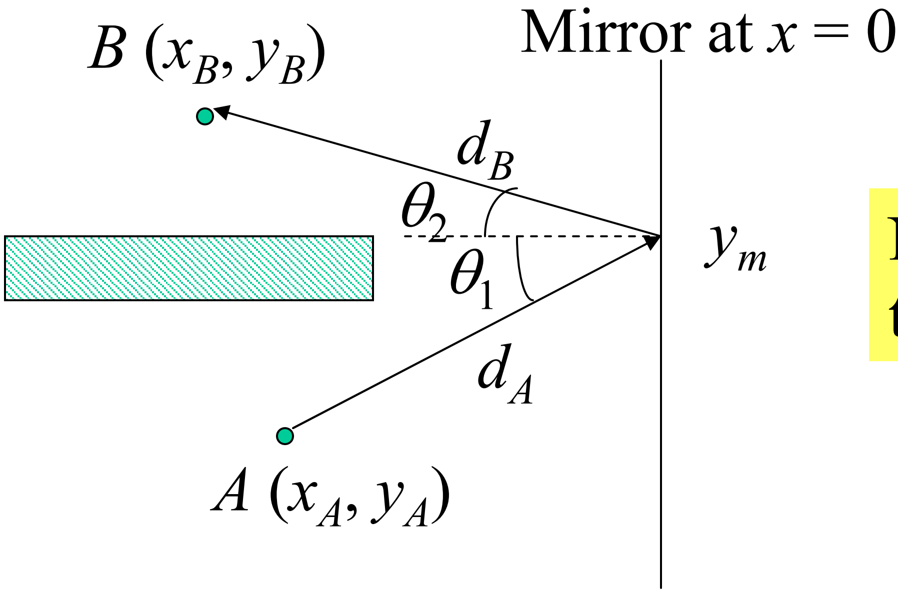
Lecture 15: Fermat's Principle, and Image Formation

- One of the goals of physics is to explain many phenomena starting from a few fundamental principles
  - Newton's Law of Gravity, for example, explained both the motion of the planets and the falling of apples on Earth
- Fermat came up with a principle that explained the propagation, reflection and refraction of light:
  - light always travels in the path that minimizes the time it takes to get from point A to point B
- This clearly is consistent with the fact that light travels in straight lines within a given medium
- The text shows how Snell's Law arises from Fermat's principle

Reflection According to Fermat

- Let's say that a set of light rays start at point A
  - where will the ones that end up at point B hit the mirror?



Need to find y_m that minimizes total distance traveled

$$d = d_A + d_B$$

$$= \sqrt{x_A^2 + (y_m - y_A)^2} + \sqrt{x_B^2 + (y_B - y_m)^2}$$

- To find the minimum, we need to take the derivative with respect to y_m and set it equal to 0:

$$\frac{dd}{dy_m} = \frac{y_m - y_A}{\sqrt{x_A^2 + \left(y_m - y_A\right)^2}} + \frac{y_B - y_m}{\sqrt{x_B^2 + \left(y_B - y_m\right)^2}} = 0$$

$$\frac{y_m - y_A}{\sqrt{x_A^2 + \left(y_m - y_A\right)^2}} = \frac{y_B - y_m}{\sqrt{x_B^2 + \left(y_B - y_m\right)^2}}$$

- From the figure, we see that this means:

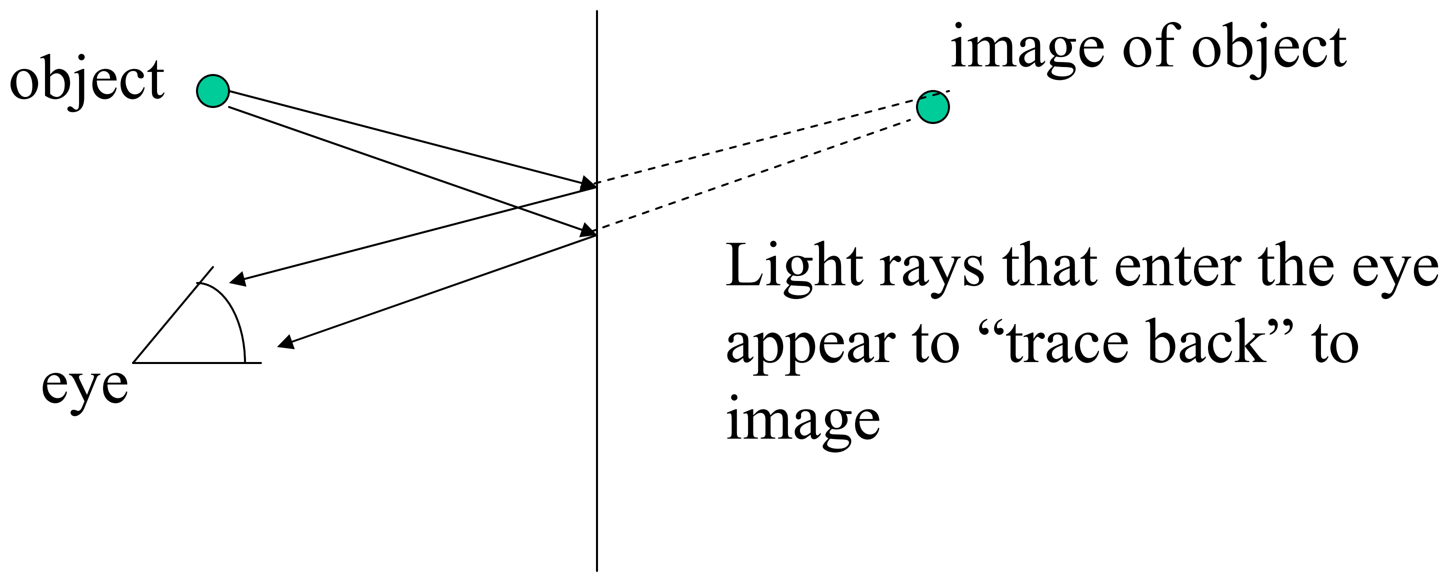
$$\sin\theta_1 = \sin\theta_2$$

$$\theta_1 = \theta_2$$

exactly as we expected!

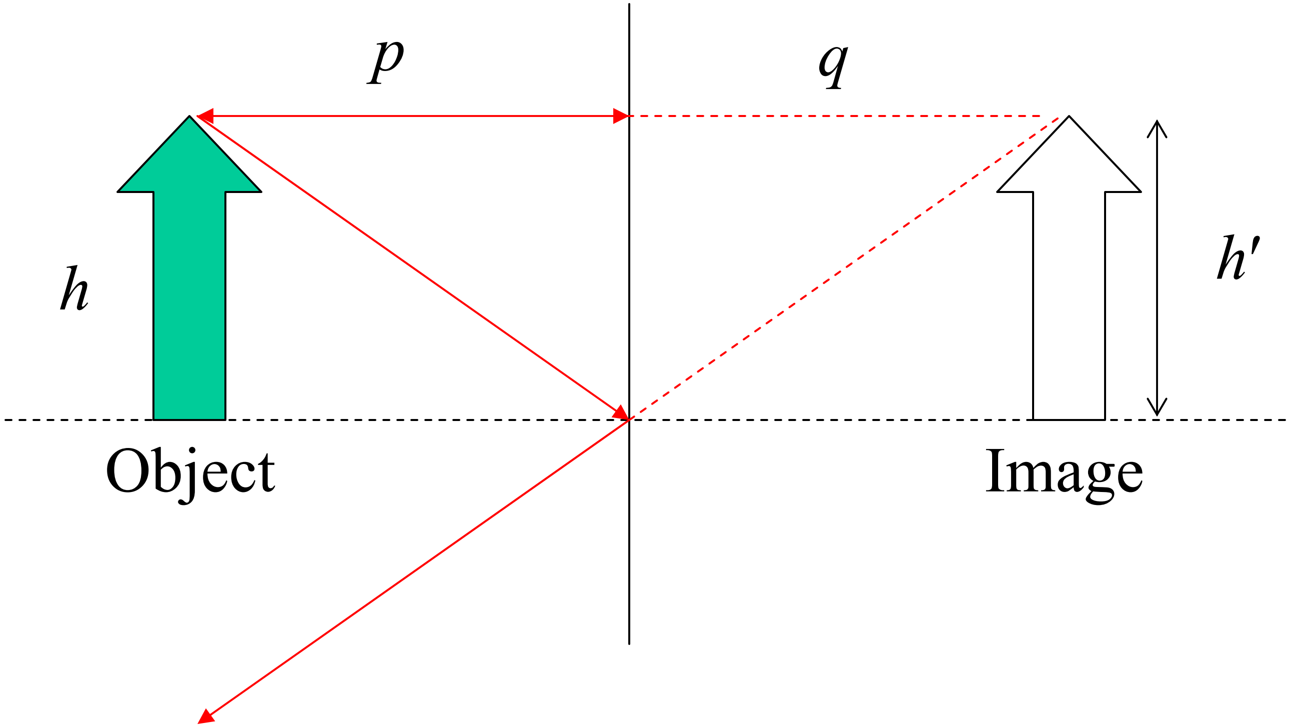
Image Formation

- The next topic we'll explore is the formation of *images*
  - images occur when light appears to originate in some place different from the object that reflected the light
- Probably the most common image is that formed by a flat mirror
  - when we stand in front of the mirror, it appears to us that there's a copy of us standing behind the mirror
- Here's how it works:



Size and position of the image

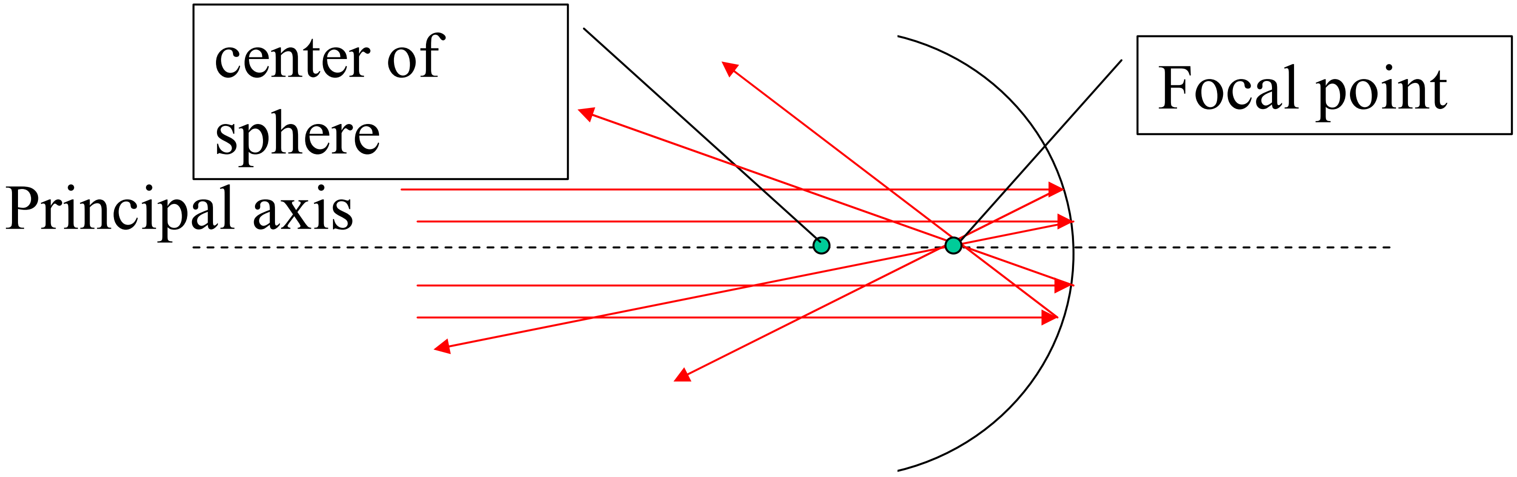
- We'll see many more examples of image formation
  - both from mirrors with different shapes and from lenses
- We want to determine the size and position of an image
- We can do this by considering only a few of the rays of light coming from the object and forming the image



- From the diagram on the previous slide we learn the following about the image formed by a flat mirror:
  - the image is always behind the mirror. Such images are called *virtual images*
  - the image has the same size as the object
  - the image is upright (arrow points upward for both object and image)
  - the image appears the same distance behind the mirror as the object is in front of it ($p = q$)
- Note that an image (even a virtual one) has a definite size and position
  - i.e. these quantities don't depend on the position of the person observing the image

Images formed by spherical mirrors

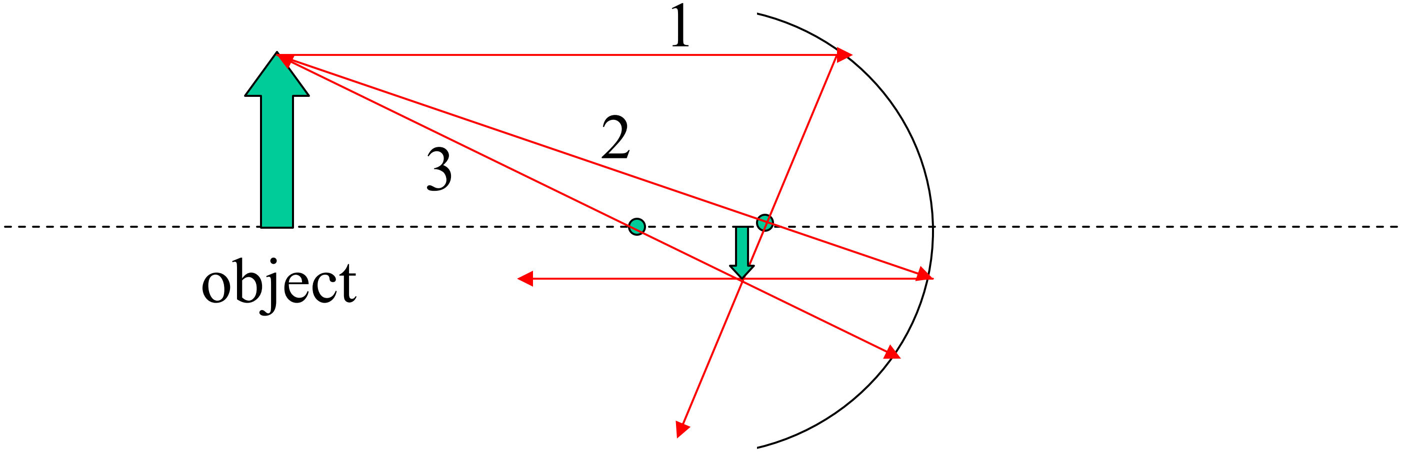
- By changing the geometry of the mirror, we can alter the behavior of images
- One example is a mirror formed into the shape of a sphere
- Let's first consider a *concave* spherical mirror (this means the mirror is on the inner surface of the sphere)



- Key fact: parallel light rays close to the principal axis reflect through a common point
  - This is the *focal point* of the mirror

Ray diagrams

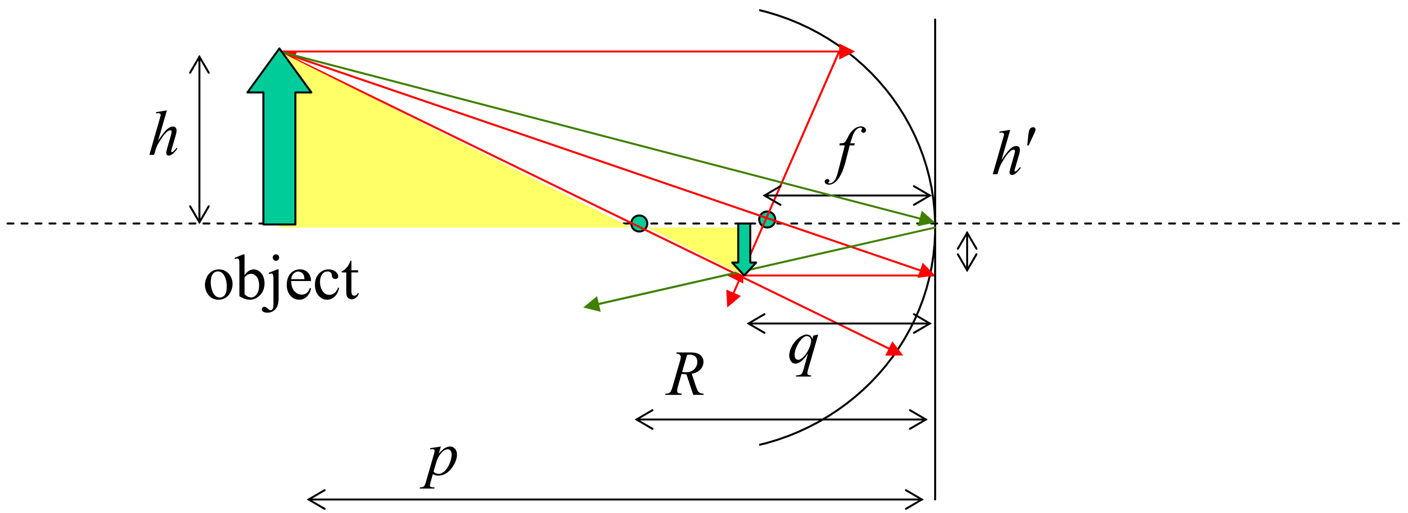
- Imagine we have an object with one end on the principal axis
  - where does the image formed by the spherical mirror appear?
- Find out by drawing three rays from the tip of the object:



1. Parallel to the principal axis, reflects through focus
2. Through focus, reflects parallel to principal axis
3. Through center, reflects back on itself

- Reflected rays meet at tip of image!

- We really only need two rays – the third acts as a cross-check
- As with the flat mirror, we'll define some distances:



- Note that I snuck in an "extra" ray here (the green one)
  - From this we see that:

$$\frac{p}{h} = -\frac{q}{h'}$$

$$\frac{q}{p} = -\frac{h'}{h} = M$$

Magnification. The – sign means the image is upside-down

- The yellow triangles on the previous slide are similar, so we know that:

$$\frac{h}{p-R} = -\frac{h'}{R-q}$$

$$\frac{h'}{h} = \frac{q-R}{p-R} = -\frac{q}{p}$$

$$pq - pR = qR - pq$$

$$\frac{1}{R} - \frac{1}{q} = \frac{1}{p} - \frac{1}{R}$$

- This leads to the *mirror equation*:

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

- For the spherical mirror it happens that $f = R/2$. The general form of the mirror equation is:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Sign Conventions

- Clearly p is always a positive number
- As I drew the diagram, q was also positive
  - And the image was *real*, not virtual like the one from a flat mirror
- But in some cases the mirror equation will tell us that q is negative
  - This means the image is *behind* the mirror (and is therefore virtual)
  - This happens when $p < f$
  - Note that the image will be upright in this case
- The equation also holds when R is negative
  - This means the mirror is convex rather than concave