Lesson Number 2. in an Oklahoma Wind Power Tutorial Series

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Estimating and Calculating Air Density

As determined in Lesson 1., the wind power density term is directly proportional to air density:

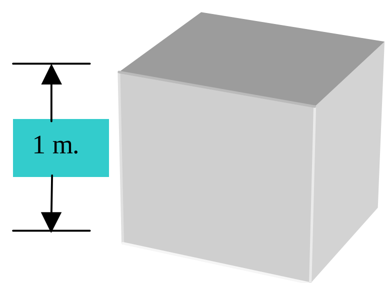
$$\mathbf{WPD} = \tfrac{1}{2} * \rho * \mathbf{V}^3$$

where the greek letter ρ stands for air density**,** which is defined by:

$$\rho = \mathbf{M} \ / \ \mathbf{Vol}$$

Air density can be defined as the mass of a given sample of air, divided by that sample's volume.

Example: for the cube on the right, measuring 1 meter on each side, the density of the air inside would be determined by its mass divided by 1 cubic meter (or per m^3). At sea level and under standard conditions (temperature of 25 degrees C and pressure = 1 atmosphere), the mass of air in this cube would be 1.225 kg, so the density of the air is **1.225 kg per m^3.**



Air density can be approximated or calculated in the following ways.

Approximating air density.

Method 1. If your area of study is close to sea level, and is in a region with moderate climate, using the value of 1.225 kg./m^3 will not introduce a huge error into your estimate of WPD. This is especially so if you are looking at annual average values, so that changes in air density due to seasonal changes in air temperature are averaged over time and a variety of conditions.

Method 2. However, if the area is much above sea level, or if you just want to be more precise, you can use this approximation to account for elevation change:

$$\rho = \mathbf{1.225 - (1.194 * 10^{-4}) * z} \quad \text{(z=the location's elevation above sea level in meters)}$$

This approximates the U.S. Standard Atmosphere profile for air density and will give a good long-term average value of air density in temperate areas.

Calculating air density more exactly.

Method 3. If your region of interest has a less temperate climate (i.e.: it is either quite hot or quite cold most of the time), or if you are interested in seasonal estimates of wind power density, the following expression gives a much better value for air density.

ρ = **P / RT** (kg/m^3) where P = air pressure (in units of Pascals or Newtons/m^2)
R = the specific gas constant (287 J kg^{-1} Kelvin^{-1})
T = air temperature in degrees Kelvin (deg. C + 273)

Method 4. While air temperature data is not too hard to come by (and is relatively cheap to measure even if you don't have data already), air pressure data can be tougher to find. If air pressure data for your region is unobtainable, you can estimate density as just a function of site elevation and temperature with the following expression.

$$\rho = (P_o / RT) \exp(-g_* z/RT) \quad (\mathrm{kg/m^3})$$

where P_o = standard sea level atmospheric pressure (101,325 Pascals) [or you can use a sea-level adjusted pressure reading from a nearby weather station];
g = the gravitational constant (9.8 m/s^2); and
z = the region's elevation above sea level (in meters)

Another treatment of air density as it relates to wind power can be found at this web site:
http://www.windpower.org/tour/wres/enerwind.htm

Sample exercises:

1. If you were responsible for calculating the annual average WPD for a potential wind farm to go offshore in the Gulf of Mexico, but weather data was scarce, which method would you use to estimate air density? Why?

2. The Oklahoma Mesonet is a statewide network of 115 weather stations. The station with the lowest elevation is near Idabel in southeast OK and is about 110 m above sea level. The highest station is near Kenton (near the Black Mesa in the far west part of the panhandle) and is at 1322 m. If the annual average temperature is about the same in both areas (this is far from the case, but this exercise is meant to just illustrate the dependence of WPD on air density alone):
   a.) what would be the ratio of Kenton's air density to that of Idabel, on average?
   b.) If both sites had the same winds (again, this is far from the case), what is the ratio (as a percentage) in WPD for these two areas, based on air density alone?

3. For Kenton, typical values for daily average pressure and temperature in the coldest month of winter are about 101392 Pascals and 11 degrees Celsius, and those for the hottest month of summer are about 101935 Pascals and 32 degrees Celsius. Determine the ratio of air densities and WPD under these two sets of conditions (again assuming the wind data is the same).

Answers to sample exercises

1. **Ans.:** Because the climate is moderate and conditions will average to close to standard conditions over a year, and because the site will be at sea level, **using Method 1.(assume $\rho = 1.225\ kg/m^3$) will suffice.**

2. **Ans.:** a.) Since we are told to assume the same temperature conditions, Method 2. will suffice for reasonable estimates of air density.

   For Idabel: $\rho = 1.225 - (1.194*10^{-4} * 110\ m.) = 1.212\ kg/m^3$

   For Kenton: $\rho = 1.225 - (1.194*10^{-4} * 1322\ m.) = 1.067\ kg/m^3$

   The air density at Kenton would be 1.067/1.212 = **88.0%** of that at Idabel.

   b.) Since WPD is dependent only on wind and air density, and we are told to assume the wind conditions are the same at both sites, the ratio of WPD at Kenton to that at Idabel will be the same as the ratios of air density, or **88.0 %**.

3. **Ans.:** In the coldest months, Kenton will have air density values around:

   $\rho_{cold} = P / RT = (101392\ N/m^2) / [(287\ J\ K^{-1}\ kg^{-1}) * (273K + 11\ K)] = \underline{1.244\ kg/m^3}$

   In the warmest months:

   $\rho_{warm} = P / RT = (101935\ N/m^2) / [(287\ J\ K^{-1}\ kg^{-1}) * (273K + 32\ K)] = \underline{1.165\ kg/m^3}$

   Since we are to assume the wind conditions are identical, all terms except the air density cancel when we take the ratio of WPD (cold season) to WPD (warm season):

   $WPD_{cold} / WPD_{warm} = \rho_{cold} / \rho_{warm} = 1.244\ kg/m^3 / 1.165\ kg/m^3 \sim \mathbf{1.07}$

   Hence, cold weather could mean higher WPD values by about **7%**, assuming the winds blow the same (there are of course seasonal differences, but this will be covered in later lessons).

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