Missing Cycle Modulation
Pulse Position Phase Reversal Keying (3PRK)
10/2805

Pulse Position Phase reversal Keying (3PRK) and MCM are variations of the Ultra Narrow Band Modulation methods. This method reverses or removes one cycle of the RF carrier to designate the presence of a digital one. The remainder of the time, the carrier remains unchanged as a steady CW signal. The bandpass filters used must have near zero group delay to pass the RF waveform un-altered.

Figure one shows the reversal of one cycle and the resulting RF waveform.

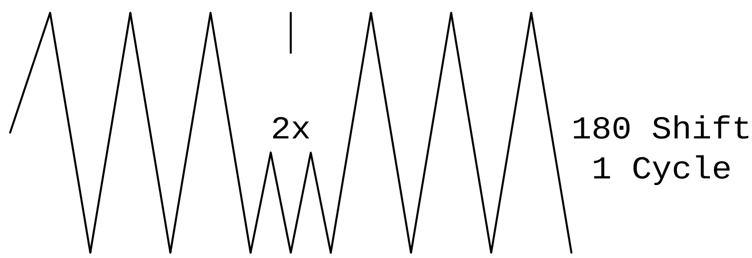


Figure 1. The RF waveform when one cycle is reversed. Note that for a period of one RF cycle (IF cycle is equally applicable), the frequency is doubled. If the IF frequency is 48 MHz, there are two cycles at 96 MHz present instead of one cycle at 48 MHz.

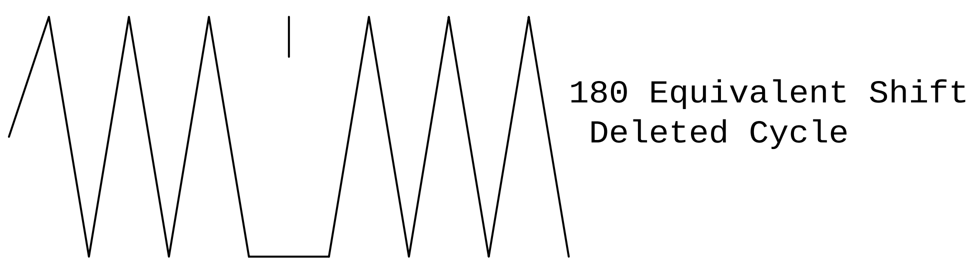


Figure 2. When the ultra narrow bandpass filter, or a low pass filter is used, the filter will not pass the 96 MHz component, resulting in energy black out, or a missing cycle. This is the same as ON/OFF keying. In fact, instead of a 180 degree phase reversal, the signal could just as well have been keyed OFF for that cycle and the result would be the same. This alternate method is referred to as 'missing cycle modulation', or MCM. At the transmitter it is merely a matter of using an XOR gate as a modulator for 3PRK, or an AND gate for MCM.

The waveform after filtering is seen in Figure 3. The missing cycle is very obvious. The lower trace is a reference waveform used in a phase detector to detect the missing cycle.

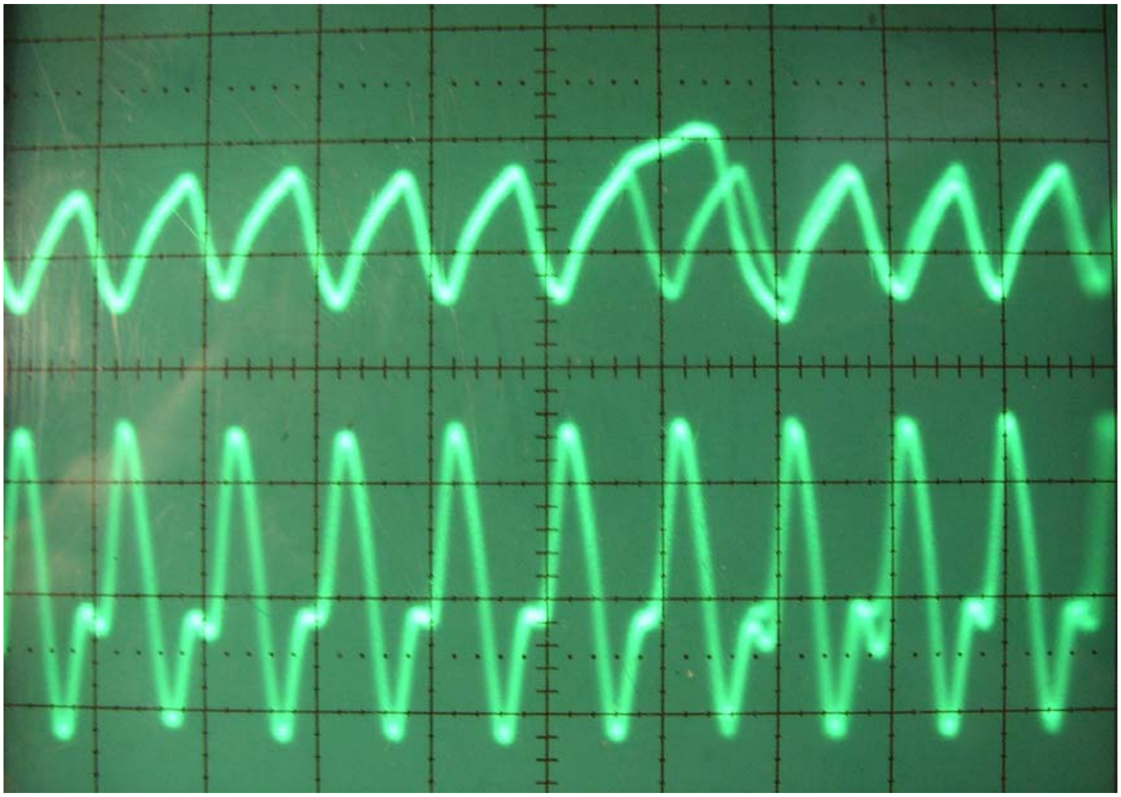


Figure 3. The Missing Cycle from 3PRK modulation or MCM modulation.

Although it is obvious from the above that the carrier is undisturbed for most of the time, in fact only once every other bit period on average over time, since digital ones occur only 50% of the time, there is more than adequate mathematical proof of the process. One paper published in 1939 by Prof. Howe is of note.

THE MSB SPECTRUM:

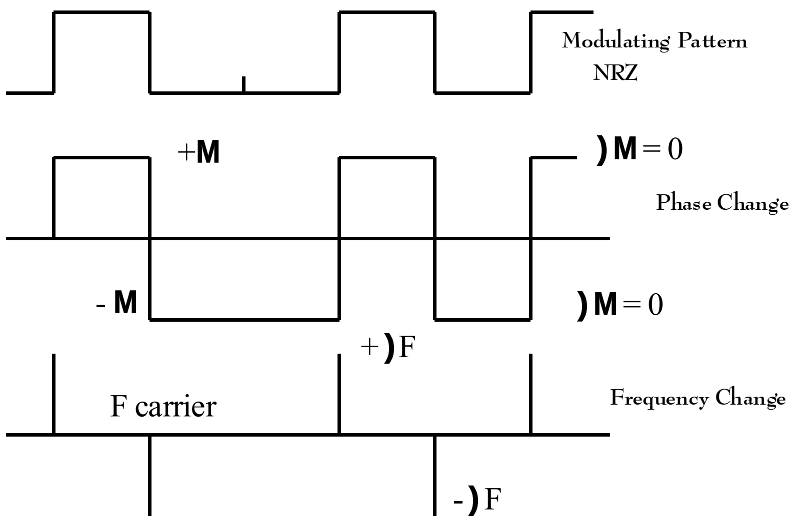
Abrupt phase change modulation has no useful sidebands. To understand how this is possible, it is necessary to study the basis of phase and frequency modulation. Refer to Figure 4 below.

The frequency change resulting from a modulating input is: $F = F_{carrier} + \Delta f$.
Δf can be calculated from the basic relationship $\omega t = \Phi = 2\pi ft$, which can be rewritten in derivative form as $\Delta f = \Delta\Phi/2\pi\Delta t$. The rise and fall time is fixed by the the circuit parameters and baseband code. $\Delta f = \Delta\Phi/2\pi\Delta t$.

There is no Δf if $\Delta\Phi = 0$.

A phase detector using $F_{carrier}$ as a phase reference will detect the phase changes as positive and negative voltages. **During the rise and fall times only**, there is a phase

change $\Delta\Phi$, which causes a frequency change Δf. During the rise time, which is assumed to be 1 cycle period in duration, there is a pulsed positive frequency shift that causes sinx/x spikes above the carrier frequency. When the phase reverses, these sinx/x spikes appear below the carrier frequency for the duration of one IF cycle.. A conventional filter would have to have a very broad bandpass to include these spikes, which Nyquist says are unnecessary for detecting the data information on the carrier.
Figure 4. Change in Frequency with Phase Change as a General Concept Using Abrupt



Phase Changes.

When the phase change is abrupt, the rise and fall times are near zero, and there is a large $\Delta\Phi/2\pi\Delta t$ value, which causes a very large Δf of very short duration.(about 1 RF cycle). At all other times, it can be seen that $\Delta\Phi$ is zero and the frequency $F = F_{carrier}$. A phase detector using $F_{carrier}$ as a phase reference will detect the phase changes as positive and negative voltages conforming to the input,

Figure 5 shows the effect when only one cycle is changed. The phase reverses for one cycle, causing a sudden shift in frequency for the duration of one cycle. The narrow bandpassfilter cannot pass the higher frequency, so for that period of time there is no enrgy transmitted. The remainder of the time, the carrier remains unchanged.

The method could be considered an ‘ultra wide pulse’ method, as oppposed to the ultra narrow pulses used with Ultra Wide Band modulation.

There is no question but that the tranmitted energy is all in the carrier. The information is in what is missing from the carrier. Any interference will have to fill in the missing cycle to cause an error.

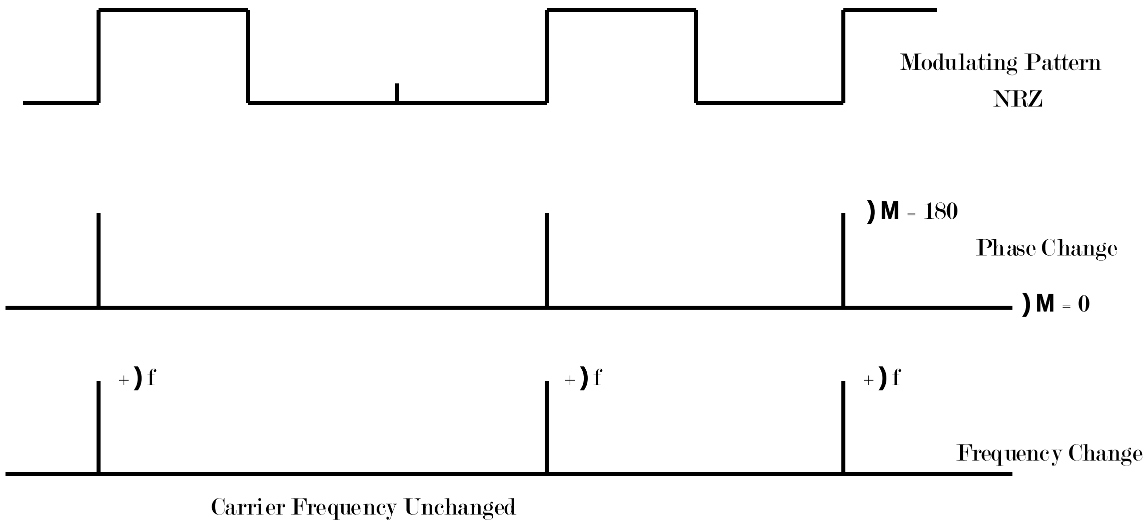


Figure 5. Spectrum for one cycle changed in the bit stream for a digiatl one.

There is no FM, and there <u>are no Bessel or other sidebands</u> during most of the bit period. There are sinx/x spikes lasting one IF cycle. All spectral components outside the carrier are Fourier amplitude products. Amplitude products do not contribute to PM.

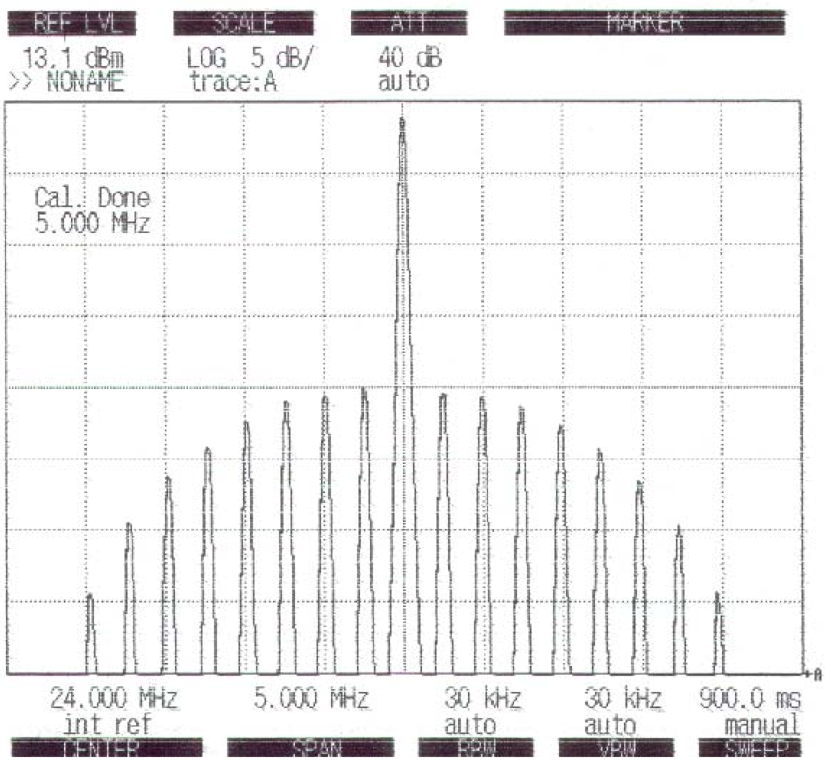


Figure 6. Spectrum of the MCM modulation when one cycle in 10 is altered.

NOTE: Taub and Schilling explain the absence of Bessel products and the presence of Fourier amplitude products---" The power spectral density and the correlation function of a waveform are a Fourier transform series pair". Result- sinx/x spectrum.

The spectrum in Fig. 6 is that given by the Fourier transform for a rectangular pulse having the time off 't' and the time ON 'T' above.

Fourier Pulse Train Expansion:

The equation-

$$A_n = \frac{2At}{T_0}\frac{Sin(n\pi t/T_0)}{(n\pi t/T_0)} \qquad = \sin/x/x$$

can be expanded into--

$v(t) = \mathcal{F}^{-1}$ {2 A $_{av}$ [½ +(2/π) Cos θ - (2/3π) Cos 3θ +(2/5π) Cos 5θ - + -----] **}**

$\theta = 2\pi\ (t/T_0) = 2\pi\ (1/T_0)t = 2\pi ft = \omega t$ $\qquad A_{av} = A(t/T_0)$

Note that it has harmonics at 3θ, 5**θ,** 7**θ,** 9θ **etc. ---** with levels of:

(2/3π)(2/5π)(2/7π)(2/9π)(2/11π)--. There are no even harmonics. Note also, the term $A_{av.}$ This is an amplitude term that varies with the time spent above or below an average level.

All these harmonics, which appear at intervals equal to the data rate, are removable with a Nyquist bandpass filter without any effect on the data recovery. The filter must have near zero group delay to pass the waveform. This in turn means the Nyquist bandwidth of the filter must be equal to the Intermediate frequency. To pass one missing cycle at 48 MHz, means the Nyquist bandwidth must be equal to 48 MHz. The **NOISE** bandwidth for a near zero group delay filter is only a few kHz wide, not the Nyquist BW.

The **POINT is : These sinx/x products are not necessary.**

In order to use this modulation method, the spectrum must be made compatible with FCC regulations. Specifically, these sinx/x products must be below certain levels to comply.

Calculating Mean Power Levels:

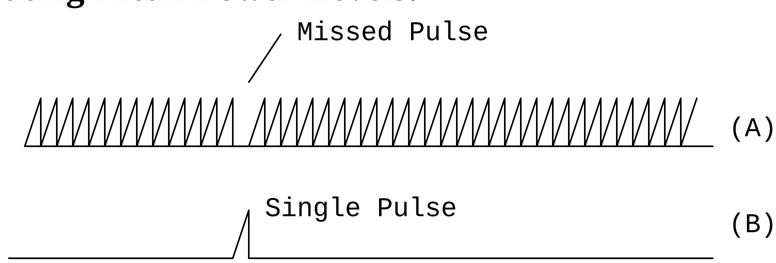


Figure 7.

Assume an RF frequency with 10 cycles per bit period. The main power spike has 9 cycles and the single pulse 1 cycle. The ratio is 9/1 in time difference. Assume 10/1.

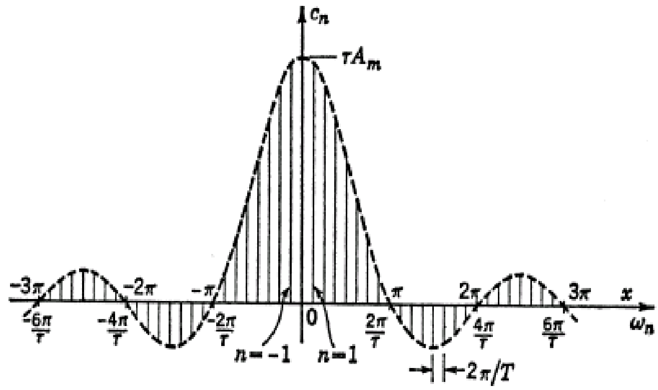


FIG. 2-8. Frequency spectrum, rectangular pulses ($\tau \ll T$).

Figure 8. The approximate sinx/x spectrum for the assumed timing.

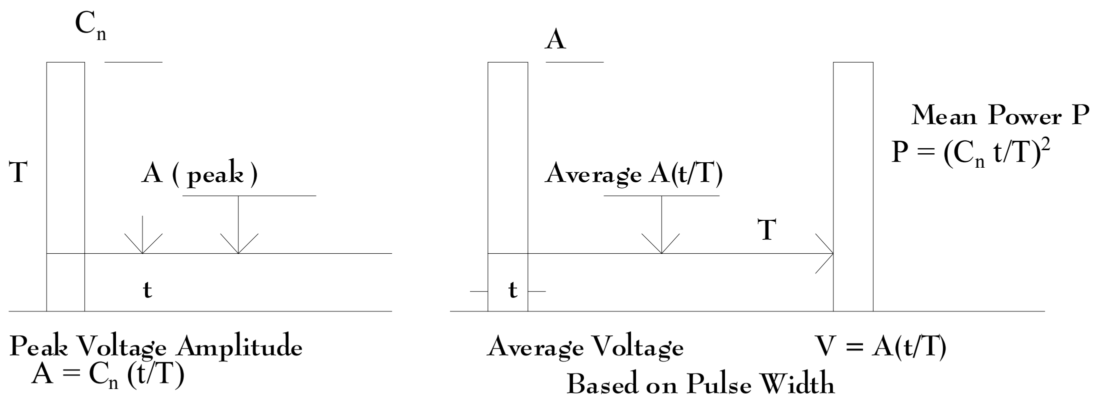


Figure 9.

This is classical pulse analysis. There will be minor spikes spread as shown above over a wide spectral range.

The minor pulses will have a peak voltage value of 1/10 the peak of the major pulse (-20 dB). But they will have only 1/10 of that level as an average voltage.
The mean (RMS) power from E^2/R is at -40 dB.

This is verified by true RMS measurements which show the strongest peaks are at $-20\mathrm{Log}_{10}(T/t)$dB. The RMS value is $-40\mathrm{Log}_{10}(T/t)$dB. Since ones only are transmitted, there is an additional 6 dB reduction in voltage average (-26 dB for the example above). RMS power = $[-40\mathrm{Log}_{10}(T/t) + 12]$dB. Or, -52 dB in this example.

In RMS terms, this spectrum could comply with FCC specifications without further filtering at the transmitter for power levels below a few Watts. They are easily reduced further with the addition of a zero group delay filter.

The special low group delay filters for this modulation method have little or no phase loss as seen in Figure 10.

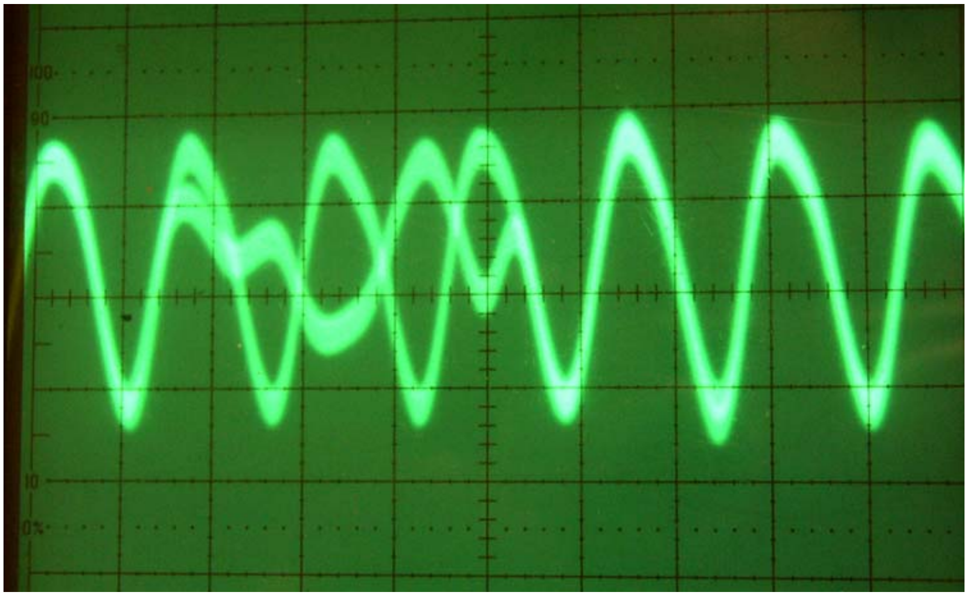


Figure 10. The output of the near zero group delay filter to a phase reversal waveform.

Interference:

Phase reversal of 180 degrees is also employed by the well known BPSK modulation method, which is transmitted ‘Double Sideband with Suppressed Carrier’

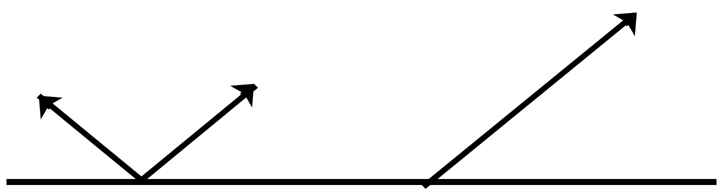


Figure 11. Vectors for DSB-SC and Carrier alone.

With two sidebands, any interference level that exceeds either sideband will cause an error. With the carrier alone, there is only one vector and the interference must exceed that vector, which is twice as strong as either sideband alone for the same power output..
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Signal and Noise Angles

The trigonometric relationship between signal and noise is seen above. A phase modulation angle of +-60 degrees will tolerate a noise vector maximum of .866 signal before the noise created error angle will override the modulation angle. The phase detector reference is set at 00. The angle created by the noise is Φ= ARCSIN(1/SNR).

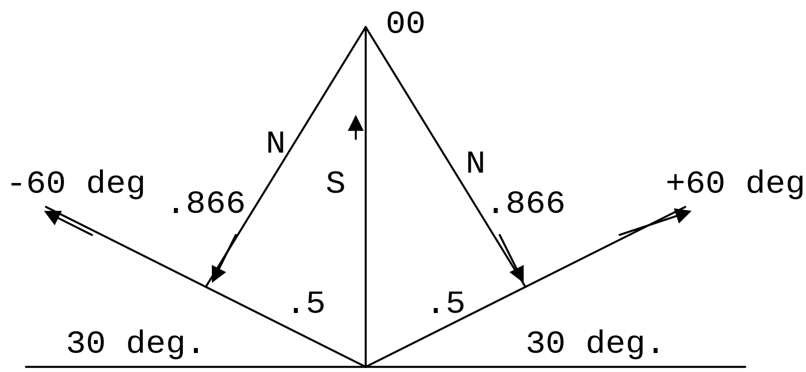

Figure 12. Signal Plus Noise Vectors.

When two level phase modulation is used, the error probability is determined by:

P_e = ½ erfc [SNR] ^{½}

SNR = $\sin\beta^2$ (E_b/n)   where β is the modulation angle. In the above plot for +-60 degrees, the SNR is reduced by .75. For 45 degrees it is .5. For 30 degrees it is .25. **There is no loss when the modulation phase angle is 90 degrees. Small phase angles can be costly in bit error rate.**

Summary: If the modulation angle is +- 90 =180, a noise vector equal to the signal vector is required to cause a signal black out. The phase angle distortion when they are equal is usually less than the indicated 90 degrees. In a practical embodiment, there will be both amplitude and phase distortion. The amplitude distortion can be largely overcome with a limiter, leaving the phase distortion as the major interference problem. In the above illustration, the noise is about 1.3 dB lower than the signal. The maximum phase angle distortion in that case is 60 degrees.

This condition can be simulated with a CW signal at a frequency near the carrier frequency while adjusting the level until errors are caused. This simulation is shown in Fig. 13. Because the interference is not AWGN, but close to peak AWGN, the AWGN result can be predicted in terms of P_e.

In Fig. 13, a frequency modulated interfering signal having the same level as the desired signal is superimposed. The FM signal uses 400 Hz modulation. (C/I = 0 dB). The deviation is +-2.7 kHz. The BER is approximately 10^{-2}. This indicates there is a 50/50 error when the interfering signal is at the same level as the desired signal. The filter noise BW is 1 kHz at the 3 dB points.

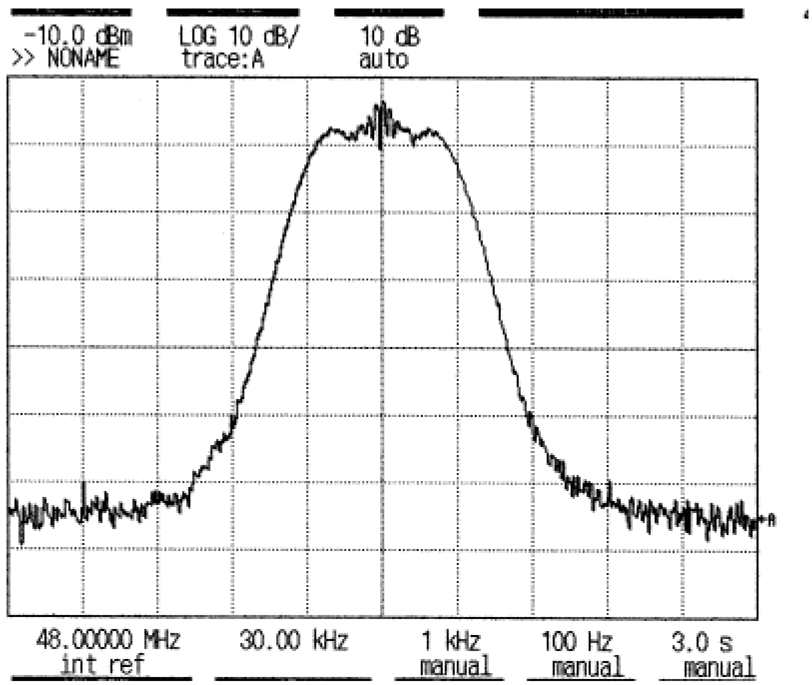

Figure 13. Spectrum showing Narrow Band FM at same level as desired Signal at same center frequency, with the two signals superimposed.

Using a CW interfering signal close to the peak filter response, a signal/noise level of 1.5 dB or less was required to cause errors.

References:

(1) H.R. Walker, U.S. Pat 6,445,737 " Digital Modulation Device In A System And Method Of Using The Same". Covers the MSB methods 3PRK and MCM.

(2) Bellamy, J.C., "Digital Telephony" John Wiley.
Quote, *" Except for a few relatively uncommon frequency modulation systems, digitally modulated carrier systems can be designed and analyzed with baseband equivalent channels".* Most Ultra Narrow Band methods fit into this **exception** category. There is no known way to simulate these methods at baseband.

(3) Taub and Schilling, "Principles of Communications Systems", McGraw Hill.

(4) Prof. Howe. "Wireless Engineer", Nov. 1939. pp 547.