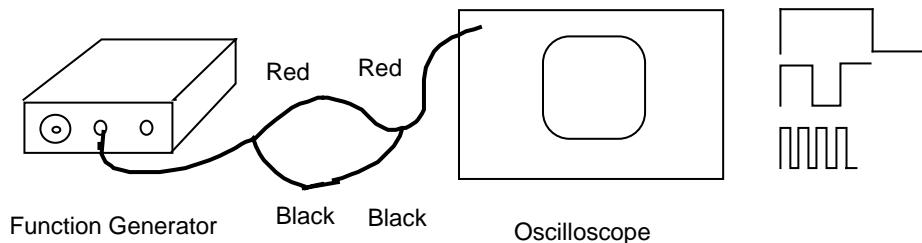


Figure 3. Observing the output of the square-wave generator.

2. Resistance-Capacitance Circuits

Introduction

We have previously studied the behavior of capacitors and looked at the way capacitor discharges through a resistor. Theory (see textbook) shows that for a capacitor, C , charging through a resistor, R , the voltage across the capacitor, V , varies with time according to

$$V(t) = V_0 (1 - e^{-t/RC}), \quad (1)$$

where V_0 is the final, equilibrium voltage.

When the same capacitor discharges through the same resistor,

$$V(t) = V_0 e^{-t/RC} \quad (2)$$

The product of the resistance and capacitance, RC , governs the time scale with which the changes take place. For this reason it is called the time constant, which we call τ (tau). It can be found indirectly by measuring the time required for the voltage to fall to $V_0 / 2$ (see Figure 4 below). This time interval is called the half-life, $T_{1/2}$, and is given by the equation $T_{1/2} = (\ln 2)\tau$, so

$$\tau = T_{1/2} / \ln 2 = T_{1/2} / (0.693) \quad (3)$$

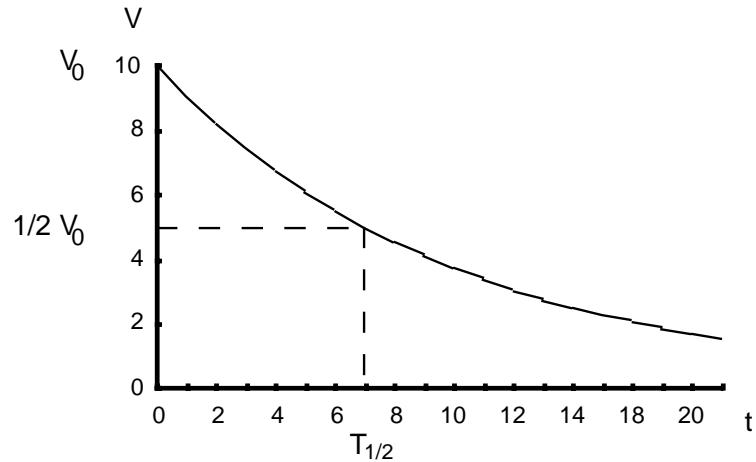


Figure 4. Discharge of a capacitor

Procedure

- Assemble the circuit shown in Figure 5.
- With initial values $R = 10 \text{ k}\Omega$, $C = 0.1 \mu\text{F}$, and $f = 100 \text{ Hz}$, observe one period of the charge and discharge of the capacitor.
- Make sure the repetition frequency is low enough so that the voltage across the capacitor has time to reach its final values, V_0 and 0.
- Sketch the waveform you observed on the data sheet.
- Use the ohmmeter to measure R .

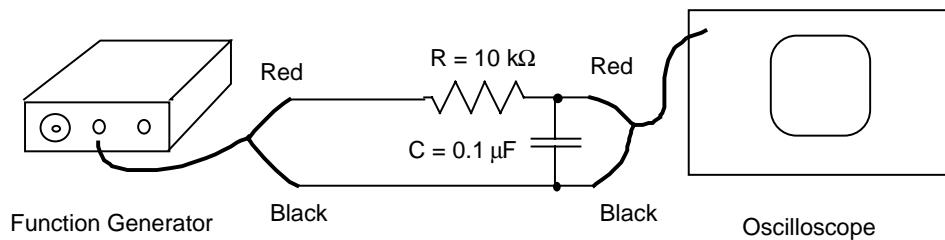


Figure 5. Investigating an RC circuit

Techniques

Here is a method for finding $T_{1/2}$.

- Change oscilloscope gain (volts/cm) and sweep rate (ms/cm) until you have a large pattern on the screen. Make sure the sweep speed is in the “calibrated” position so the time can be read off the x -axis.
- Center the pattern on the screen so that the horizontal axis is in the center of the pattern. That is, so that the waveform extends equal distances above and below the axis.
- Move the waveform to the right until the start of the discharge of the capacitor is on the vertical axis (Figure 6b). You may find it helpful to expand, or magnify, the trace. The sweep time is now a factor of five or ten faster than indicated on the dial. Ask your instructor for details.
- The half-life, $T_{1/2}$, is just the distance shown on Figure 6b.

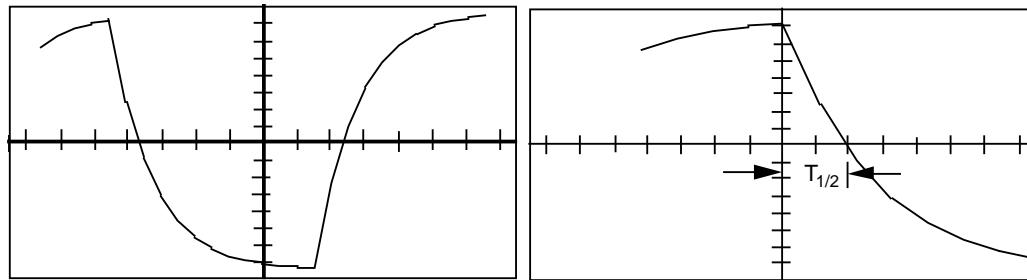


Figure 6a and b. Measuring the half-life

Procedure

- Measure the half-life, $T_{1/2}$ and from this compute the time constant τ using Equation 3.
- Compute the value of RC from component values. Note that, as described above, the square-wave generator has an internal resistance of 50Ω . Thus the total resistance through which the RC circuit charges and discharges is $(R + 50 \Omega)$.
- Try different values of f and hence, T , while keeping τ fixed by not changing either R or C .
- Sketch what you saw when the period T of the square wave is much larger than the time constant τ . Repeat when it is much smaller.

3. Resistance-Inductance Circuits

Introduction

In this part we conduct a similar study of a circuit containing a resistor and an inductor, L . First consider the circuit shown in Figure 7, below. The text shows that if we start with the battery connected to the LR circuit, after a long time the current reaches a steady-state value, $i_0 = V_0/R$.

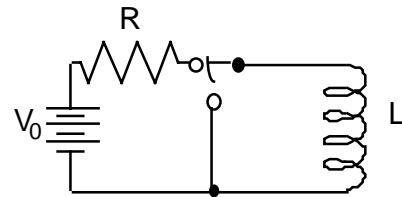


Figure 7. A model circuit with an inductor and resistor

If we call $t = 0$ the time when we suddenly throw the switch to remove the battery, allowing current to flow to ground. The current changes with time according to the equation

$$i(t) = i_0 e^{-(R/L)t} \quad (4)$$

If at a new $t = 0$ we throw the switch so the battery is connected, the current increases according to the equation

$$i(t) = i_0(1 - e^{-(R/L)t}) \quad (5)$$

The time constant for both equations is L/R and

$$\frac{L}{R} = \tau = \frac{T_{1/2}}{0.693} \quad (6)$$

We can find the current by measuring the voltage across the resistor and using the relationship $i = V/R$.

Note that what we would first see is the growth of current given by Equation 5, where the final current depends on the square-wave amplitude V_0 . Then, when the square wave drops to zero, the current decays according to Equation 4. The time constant should be the same in both cases.

Procedure

- Set up the circuit shown in Figure 8 below.
- With initial values $R = 1\text{k}\Omega$ and $L = 25\text{mH}$, set the oscilloscope to view one period of exponential growth and decay. Again, make sure that f is low enough for the current to reach its final values, i_0 and 0. Start with $f = 5\text{ kHz}$. Sketch the pattern.
- Measure the half-life. From this value, compute the time constant τ .
- Measure the value of R and the dc resistance of the inductor with an ohmmeter. Finally add the internal resistance of the square-wave generator to obtain the total resistance. Compute the value of L/R from the components and compare with τ found from the indirect measurement above.
- Try different values of T . In particular, sketch the waveform when τ is much larger than the period T of the square wave and when it is much smaller.

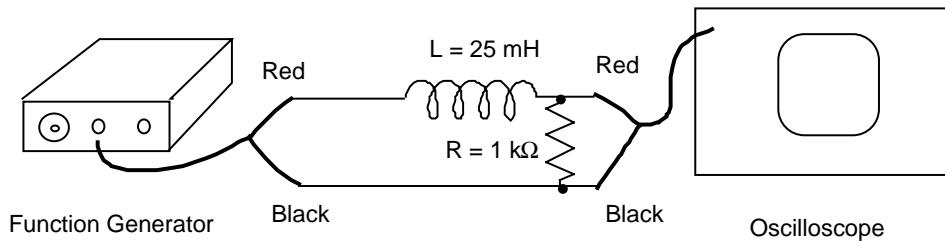


Figure 8. Investigating the LR circuit

4. Resistance Inductance Capacitance circuits

Introduction

As discussed in the textbook, a circuit containing an inductor and a capacitor, an LC circuit, is an electrical analog to a simple harmonic oscillator, consisting of a block on a spring fastened to a rigid wall.

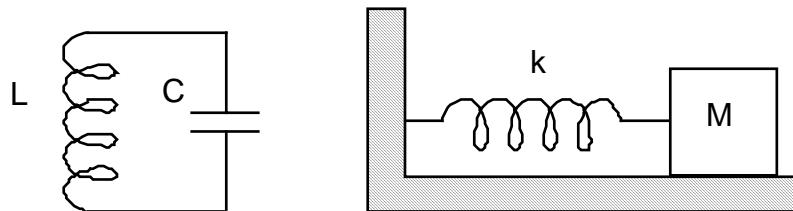


Figure 1. LC Circuit and its analog, a mechanical SHM System

In the same way that, in the mechanical system, energy can be in the form of kinetic energy of the block of mass M , or potential energy of the spring with

constant k , in the LC circuit energy can reside in the magnetic field of the inductor $U = \frac{1}{2} Li^2$, or the capacitor, $U = \frac{1}{2} q^2/C$. Both the current and the charge then change in a sinusoidal manner. The frequency of the oscillation is given by

$$\omega_0 = 1/\sqrt{LC} \quad (7)$$

All circuits have some resistance, and in the same way frictional forces damp mechanical SHM , resistance causes energy loss (i^2R) which makes the charge decay in time.

$$q(t) = q_0 e^{-t/\tau} \cos \omega_1 t \quad (8)$$

$$\omega_1 = (\omega_0^2 - 1/\tau^2)^{1/2} \quad (9)$$

where $\tau = 2L/R$ or

$$T_{1/2} = \ln 2 (2L/R) = (0.693)\tau \quad (10)$$

For large τ the system is underdamped and the charge oscillates, taking a long time to return to zero.

Note that in Equation (3) when $\omega_0^2 = 1/\tau^2$, ω_1 , the argument of the cosine function of Equation (2), is zero at all times. This condition is called critical damping. This condition exists when $R = 2\sqrt{L/C}$

When the resistor is larger than the critical value the system is overdamped. The charge actually takes longer to return to zero than in the critically damped case.

Experiment

The decaying oscillations in the LRC circuit can be observed using the same technique as used to observe exponential decay. Again, a square-wave generator produces the same effect as a battery switched on and off periodically. The scope measures the voltage across C as a function of time.

Procedure

- Assemble the circuit of Figure 2. Use a small value of R , say, 47Ω . Be sure to reduce the signal generator frequency to 100 Hz or below so you can see the entire damped oscillation.

- Measure the period and calculate the frequency of the oscillations. The period is NOT $0.01 \text{ s} = 1/100 \text{ Hz}$, the repetition frequency of the square wave.

NOTE: You have actually measured ω_1 , the damped frequency. This is slightly less than ω_0 , the undamped frequency of Equation 1 but the difference is negligible.

- Compare the ω_1 you measured with the calculated $\omega_0 = 1/\sqrt{LC}$.
- Note that in the equations, R represents the sum of the resistance of the inductor, the internal resistance of the square-wave generator, 50Ω , and the resistor you put in.
- To study critical damping and overdamping, remove your fixed resistor and put in its place a $5-\text{k}\Omega$ variable resistor.
- Start with a small value of R . Sketch the underdamped oscillations.
- Increase R until critical damping is reached; that is, until the oscillations disappear. Sketch this curve. Use the ohmmeter to measure the value of the variable (potentiometer) resistor. Add the dc resistance of the inductor and 50Ω for the generator and compare with the predicted value for critical damping, $R = 2\sqrt{L/C}$.
- What happens when the resistance is larger than the critical-damping value? Sketch your results.

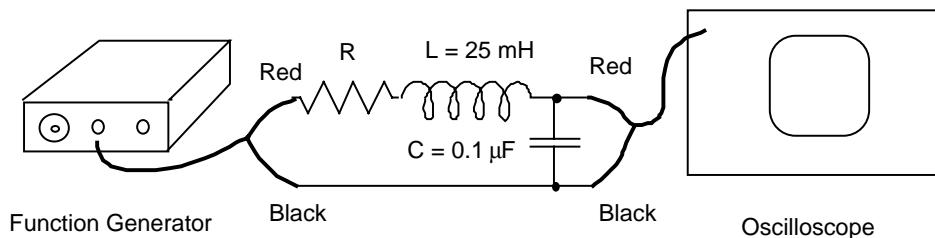


Figure 2. Investigating the LRC circuit

When the circuit is underdamped, Equation (2) applies. This means that the amplitude of the oscillation will decay exponentially, with the time constant for the decay being:

$$\tau = 2L/R. \quad (11)$$

Recall that when an exponential decay is plotted on a semi-log scale the resulting graph is a straight line with a slope equal to $-1/\tau$.

You can find the slope of a line on a semi-log graph by identifying the two end points of the line. Note the time and voltage at each point t_1 and V_1 , t_2 and V_2 .

Calculate the natural log of the two voltages.

Then $(\ln V_2 - \ln V_1)/(t_2 - t_1) = -1/\tau$

To measure the time constant of the decay of the oscillations, follow this procedure:

Procedure

- Adjust the variable resistor so that the circuit is underdamped and oscillates about seven or eight times before the oscillations become too small to be easily seen on the oscilloscope.
- Center the oscillation pattern vertically on the screen so that when the oscillations have decayed the line on the oscilloscope coincides with the time axis.

Make a data table recording the voltage of each oscillation peak, and the corresponding time for each peak. When your table is complete you should have six or seven sets of data recorded.
- Plot your data on a semi-log graph.
- Following the procedure described above, determine the time constant for your circuit. This is your experimental value for the time constant.
- Remove the variable resistor from the circuit and measure its resistance. Add this value to the resistance of the square-wave generator (50 ohms) and the resistance of the inductor to get the total resistance of your circuit.
- Using Equation (5), calculate the theoretical decay time constant for your circuit. Compare this theoretical value to the experimental value you found above. They should agree within ten or twenty percent. If they do not, consult your instructor.

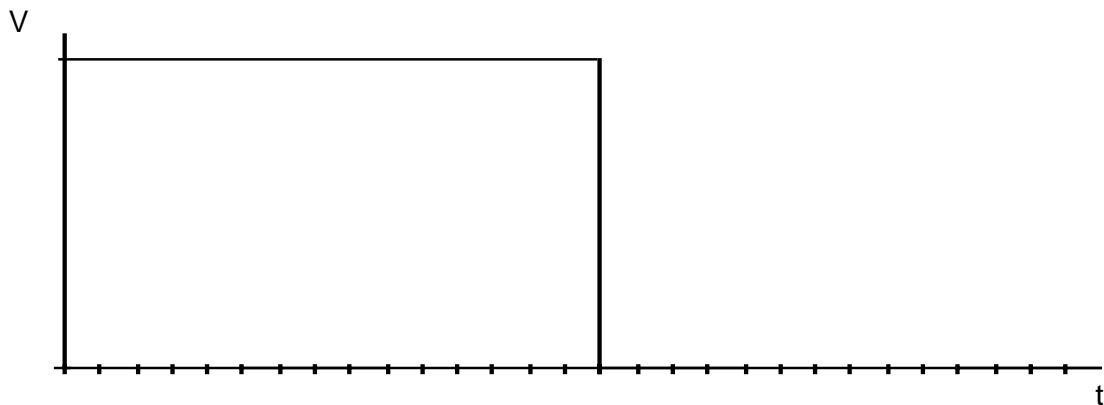
RC, RL, and RLC Circuits

Name_____ Partner_____ Section:_____

2. RC Circuits

1. Using dimensional analysis, show that the product of an ohm times a farad (RC) has units of time.

2. Sketch observed pattern produced by the RC circuit on top of the square wave.



3. Measured $T_{1/2}$ _____ s $\tau = T_{1/2} / 0.693 =$ _____ s

RC from component values _____ s

Within the uncertainties of the tolerances (10%) of the resistor and capacitor, do your measurements support the equation $\tau = RC$?

If there is more than 20% disagreement, see your instructor.

4. What is the largest voltage across the capacitor (V_o)?

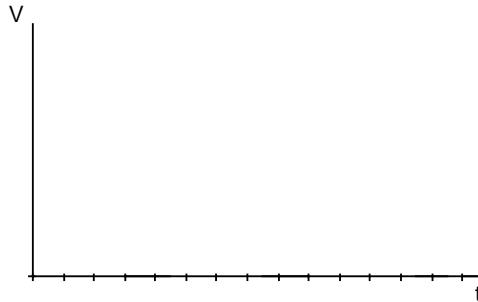
What is the largest charge on the capacitor ($q_o = C V_o$)

5. Sketches.

$\tau \gg T$

$$\tau = \underline{\hspace{2cm}}$$

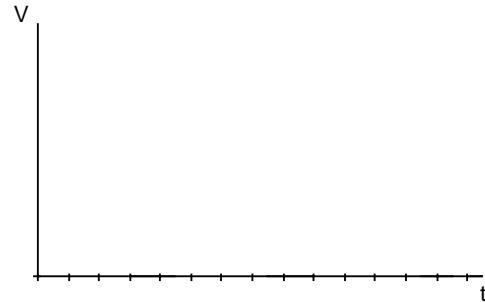
$$T = \underline{\hspace{2cm}}$$



$\tau \ll T$

$$\tau = \underline{\hspace{2cm}}$$

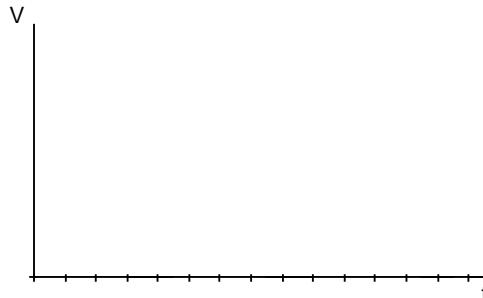
$$T = \underline{\hspace{2cm}}$$



6. How might you measure the internal resistance of your function generator?

3. RL Circuits

1. Using dimensional analysis, show L/R has units of time.



2. Sketch of V across R .

3. Measured $T_{1/2} = \underline{\hspace{2cm}}$ s; $\tau = T_{1/2}/0.693 = \underline{\hspace{2cm}}$ s

L/R from component values $\underline{\hspace{2cm}}$ s

Within the uncertainties of the manufacturing tolerances (10%) of the resistor and inductance, do your measurements support the equation $\tau = L/R$?

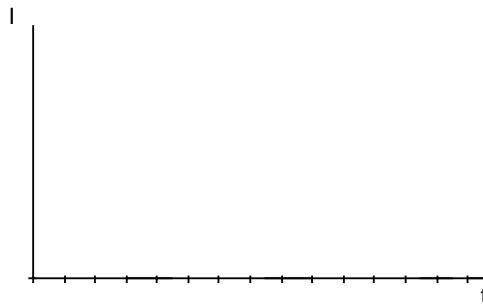
4. What is the largest current through the inductor (i_0)?

5. Sketches

$\tau \gg T$

$$\tau = \underline{\hspace{2cm}}$$

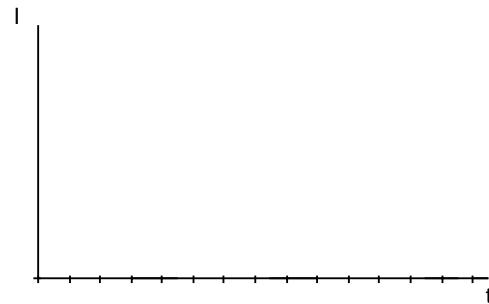
$$T = \underline{\hspace{2cm}}$$



$\tau \ll T$

$$\tau = \underline{\hspace{2cm}}$$

$$T = \underline{\hspace{2cm}}$$



4 RLC Circuits

1. Measured period s.

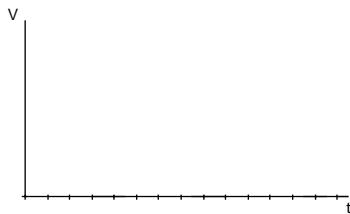
$$\text{Calculated } f_1 = \underline{\hspace{2cm}} \text{ Hz}; \quad \omega_1 = 2\pi f_1 = \underline{\hspace{2cm}} \text{ rad/s.}$$

Compare with the frequency calculated from component values,

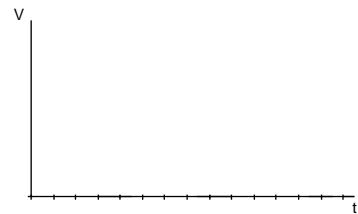
$$\omega_0 = 1/\sqrt{LC} = \underline{\hspace{2cm}} \text{ rad/s.}$$

2. Sketch of patterns

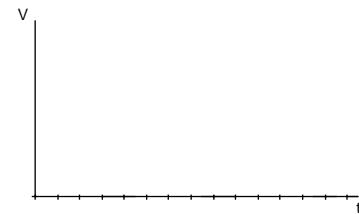
Underdamped



Critically damped



Overdamped



3. When critically damped, resistance of potentiometer = Ω

$$\text{Sum of resistances of potentiometer, inductor, and generator} = \underline{\hspace{2cm}} \Omega$$

$$\text{Theoretical value } R = 2\sqrt{L/C} = \underline{\hspace{2cm}} \Omega.$$

Comparison and discussion of any disagreement: _____

4. Experimental decay constant (from graph) = _____

$$\text{Theoretical decay constant (Equation(11))} = \underline{\hspace{2cm}}$$

Comparison and discussion of any disagreement: _____