Lecture 5: Using electronics to make measurements

- As physicists, we're not really interested in electronics for its own sake
- We want to use it to measure something
  - often, something too small to be directly sensed
- As an example, we'll assume we want to measure the strain on an object
  - strain is the degree to which an object changes its shape due to an external force:

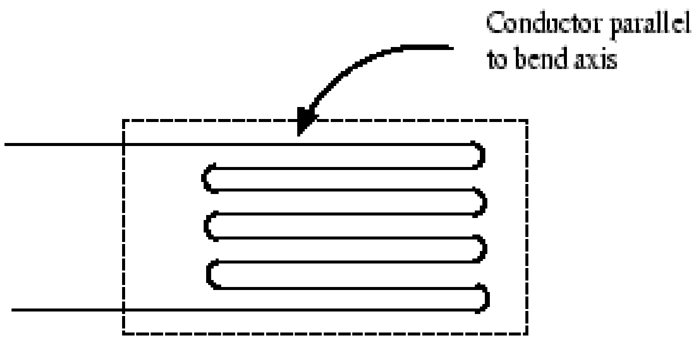
$$\text{strain} = \frac{\Delta L}{L}$$

- We can convert this to an electrical signal by using a *strain gauge*, a device that changes its resistance under strain

- The measuring power of a strain gauge is quantified by the *gauge factor*:

$$GF = \frac{\Delta R / R}{\text{strain}} = \frac{\Delta R / R}{\Delta L / L}$$

- A higher gauge factor makes a better gauge
- One example is a metallic wire bonded to a piece of material:



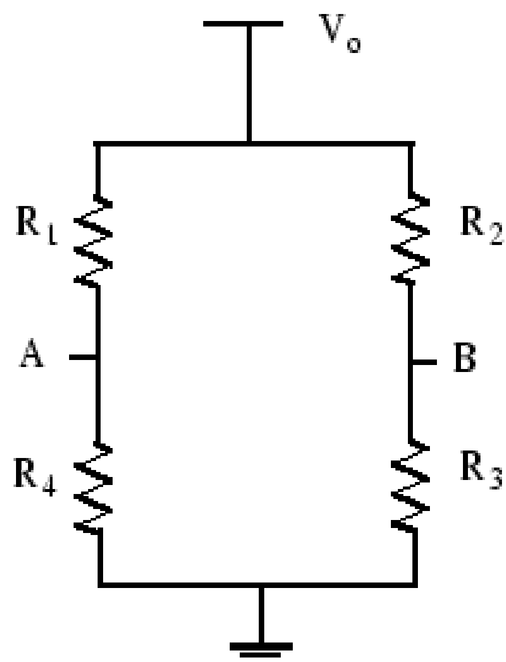
wire become thinner and longer as material stretches – resistance increases

- Typically these have resistance of 120Ω and GF ≈ 2

- Let's say we want to be sensitive to strains of the order 10^{-5} or so using this gauge
- That means the change in resistance is:

$$\Delta R = R \cdot S \cdot GF = 120\Omega \cdot 10^{-5} \cdot 2 = 0.0024\Omega$$

- How can we measure such a small change in resistance?
- One answer is the *Wheatstone bridge*:



Analysis of Wheatstone bridge

- Think of it as a set of two voltage dividers:

$$V_A = \frac{R_4}{R_1 + R_4} V_o$$

$$V_B = \frac{R_3}{R_2 + R_3} V_o$$

- So the difference in voltages is:

$$V_A - V_B = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) V_o$$

- Note that when $R_4 R_2 = R_3 R_1$,

$$V_A - V_B = 0$$

- The bridge is *balanced* when this happens

- For simplicity, we'll choose $R_2 = R_3 = R_4 = R$
- The bridge is then balanced when $R_1 = R$
- Let's see what happens when R_1 is close to, but not equal to, R:

$$R_1 = R + \delta$$

$$\Delta V = V_A - V_B = \left(\frac{R}{R+\delta+R} - \frac{R}{R+R} \right) V_o$$

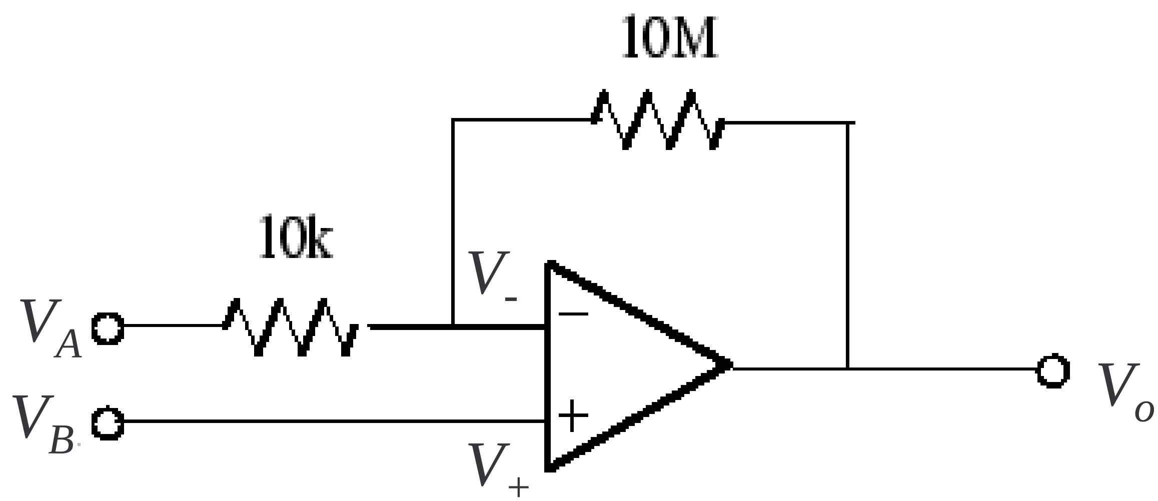
$$= \frac{1}{2} \left(\frac{1}{\delta/(2R)+1} - 1 \right) V_o$$

$$\approx \frac{1}{2} \left(1 - \frac{\delta}{2R} - 1 \right) V_o = -\frac{\delta}{4R} V_o$$

- So we see that the voltage difference varies linearly with δ
  - But since the change in resistance is small, so is the change in voltage

$$\frac{\Delta V}{V_0} \text{ can be } \sim 10^{-6}$$

- So we need to measure a very small voltage difference, without being sensitive to fluctuations in V_o itself
  - sounds like a job for a differential amplifier!
- We might try a variation on the inverting amplifier discussed in the last lecture
  - This is called a “follower with gain”:



- This has some nice features
  - for example, the input impedance is very high (equal to the internal resistance of the op-amp)
- Here's how it works as an amplifier:
  - The op-amp makes sure that $V_+ = V_-$
  - We also know that:

$$V_A - I \cdot 10\text{k}\Omega = V_- = V_B$$

$$V_- = V_B = V_o + I \cdot 10\text{M}\Omega$$

  - Current is the same through both resistors since no current flows into the op-amp
  - So we have two expressions for I that must be equal:

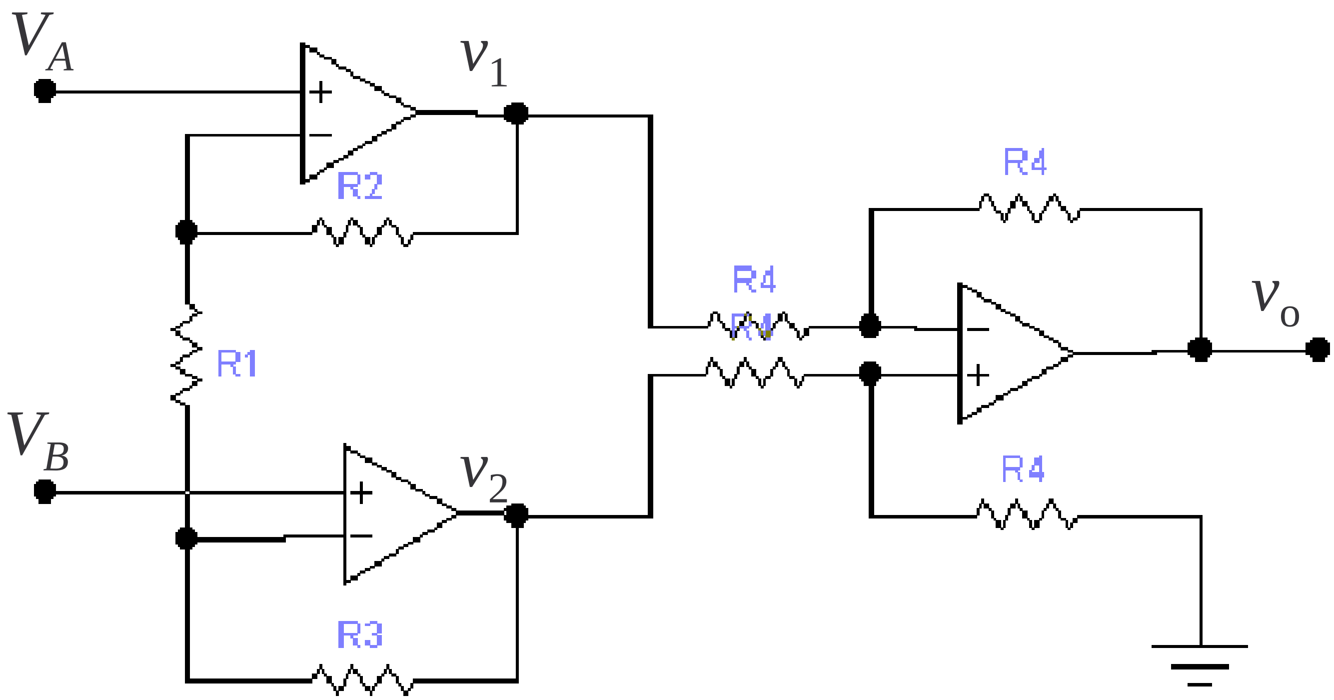
$$I = \frac{V_A - V_B}{10\text{k}\Omega} = \frac{V_B - V_o}{10\text{M}\Omega}$$

$$V_o = \frac{10\text{M}\Omega}{10\text{k}\Omega}\left(V_B - V_A\right) + V_B \approx \frac{10\text{M}\Omega}{10\text{k}\Omega}\left(V_B - V_A\right)$$

- This has high gain (1000), depending only on resistor values
  - that's good!
- But the output is only *approximately* equal to the difference between the inputs
  - there was still that V_B term all by itself
  - means there is some common-mode gain as well
- This circuit is good enough for many differential-amplification uses
  - but not good enough for our strain gauge!

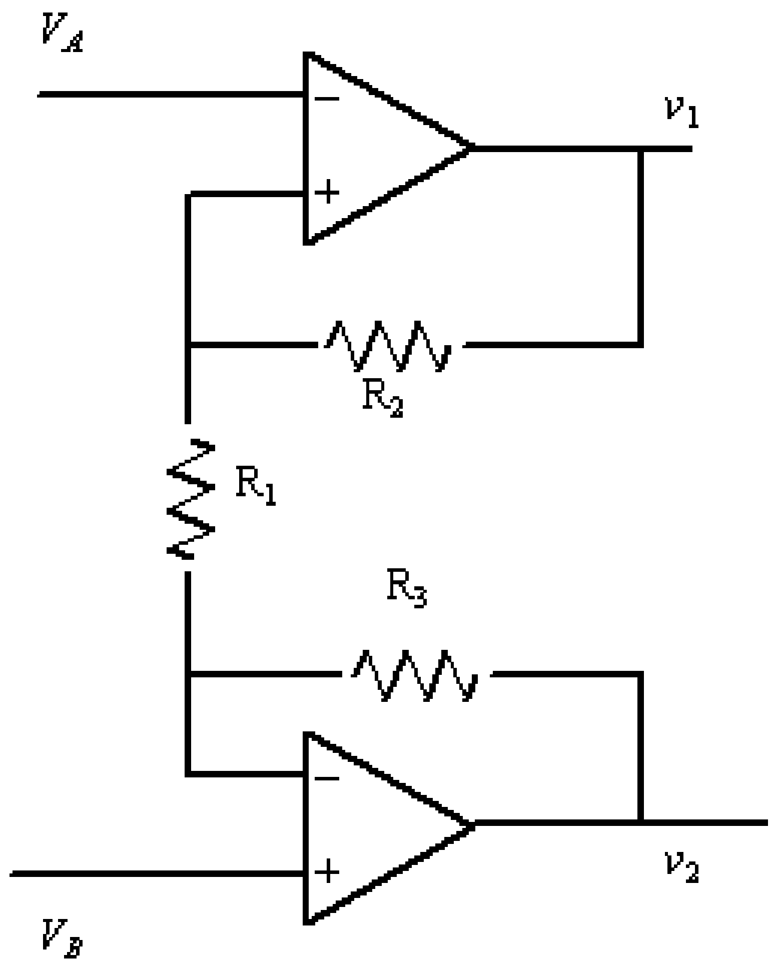
Instrumentation Amplifier

- To do the job we want, we need a circuit like this (called an *instrumentation amplifier*):



- We’ll break this up into pieces to see how it works

- We'll start with the two left op-amps:



- Each op-amp has equal voltage at its inputs
  - so, voltage at top of R_1 is V_A, and at bottom it's V_B
- Current through R_1 is:

$$I = \frac{V_A - V_B}{R_1}$$

- This current can't go into op-amps
  - must go across R_2 and R_3

- Putting that information together, we have:

$$I = \frac{V_A - V_B}{R_1}$$

$$v_1 - IR_2 = V_A$$

$$v_2 + IR_3 = V_B$$

$$v_1 = V_A + \frac{R_2}{R_1}(V_A - V_B)$$

$$v_2 = V_B - \frac{R_3}{R_1}(V_A - V_B)$$

$$v_1 - v_2 = (V_A - V_B)\left(1 + \frac{R_2 + R_3}{R_1}\right)$$

- So this is also a differential amplifier
  - gain determined by resistors, which is good!

- What about the common-mode gain of these two op-amps?
- Using the equations on the previous slide, we have:

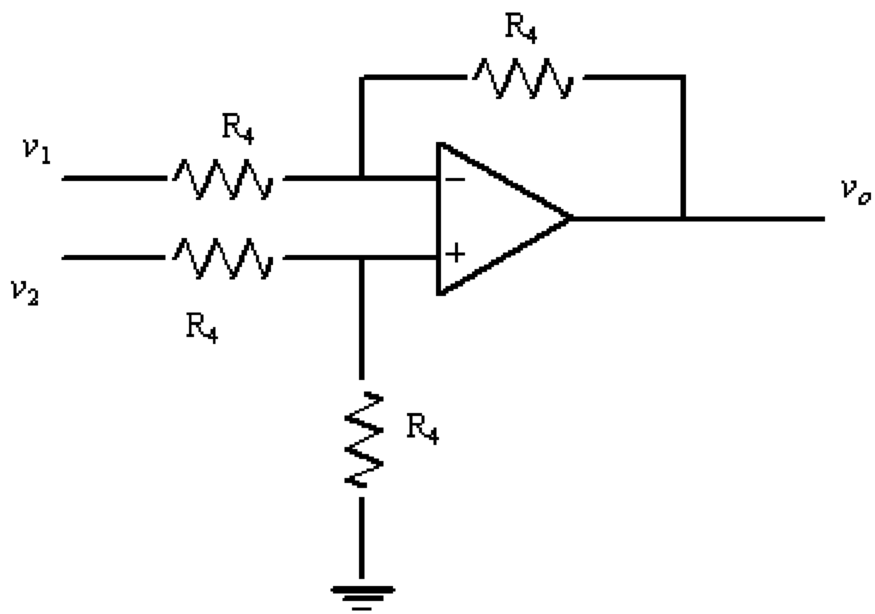
$$v_1 = V_A + \frac{R_2}{R_1}(V_A - V_B)$$

$$v_2 = V_B - \frac{R_3}{R_1}(V_A - V_B)$$

$$v_1 + v_2 = V_A + V_B + \left(\frac{R_2 - R_3}{R_1}\right)(V_A - V_B)$$

- If we choose R_1 and R_2 to be equal, the common-mode output is the same as the input
  - in other words, common-mode gain is one
- CMRR is already pretty good for this circuit
  - but not good enough for our strain gauge!

- That brings us to the final op-amp in our instrumentation amplifier:



- Applying the ideal op-amp rules here, we have:

$$V_- = v_1 - I_1 R_4 = v_o + I_1 R_4$$
$$V_+ = v_2 - I_2 R_4 = I_2 R_4$$
$$V_+ = V_-$$

- We can solve for the currents:

$$I_2 = \frac{v_2}{2R_4}$$
$$I_1 R_4 = v_1 - v_2 + I_2 R_4 = v_1 - v_2 / 2$$
$$I_1 = \frac{2v_1 - v_2}{2R_4}$$

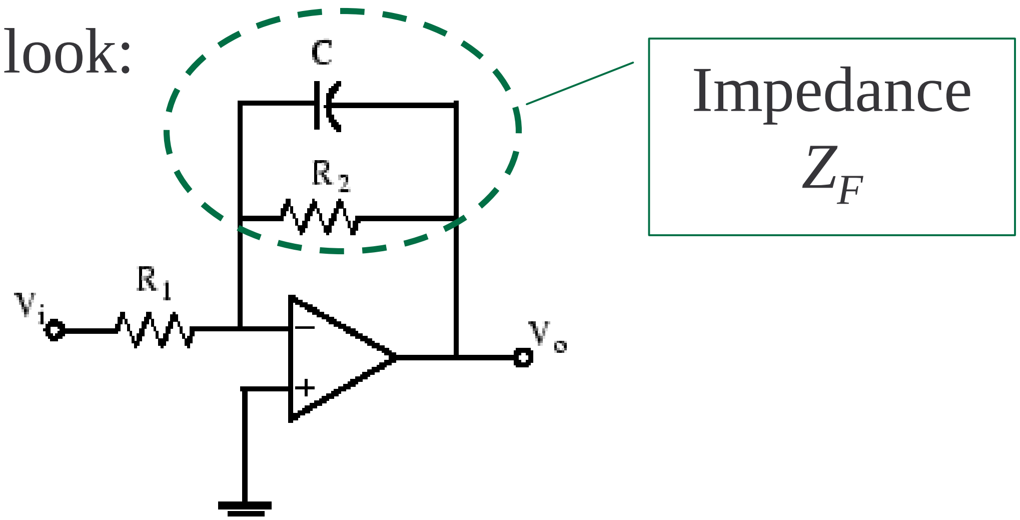
- With this, we can solve for v_o:

$$v_o = v_1 - 2\left(v_1 - \frac{v_2}{2}\right) = v_2 - v_1$$

- This part of the circuit is a *subtractor*
  - Gets rid of the common-mode signal that makes it through the first set of op-amps
- Note that exact subtraction requires all four resistors to have the same value
  - Large CMRR will require the use of high-precision resistors!

Active low-pass filter

- We studied filters in the first lecture, and built them in the first lab
- Those were “passive” filters
  - they could transmit or supress a signal, but they couldn’t amplify it
- With an op-amp, we can build an “active filter”
  - an amplifier where the gain depends on the frequency of the signal
- Here’s how it might look:



- This looks like an inverting amplifier, so we know the gain is:

$$\frac{V_o}{V_i} = -\frac{Z_F}{R_1}$$

- The impedance is given by:

$$\frac{1}{Z_F} = \frac{1}{R_2} + \frac{1}{Z_c} = \frac{1}{R_2} + i\omega C = \frac{1 + i\omega R_2 C}{R_2}$$

$$Z_F = \frac{R_2}{1 + i\omega R_2 C}$$

- So the gain is:

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\left(\frac{1}{1 + i\omega R_2 C}\right)$$