Lesson Number 3. in an Oklahoma Windpower Tutorial Series

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Determining Wind Power Density and Wind Power Classes From Wind Speed Information

Review: In lessons 1. and 2. we determined this expression for wind power density:

$$\mathbf{WPD} = \tfrac{1}{2} * \rho * V^3 \qquad \textbf{(1)}$$

And that air density can be determined to varying degrees of accuracy with the following.

Methods

1.) $\rho = 1.225\ \mathrm{kg/m^3}$ (constant value based on U.S. Std. Atmosphere, at sea level)

2.) $\rho = 1.225 - (1.194 * 10^{-4}) * z$ (z=the location's elevation above sea level in m.)

3.) If you have pressure and temperature data:

$\rho = P / RT$ ($\mathrm{kg/m^3}$)

where P = air pressure (in units of Pascals or Newtons/m^2)
R = the specific gas constant (287 J kg^{-1} $Kelvin^{-1}$)
T = air temperature in degrees Kelvin (deg. C + 273)

4.) If you have temperature data but not pressure data:

$\rho = (P_o / RT) * \exp(-g * z/RT)$ ($\mathrm{kg/m^3}$)

where P_o = std. sea level atmospheric pressure (101,325 Pascals)
g = the gravitational constant (9.8 m/s^2); and
z = the region's elevation above sea level (in meters)

New lesson: Note that the expression (1) for WPD is a simplification that held for our example in Tutorial 1 because we made the tacit assumption that the wind blew with speed **V** *all the time.* In reality, varying winds mean we must work a little harder to find the true WPD. To get the most accurate estimate for Wind Power Density, one must actually perform a summation using data taken over time, as follows.

$$\mathbf{WPD} = 0.5 * 1/n * \sum_{j=1}^{n} (\rho_j * V_j^3) \qquad \textbf{(2)}$$

where n is the number of wind speed readings and ρ_j and $\mathbf{V_j}$ are the j^{th} (1^{st}, 2^{nd}, 3^{rd}, etc.) readings of the air density and wind speed.

Since air density ρ and wind speed **V** will change with every data reading, the most accurate result would entail a calculation for every data interval. For example, to calculate the best possible value for WPD for an Oklahoma Mesonet weather station location for a year, one would need to perform calculations for ρ and **V** for 105,120 data intervals! (288 observations per day times 365 days per year). Clearly this must involve running computer programs and is one method by which the Oklahoma Wind Power Assessment Initiative will assess WPD for our state.

Fortunately, there are two ways to get reasonable estimates for WPD without doing all the calculations described above:

Method 1.) The best way to approximate WPD uses the results of a wind speed frequency distribution (this is like a histogram - a sample table of wind speed frequency occurrence from a Mesonet weather station will be shown in the next tutorial). Using such distribution information, the following summation can be applied:

$$\textbf{WPD} = 0.5 * \sum_{j=1}^{n} [\rho * (\text{median } \mathbf{V}^3 \text{ in class j}) * (\text{frequency of occurrence in class j })]$$

If one uses a value for air density that does not change over time (like values from methods no. 1 or 2) , then air density can come out of the expression to give:

$$\textbf{WPD} = 0.5 * \rho * \sum_{j=1}^{n} [(\text{median } \mathbf{V}^3 \text{ in class j}) * (\text{frequency of occurrence in class j})]$$

By using wind summary products that are available from the Oklahoma Mesonet, one can get a reasonable estimate of WPD that does not involve hundreds of thousands of calculations. In fact, it can be done by summing only about 8 terms in the above expression. This method will be covered in more detail in the following tutorial (no. 4.)

Method 2.) For the rest of this tutorial, we will focus on a simpler method by which you can estimate the WPD in your area of interest.

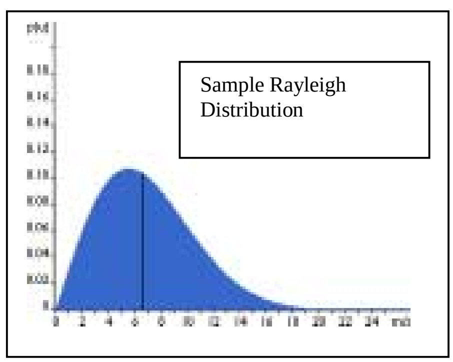
If one makes an assumption about how wind speeds are distributed in the wind speed frequency diagram, one can approximate WPD with the following:

$$\textbf{WPD} = 0.5 * K * \rho * (\text{mean wind speed})^3 \qquad \textbf{(3)}$$

where K = a value determined by the shape of the distribution pattern that the wind speeds follow.

For example, some wind speed frequency patterns follow a "Rayleigh" distribution, which looks like the figure to the right. For some wind speed distributions, K will have the value 1.91. Inserting that into the expression above gives:

WPD = 0.955 * ρ* (mean wind speed)^3



Now, all that is needed is knowledge of your nearest weather station's mean wind speed and your elevation (to use method 2. for estimating air density) and you can find a reasonable estimate for WPD in your area.

An important note: it may be tempting to simply take the mean wind speed for your area and plug that in for **V** in the expression **WPD** = ½ * ρ * **V**^{3}, but this will give you an erroneous value. This is basically because *the mean of the cubes of wind velocities will almost always be greater than (mean wind speed)^3*. That's a mouthful. To understand this better, do exercises 1 and 2 below.

Determining "Wind Classes"

Areas are often described by their "wind class" ranking, rather then their range of Wind Power Densities or mean wind speeds. Below is a table that shows the ranges of WPD and associated classes, at 10 meters height above ground (a typical wind speed measurement height and small turbine height) and at 50 meters (the industry standard level for WPD determinations for large wind turbines). Note that it also gives the ranges of mean wind speed for each class.

	10 m (33 ft)		50 m (164 ft)	
Wind Power Class	Wind Power Density (W/m^2)	Mean Speed range (b) m/s (mph)	Wind Power Density (W/m^2)	Mean Speed range (b) m/s (mph)
1	<100	<4.4 (9.8)	<200	<5.6(12.5)
2	100 - 150	4.4 (9.8)/5.1 (11.5)	200 - 300	5.6 (12.5)/6.4 (14
3	150 - 200	5.1 (11.5)/5.6 (12.5)	300 - 400	6.4 (14.3)/7.0 (15
4	200 - 250	5.6 (12.5)/6.0 (13.4)	400 - 500	7.0 (15.7)/7.5 (16
5	250 - 300	6.0 (13.4)/6.4 (14.3)	500 - 600	7.5 (16.8)/8.0 (17
6	300 - 400	6.4 (14.3)/7.0 (15.7)	600 - 700	8.0 (17.9)/8.8 (19
7	>400	>7.0 (15.7)	>800	>8.8 (19.7)

(a) Vertical extrapolation of wind speed based on the 1/7 power law
(b) Mean wind speed is based on the Rayleigh speed distribution of equivalent wind power density. Wind speed is for standard sea-level conditions. To maintain the same power density, mean wind speed must increase 3%/1000 m (5%/5000 ft) of elevation. (from the Battelle Wind Energy Resource Atlas)

This table and more explanation of wind power classes can be found on the web pages of the American Wind Energy Association (www.awea.org/faq/basicwr.html).

Sample Exercises

1.) Imagine that you have just 2 readings of wind speed: 5 m/s and 15 m/s. Calculate the WPD over the interval of these readings (assume $\rho = 1.0$ kg/m^3 to make the math easier).
2.) Calculate the mean of the two readings given in exercise 1, plug that value into expression (1), and compare to your answer above to see the error in WPD that would result.
3.) Let's say you are interested in buying and installing a small wind turbine, and that you live near the Cheyenne Mesonet station (CHEY). But first you want to get some idea of the WPD (at 10 meters). Given that CHEY's elevation is 692 m. and its long term mean wind speed (at 10 meters) is 5.8 m/s, estimate the WPD in that area (assume a Rayleigh distribution for the winds, with K=1.91). What wind class does this represent?

Answers to sample exercises

1.) **Ans.** If you do this the correct way (using expression 2), you will have:

$$\mathbf{WPD} = 0.5 * 1/2 * \sum_{j=1}^{2} (\rho_j * V_j^3) = 0.25 * [(1.0 * 5^3) + (1.0 * 15^3)] = \underline{\mathbf{875\ W/m^2}}$$

2.) **Ans.** While if you mistakenly plug the mean wind speed (the mean of 5 m/s and 15 m/s is 10 m/s) into expression 1, you will get:

$$\mathbf{WPD} = ½ * \rho * V^3 = ½ * 1.0 * (10)^3 = \underline{\mathbf{500\ W/m^2}}$$

Clearly, the incorrect method gives a much lower value of WPD, and this will almost always be the case.

This is another illustration of the significance of the dependence of WPD on the cube of the wind speeds. The more wind speeds that fall into the high end of the wind speed frequency distribution, the higher your value of WPD will be.

3.) **Ans.** Use expression 3: WPD = 0.5 * K * ρ * (mean wind speed)^3, where K = 1.91.

First calculate air density using the adjustment for elevation only, since you have no other data.

$$\rho = 1.225 - (1.194 * 10^{-4}) * z = 1.225 - (1.194*10^{-4}) * 692 \text{ m} = 1.091 \text{ kg/m}^3$$

Then WPD = 0.5 * 1.91 * (1.091 kg/m^3) * (5.8 m/s)^3 = **203 W/m^2**.

Looking at the table above, you will see that wind class 4 at 10 meters has WPD values between 200 and 250 W/m2. Therefore, your area near Cheyenne is **wind class 4**. This is a very good wind resource, by the way.

A note for later study:
The assumption that your wind resource is close to that of the Cheyenne Mesonet site depends heavily on your elevation and vegetation being similar to that of the site. That is, if you live in a valley (a relative low spot) while the site is at a relative high spot (as CHEY is), and/or if your site is surrounded by trees or other heavy vegetation (while CHEY is clear), your wind resource will not be as good. The dependence of wind resource on elevation and vegetation will be covered in later tutorials.

You can learn more about the terrain and vegetation around Cheyenne and other mesonet sites by visiting the OK Mesonet web site: http://okmesonet.ocs.ou.edu/siteinfo/