

## Lesson Number 1. in an Oklahoma Wind Power Tutorial Series

By Tim Hughes, Environmental Verification and Analysis Center, The University of Oklahoma

### Calculation of Wind Energy and Power

Calculating the energy (and later power) available in the wind relies on knowledge of basic geometry and the physics behind kinetic energy. The kinetic energy (**KE**) of an object (or collection of objects) with total mass **M** and velocity **V** is given by the expression:

$$\mathbf{KE} = \frac{1}{2} * \mathbf{M} * \mathbf{V}^2 \quad (1)$$

Now, for purposes of finding the kinetic energy of moving air molecules (i.e.:wind), let's say one has a large air parcel with the shape of a huge hockey puck: that is, it has the geometry of a collection of air molecules passing through the plane of a wind turbine's blades (which sweep out a cross-sectional area **A**), with thickness (**D**) passing through the plane over a given time.

The volume (**Vol**) of this parcel is determined by the parcel's area multiplied by its thickness:

$$\mathbf{Vol} = \mathbf{A} * \mathbf{D}$$

Let  $\rho$  (the greek letter 'rho') represent the density of the air in this parcel. Note that density is mass per volume and is expressed as:

$$\rho = \mathbf{M} / \mathbf{Vol}$$

and a little algebra gives:  $\mathbf{M} = \rho * \mathbf{Vol}$

Now let's consider how the velocity (**V**) of our air parcel can be expressed. If a time **T** is required for this parcel (of thickness **D**) to move through the plane of the wind turbine blades, then the parcel's velocity can be expressed as  $\mathbf{V} = \mathbf{D} / \mathbf{T}$ , and a little algebra gives  $\mathbf{D} = \mathbf{V} * \mathbf{T}$ .

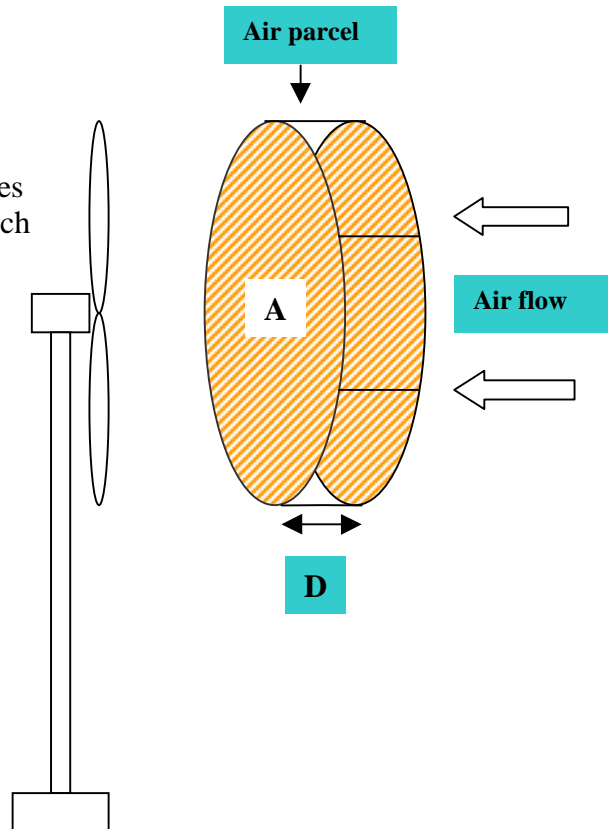
Let's make some substitutions in expression no. 1 ( $\mathbf{KE} = \frac{1}{2} * \mathbf{M} * \mathbf{V}^2$ )

Substitute for **M** ( $= \rho * \mathbf{Vol}$ ) to obtain:  $\mathbf{KE} = \frac{1}{2} * (\rho * \mathbf{Vol}) * \mathbf{V}^2$

And **Vol** can be replaced by  $\mathbf{A} * \mathbf{D}$  to give:  $\mathbf{KE} = \frac{1}{2} * (\rho * \mathbf{A} * \mathbf{D}) * \mathbf{V}^2$

And **D** can be replaced by  $\mathbf{V} * \mathbf{T}$  to give:  $\mathbf{KE} = \frac{1}{2} * (\rho * \mathbf{A} * \mathbf{V} * \mathbf{T}) * \mathbf{V}^2$

Leaving us with:  $\mathbf{KE} = \frac{1}{2} * \rho * \mathbf{V}^3 * \mathbf{A} * \mathbf{T}$



Now, power is just energy divided by time, so the power available from our air parcel can be expressed as:

$$\underline{\mathbf{Pwr}} = \mathbf{KE} / \mathbf{T} = (1/2 * \rho * \mathbf{V}^3 * \mathbf{A} * \mathbf{T}) / \mathbf{T} = \underline{1/2 * \rho * \mathbf{V}^3 * \mathbf{A}}$$

And if we divide **Pwr** by the cross-sectional area (**A**) of the parcel, then we are left with the expression:

$$\underline{\mathbf{Pwr} / \mathbf{A}} = \underline{1/2 * \rho * \mathbf{V}^3}$$

Note two important things about this expression: one is that the power is proportional to the **cube** of the wind speed. The other is that by dividing power by the area, we have an expression on the right that is independent of the size of a wind turbine's rotor. In other words, **Pwr/A** only depends on (1) the density of the air and (2) the wind speed. In fact, there is no dependence on size, efficiency or other characteristics of wind turbines when determining **Pwr/A**.

The term for the quotient **Pwr/A** is called the "**Wind Power Density**" (**WPD**) and has units of watts/m<sup>2</sup>. **WPD** will be discussed more in later lessons.

An excellent treatment of Wind Power Density and other wind power related concepts can be found at this web location: <http://www.windpower.org/tour/wres/enerwind.htm>

### **Sample exercises:**

1. For a wind turbine with rotor diameter 43 meters (a typical size for a 600 kW turbine), calculate the volume and mass of a 1 meter thick parcel of air passing through the plane of the turbine blades (for this exercise, assume a value for the air density of 1.225 kg/m<sup>3</sup>).
2. Assume there is a wind blowing with a constant velocity **V** of 10.0 m/s through the blades of the turbine described in no. 1. What is the wind power density? (again, assume **ρ** = 1.225 kg/m<sup>3</sup>)
3. A smaller wind turbine which produces about 50kW has a blade rotor radius of 7 m. (14 m. diameter). Calculate **WPD** for this wind turbine, assuming the same conditions as given in Exercises 1 and 2.

Future lessons will cover these topics.

- **Air Density**
- **Determining Estimates of Wind Power Density (WPD) and Wind Power Classes**
- **Determining More Accurate Estimates of Wind Power Density**
- **How Turbine Differences Affect Power Output**

## Answers to sample exercises

**1.) Ans.:** the volume of this disk of air equals its cross-sectional area  $A$  ( $= \frac{1}{4} \text{PI} * \text{Diameter}^2$ ) times the disk's depth ( $D$ ):  $\text{Vol} = \frac{1}{4} * 3.1416 * (43 \text{ m})^2 * 1 \text{ m} = \underline{\underline{1451 \text{ m}^3}}$ .

And its mass equals the volume of air times air density:

$$\text{M} = \rho * \text{Vol} = 1.225 \text{ kg/m}^3 * 1451 \text{ m}^3 = \underline{\underline{1780 \text{ kg.}}} \text{ (or } \sim 3900 \text{ pounds - about the weight of an intermediate car!)}$$

**2.) Ans.:**  $\text{WPD} = \text{Pwr} / \text{A} = \frac{1}{2} * \rho * V^3$   
 $= \frac{1}{2} * 1.225 \text{ kg/m}^3 * (10.0 \text{ m/s})^3$   
 $= \underline{\underline{613 \text{ watts/m}^2}}$  (this is an excellent value for WPD, as will be discovered in future lessons)

**3.) Ans.:** Easy! Remember that WPD is independent of turbine type or size, and only depends on wind speed and density. Hence, the WPD is still 613 watts/m<sup>2</sup>, the same as calculated in exercise no. 2.

Vers. Date:11/11/00