Operational Amplifiers

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Operational Amplifiers

The theory of electrical signal processing requires amplifiers to perform, with electrical signals, **mathematical operations** such as addition, subtraction, multiplication, division, differentiation, integration, etc.
These amplifiers must fulfil the following requirements:

- Differential inputs
- D.C. amplification
- Very high voltage gain
- Very high input resistance
- Very low output resistance

They are then called "**operational amplifier**s" (opamps) because they are able to perform mathematical operations. With opamps, even analog computers are constructed which surpass any digital computer when high speed of signal processing is required.

The first opamps were built using discreet transistors, but it a was difficult and expensive process because of temperature drift problems. The big breakthrough came with integrated circuits. Having all circuit elements on one **monolithic silicon chip** solved most of the temperature drift problems and allowed for cheap mass production.

Today we have to consider the opamp as a circuit element. We will study its characteristics but not dwell on how it works internally.

1. Design

The basic form of an opamp is a **high gain dc-amplifier** with a **differential input** port and a single output port.

A differential input has two terminals, which are both independent of ground or common. The signal between these two terminals is the input signal, which will be amplified.
The terminals are called **non-inverting input** and **inverting input**.

The two inputs can be used in three different ways:

1. Non-Inverting Amplifier:
The input signal is applied between the non-inverting input and ground. The inverting input is connected to ground.
The output signal will be in phase with the input signal

2. Inverting Amplifier:
The input signal is applied between the inverting input and ground. The non-inverting input is connected to ground.
The output signal will be 180° out of phase with the input signal.

3. Differential Amplifier:
Two input signals are each connected to the non-inverting and the

inverting input, using both common as second terminal. The output signal will be the amplified difference between the two.

$$\mathbf{U_o = (U_{i+} - U_{i-}) \cdot g}$$

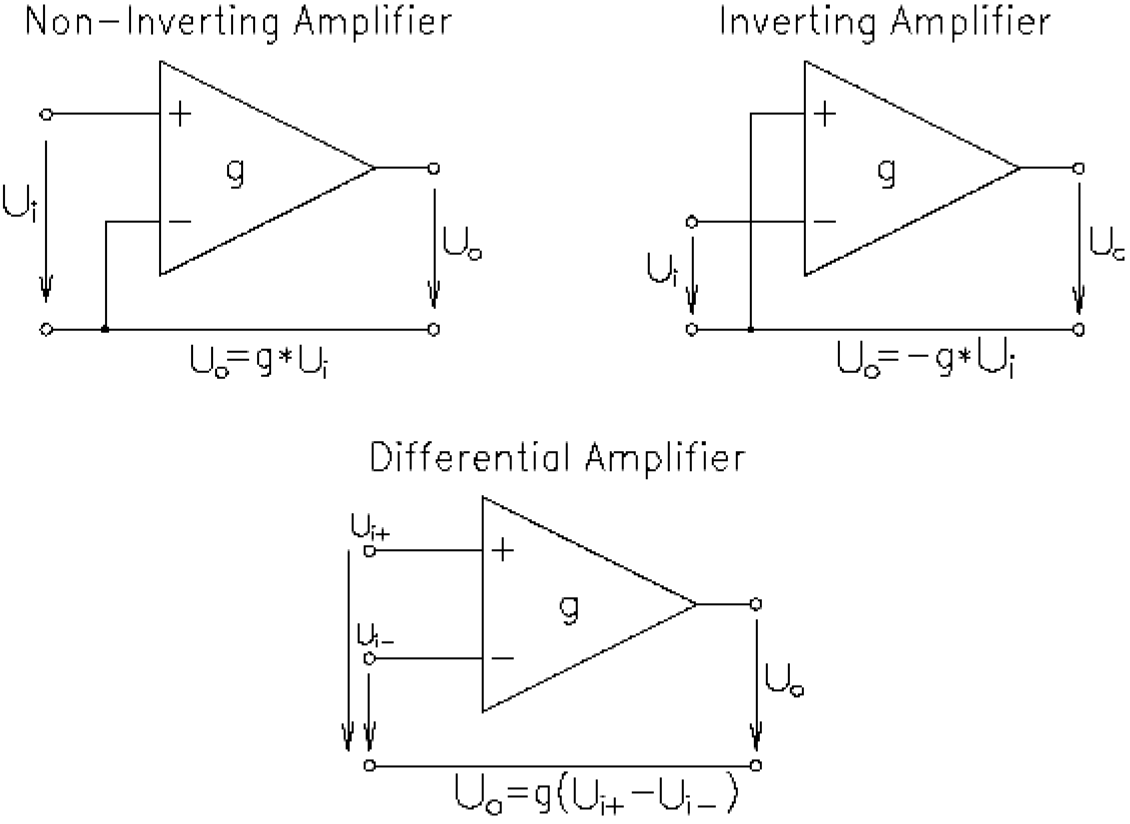


Fig. 1.1.

The three basic ways of applying input signals to the opamp.

When there is no voltage difference between the input terminals, the output voltage should be 0.

The internal circuit of opamps consists basically of three main parts:

1.1. The Differential Amplifier:

A differential amplifier stage consists of two transistors in common emitter configuration which are supplied with a common emitter current.

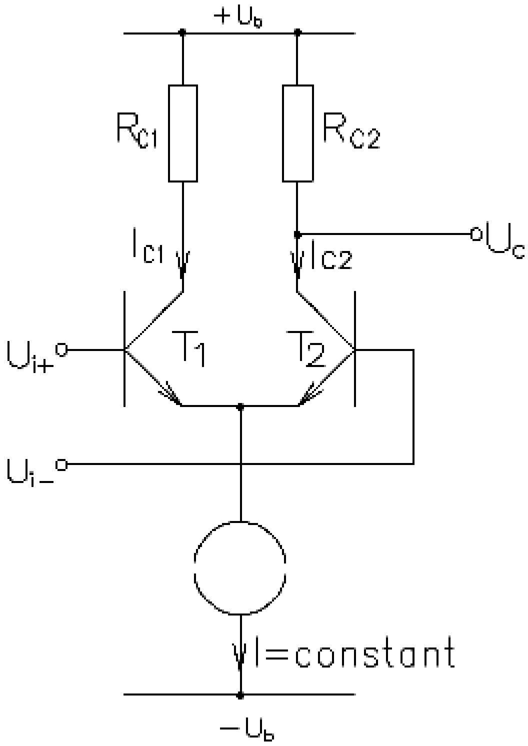


Fig. 1.1.1.

The basic design of a differential amplifier stage.

As long as there is no voltage difference between the two bases of the transistors, the two transistors will draw the same collector currents and a certain voltage will appear at the output.

If the base of T_1 becomes more positive than of T_2, T_1 will draw more current, the voltage across R_{C1} will increase. As the total current is constant, the current through T_2 will decrease by the same amount. The voltage across R_{C2} will decrease and the output voltage becomes more positive.
So the base of T_1 is the non-inverting input.

If the base of T_2 becomes more positive than that of T_1, T_2 will draw more current. The voltage across R_{C2} increases and the output voltage becomes more negative.

Thus the base of T_2 is the inverting input.

If the voltages at the bases of T_1 and T_2 are varied by the same amount, the current distribution between the two transistors does not change and no voltage results at the output.
This case is called **common mode** and should not produce an output signal.

The general requirements for the differential amplifier:

- high differential mode gain
- low common mode gain
- high input impedance

- low base currents
- temperature stability

Some opamps use FET as input transistors to achieve extremely high input resistances.

1.2. Level Shifter

The level shifter fulfils two main tasks:

- it provides most of the voltage amplification of the opamp;
- it provides dc-matching between differential amplifier and the output to obtain zero output voltage for zero input (offset voltage).

The level shifter consists mainly of a number of dc-coupled transistor stages which are arranged and biased in such a way that zero offset voltage with a high temperature stability is achieved.

Requirements to the level shifter:

- low distortion
- wide frequency range

1.3. Power Amplifier

The final stage of an opamp is in most cases a complementary push-pull amplifier. It has to provide the required output current at a low output resistance.

Requirements:

- symmetrical output swing from $+U_b$ to $-U_b$
- low output impedance
- short-circuit protection
- low distortions

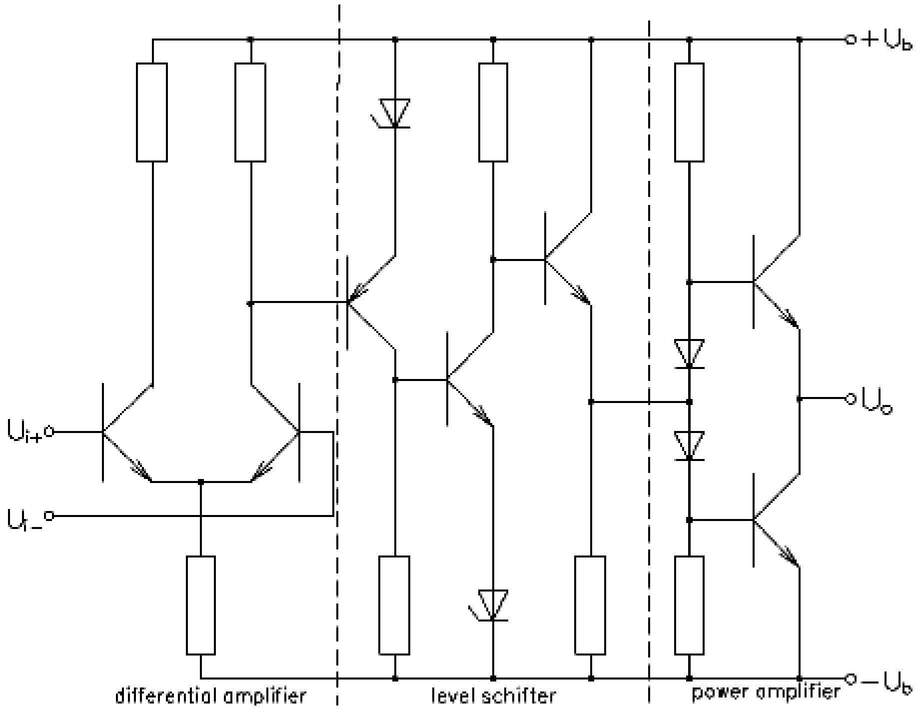


Fig. 1.3.1.

An example of the circuit of a simple integrated opamp.

The **circuit symbol** for an opamp is a triangle pointing towards the output. The input terminals are drawn to the vertical left side. Any further auxiliary terminals such as supply voltages or offset adjustment are drawn at the top and bottom slopes of the triangle.

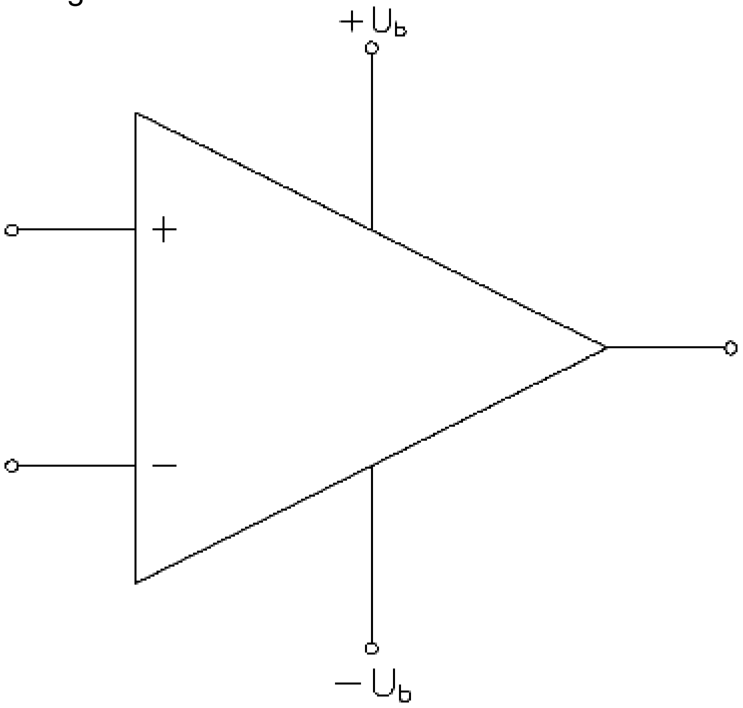


Fig. 1.3.2.

The circuit symbol for a general opamp.

2. Characteristics

Voltage gain

An ideal opamp should have an open loop voltage gain g (without NFB) which is infinite. Practical opamps may have values from

60dB to 120d, which equals **10^3 to 10^6**.

In general, all practical opamps have sufficient gain for most requirements.

Input resistance

An ideal opamp should have an input resistance R_i which is infinite. Practical opamps may have values from

10kΩ to 1MΩ

Input up to 1GΩ can be reached for opamps with MOSFET.

The input resistance of opamps will further be increased by NFB, so that the achieved values will satisfy most practical requirements.

Output Resistance

An ideal opamp should have an output resistance R_o of zero. Practical opamps may have values from

50Ω to 500Ω

These values are not made lower in order to achieve short circuit protection of the output. The output resistance will be reduced by NFB, so that the achieved values will satisfy most practical requirements.

Supply Voltage

In general, opamps require two symmetrical (equal but of opposite polarity) supply voltages $+U_b$ and $-U_b$ in respect to ground. These voltages must be large enough in order to properly bias all internal transistors. On the other hand, they may not exceed a specific maximum value.
Practical supply voltages range from ±3V to ±30V. A common value is ±15V.

Some opamps are also designed to be operated on one supply voltage only. This requires a special design for the input and output stage. Either supply terminal may then be connected to ground.

Output voltage Swing

The maximum output signal U_{sat} (saturation voltage of the output stage) will depend on the supply voltage. It is obvious that the output voltages cannot be higher than the supply voltages. As the output of the amplifier will always require a certain voltage drop, the maximum output voltage swing will be 1V to 3V lower than the supply voltage, depending on the type of opamp.

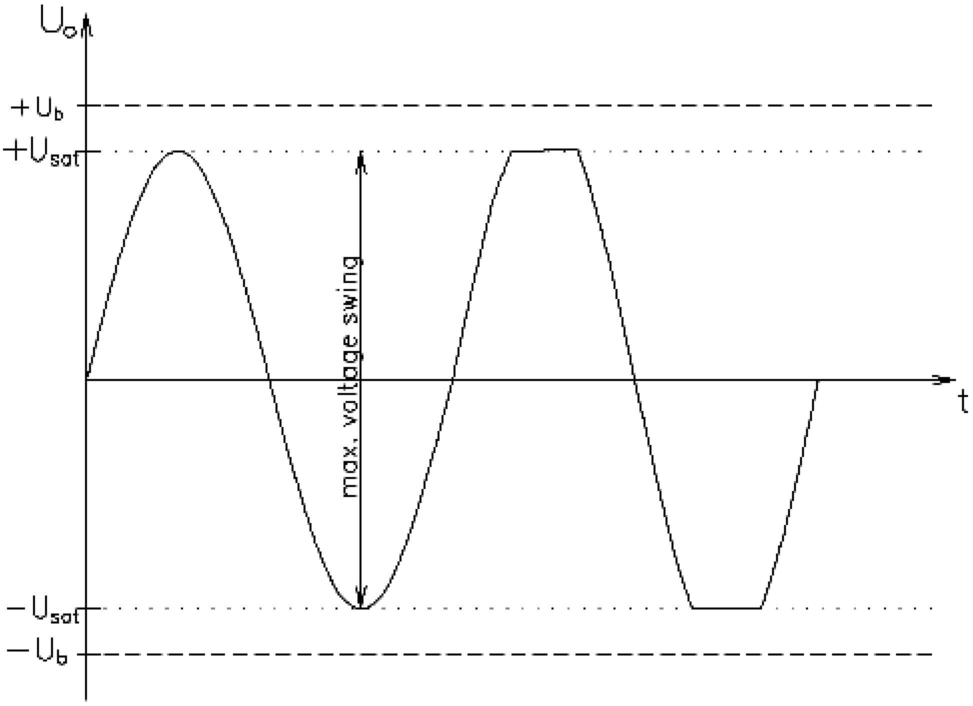


Fig. 2.1.

The relationship between supply voltage and maximum output voltage swing.

The maximum output voltage will depend on the supply voltage. The higher the supply voltage, the more output amplitude can be achieved.

As for opamps operated on one supply voltage only, the amplitude of the output signal can only be less than half of the supply voltage.

Input Offset Voltage

The output voltage of an opamp should be zero, if the input voltage is zero (input terminals shorted). In practice, there will always be some asymmetry in the differential amplifier. This voltage is then amplified through all stages and, depending on the gain, there might be a high voltage at the output of the opamp.

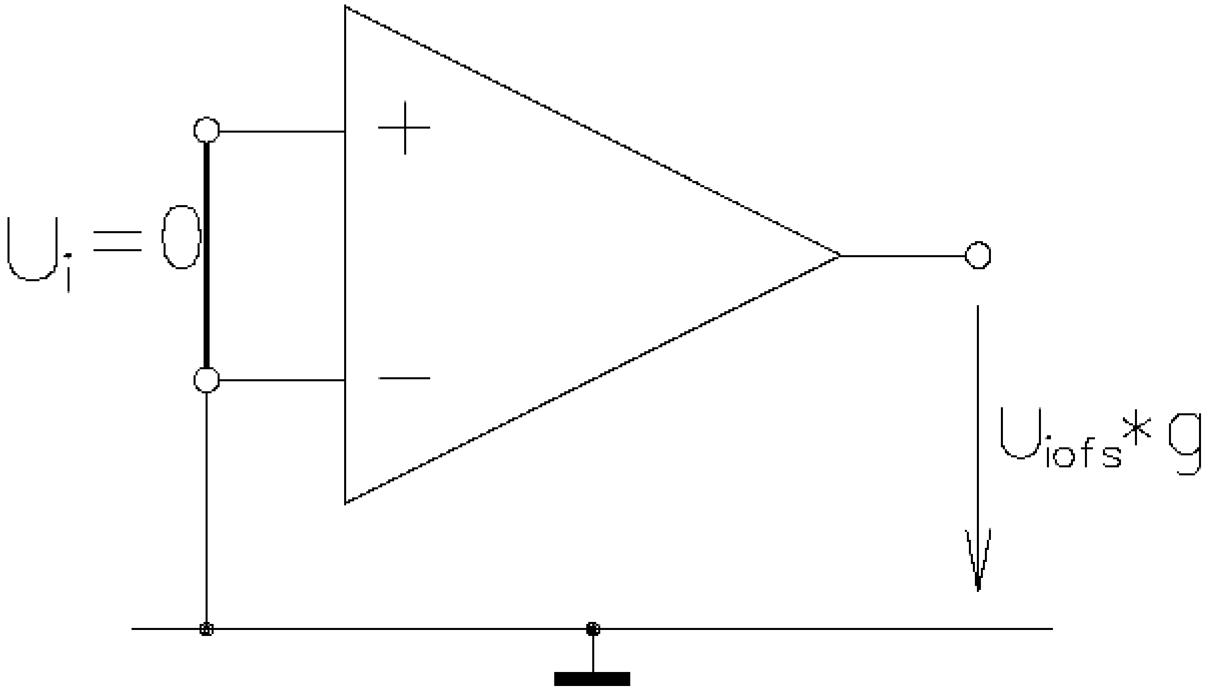


Fig. 2.2.

The output voltage which is measured at the output of an opamp with shorted input terminals is the internal offset voltage U_{iofs} multiplied by the gain g.

This voltage could be compensated by feeding a dc-voltage to the input which opposes the internal offset. This voltage is equal to the input offset voltage Uofs. This process is called offset compensation or offset null-balance. It is required for most cases of dc-amplifiers (e.g. measuring amplifiers).

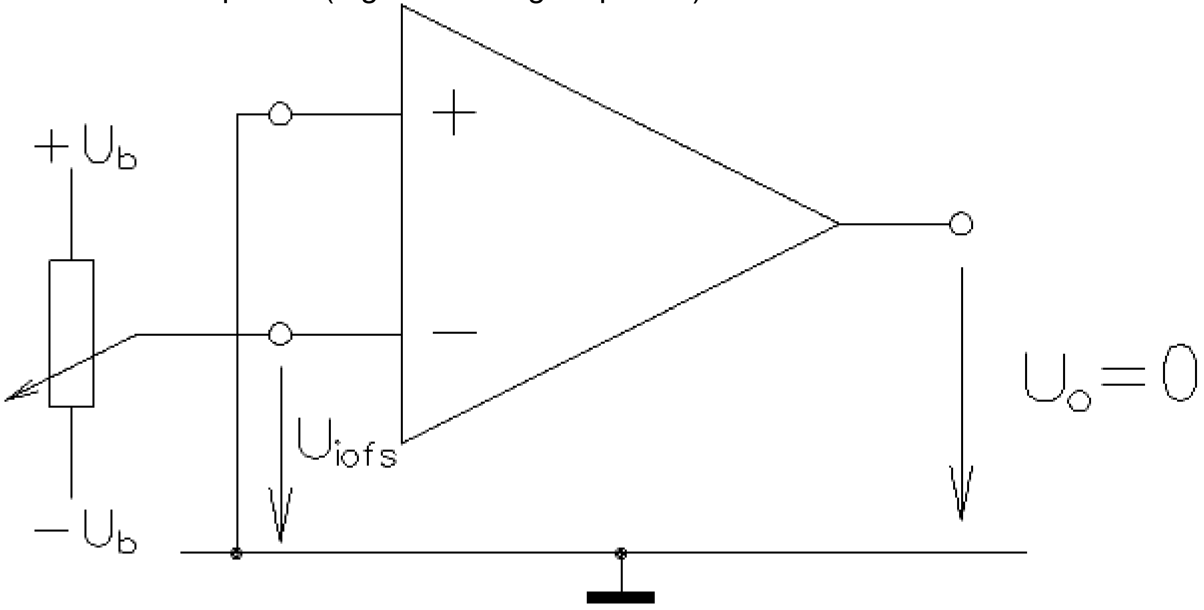


Fig. 2.3.

If the input offset voltage U_{iofs} is fed into the inverting input terminal, the output voltage can be set to zero.

In order to keep the input terminals free for the signal, some opamps provide separate terminals for offset adjustment. These offset adjustment terminals must be used according to the specifications of the data sheets.

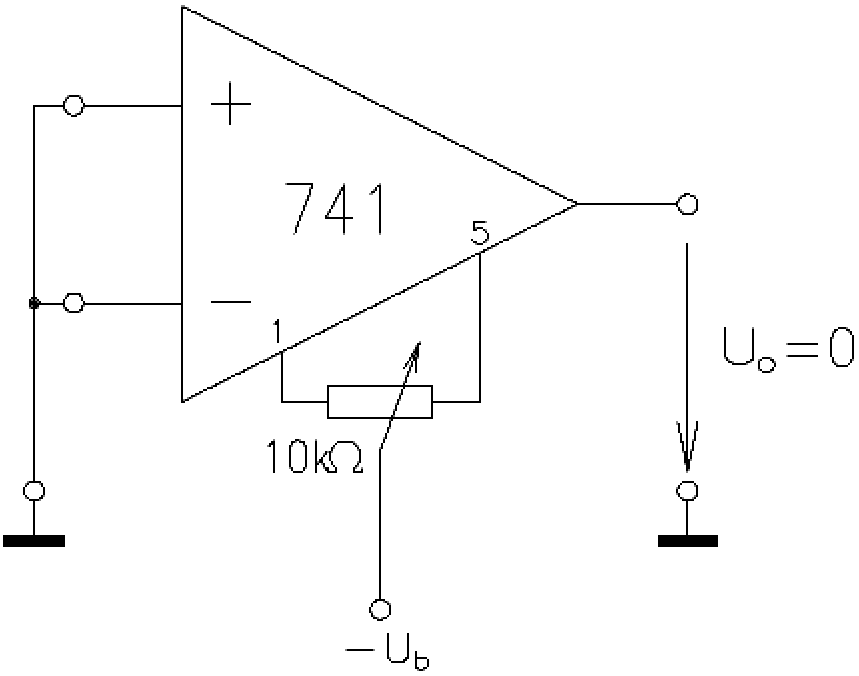


Fig. 2.4.

Example of the offset compensation using the separate terminals of an opamp (741).

Input Bias Current

The input terminals of opamps can be considered as base terminals of the transistors of a differential amplifier stage. In order to operate the transistors in the active region, they require a certain bias current I_{iB}.
For opamps with bipolar input this will be in the range of some nA or µA.

Although these currents are very small, they may produce a voltage drop across any resistance in series with the input. This is then a voltage difference at the input which again produces an offset at the output. If the two resistors are equal, the voltage drops will be equal and there will be no voltage difference at the input.

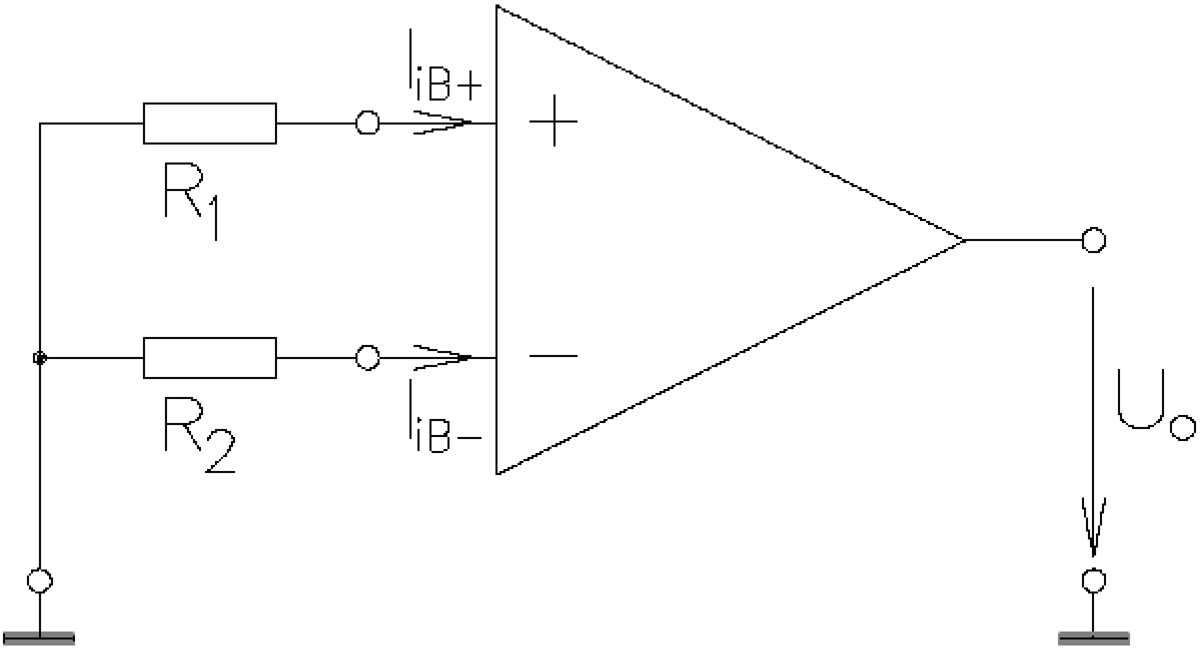


Fig. 2.5.

The input bias current I_{iB} *of the input transistors will produce a voltage drop across any resistor connected in series to the input. Making both resistors equal will cancel out the two voltages* U_{R1} *and* U_{R2}*.*

Care is therefore often taken that both inputs of the opamp have an equivalent resistance to ground to avoid offset due to bias current.

Input offset Current

The bias current of the two transistors may not be equal, so even if both inputs have equal resistors in series, there might be an offset voltage. In practice, this effect cannot be distinguished from the effect of the input offset voltage, so they will be compensated together.

3. The Opamp without NFB

Let us look at how the opamp can amplify signal. We will assume that the opamp has an open loop gain of g = 6000 = 76dB. This means an input voltage of 1mV will produce an output voltage of 6V.

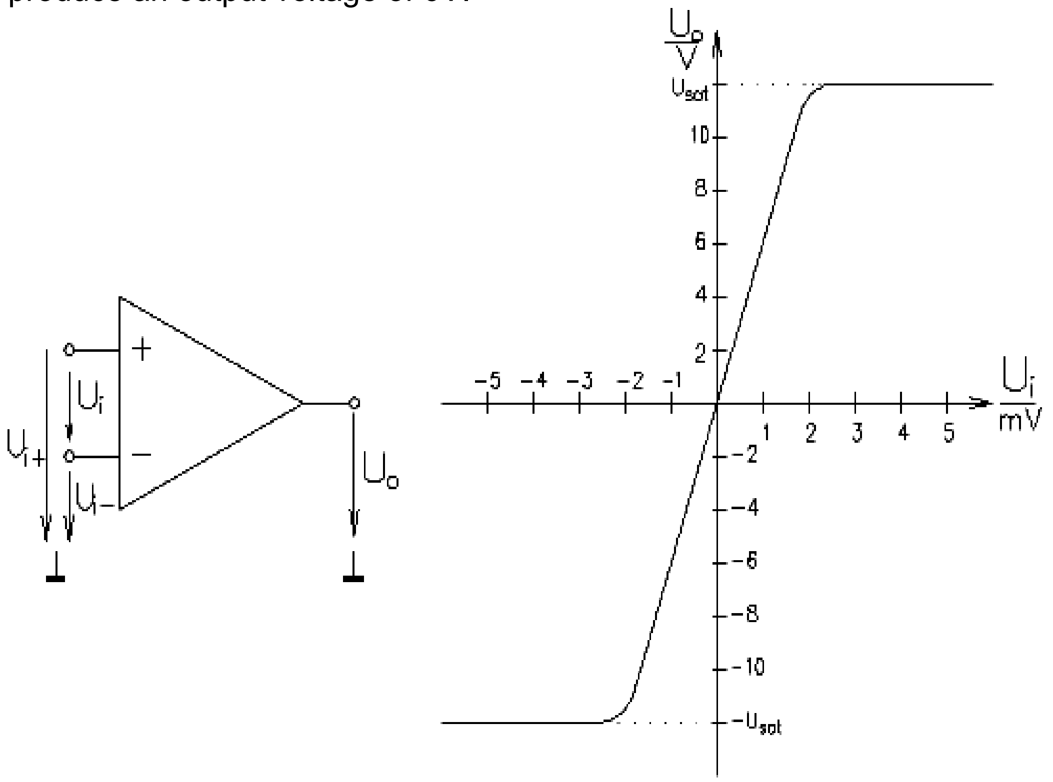


Fig. 3.1.

Opamp as amplifier with its transfer characteristic. Input voltages of more than 2mV will drive the output to saturation.

In practice, it will be found that an amplifier with such a large dc-gain will not work properly because the offset voltage drift will not allow a stable working point.

An opamp without NFB can not be used as linear amplifier.

The opamp in this "pure" form is only used as COMPARATOR. The comparator compares two input signals and provides a digital (high/low) output signal, depending on which of the two is larger.

$U_o = +U_{sat}$ (approx. $+U_b$) if $U_{i+} > U_{i-}$

$U_o = -U_{sat}$ (approx. $-U_b$)   if $U_{i+} < U_{i-}$

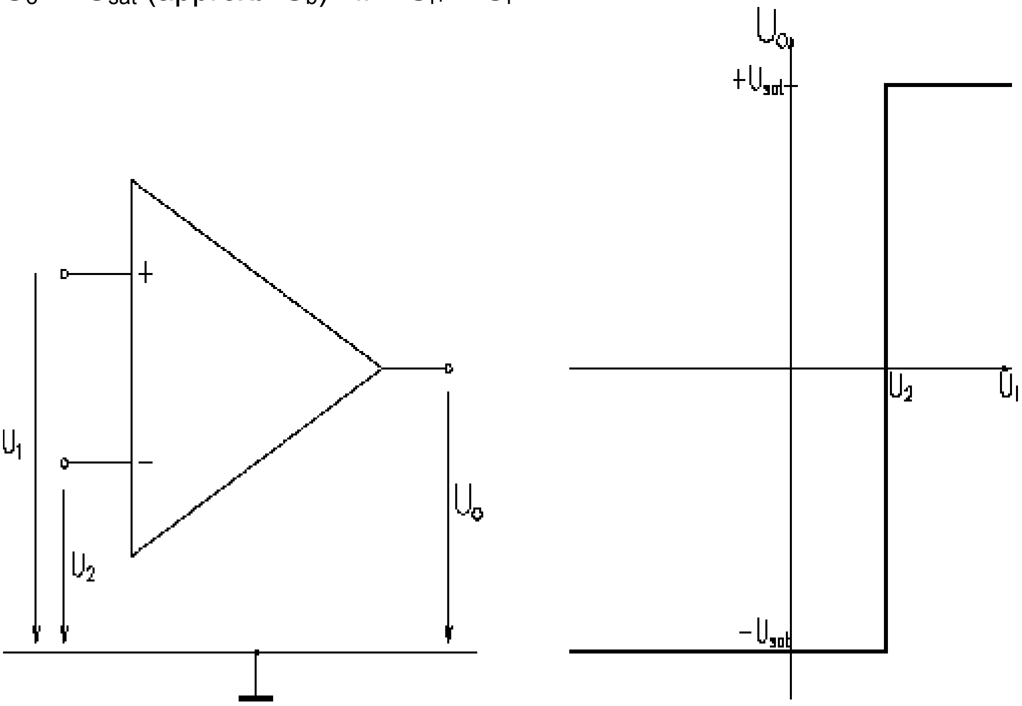


Fig. 3.2.

The opamp as comparator.
The output signal is either +U_{sat} or -U_{sat}, depending on which of the two input voltages is larger.

Normally one of the two input voltages is used as a reference or threshold for the other. If the reference voltage is connected to the inverting terminal, we will get a non-inverting comparator. If the reference voltage is connected to the non-inverting input, we will have an inverting comparator.

4. Linear Amplifiers

Opamps can only be used as linear amplifiers with external negative feedback. The NFB is achieved by a voltage divider circuit which feeds back a fraction of the output signal to the inverting input.

As opamps have a very high open loop gain, very strong NFB can be provided. This makes strong use of all of the advantages of NFB such as:
- reduction of distortion,
- favourable input and output resistances,
- stable working parameters.

Depending on how (in which form) the NFB is achieved and how the signal is fed to the input, different types of amplifiers with different characteristics are created.

4.1. The Non-Inverting Amplifier

The non-inverting amplifier feeds the input signal to the non-inverting input. The NFB-signal is derived from a voltage divider from the output signal and is fed to the inverting input.

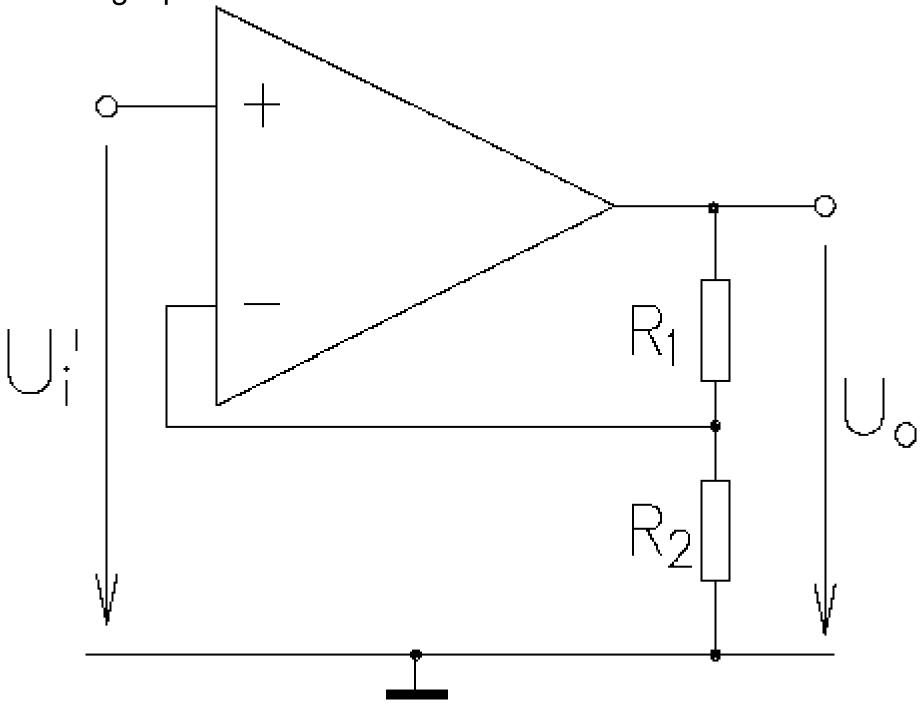


Fig. 4.1.1.

The basic configuration of the non-inverting amplifier.

The properties of this amplifier are controlled entirely by the NFB voltage divider (see chapter on NFB):

Close Loop Voltage Gain

$$g' = \frac{R_1}{R_2} + 1$$

This formula is correct if g' << g (g' is much smaller than g)

Input Resistance

$$R_i' = R_i \cdot \frac{g}{g'}$$

The input resistance is increased by the degree of reduction of gain. This factor will in practice be at least 10 or 100, so the input resistance of this amplifier will be very high (>1MΩ) in all cases.

Output Resistance

$$R_o' = R_o \cdot \frac{g'}{g}$$

The output resistance will be reduced by the same factor by which the input resistance is increased. In practice, this leads to very low values (<1Ω).

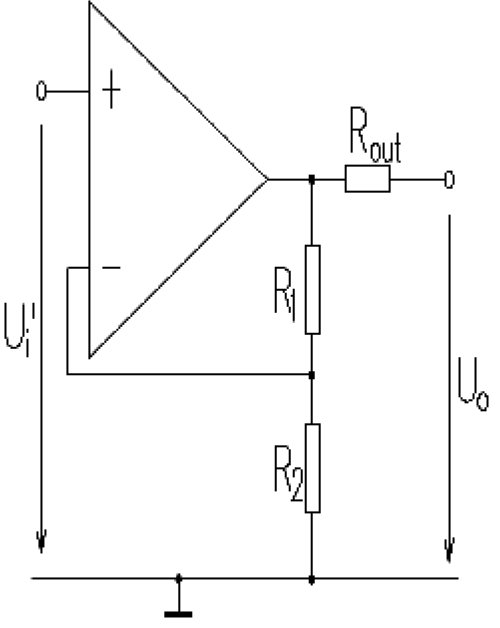
Summary of properties of the non-inverting opamp:

- the signals at input and output are in phase,
- the closed loop gain g' depends on the external elements R_1 and R_2 only,
- the input resistance is very high,
- the output resistance is very low.

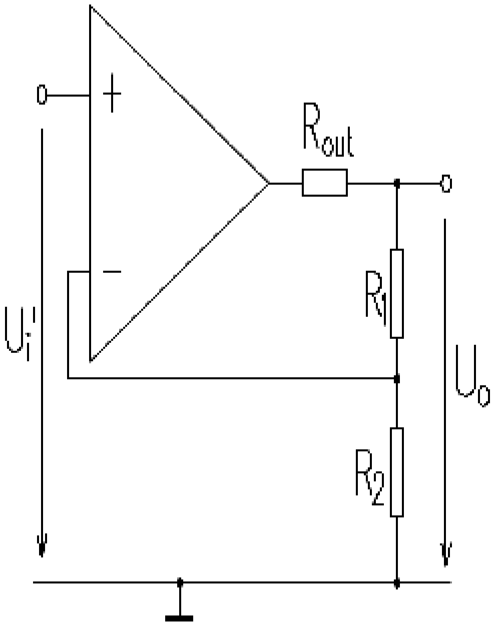
The non-inverting amplifier is used for audio amplification and as a measuring amplifier.

The NFB tends to eliminate all kinds of negative influences which appear between the input and output of the amplifier. It can be used to reduce the influence of any other circuit elements which are used in conjunction with opamps.

Any resistance which is in series with the output of the amplifier will increase the output resistance. The effect of this resistance can be reduced if the resistor is taken into the NFB-loop.



a.)



b.)

Fig. 4.1.2.

A resistance in series with the output of an amplifier.
a.) If the resistance in series with the output is outside of the NFB-loop, the resistance adds fully to the output resistance.
b.) If the resistance in series with the output is within the NFB-loop, the resistance is eliminated by the NFB.

If more output current is required, a push-pull stage can be connected to the output of the opamp. A push-pull stage can produce distortions, mainly cross-over distortions. Taking the push-pull stage into the NFB-loop will strongly reduce the distortions.

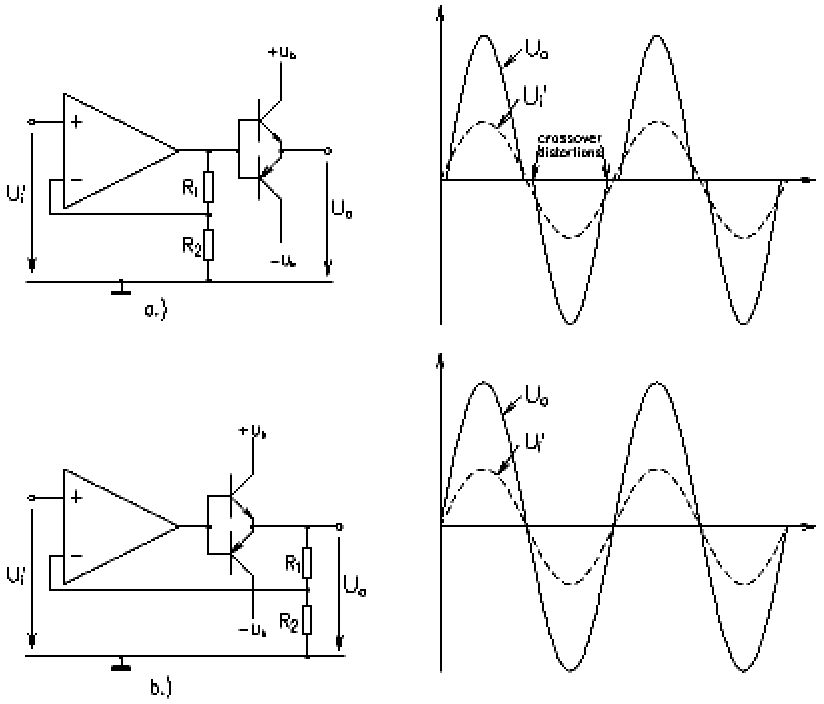


Fig. 4.1.3.

A push-pull state may be used to boost the output current of the opamp.
a.) If the push-pull stage is outside of the NFB-loop, the distortions of this stage appear at the output.
b.) If the push-pull stage is within the NFB-loop, the distortions of this stage are reduced by the NFB.

4.2. The Voltage Follower

The smallest gain to be achieved with a non-inverting amplifier is one. This is achieved if the entire output signal is fed back to the input.
Considering the formulas above, this means that $R_1 = 0\Omega$ and $R_2 = \infty\Omega$ (infinit).

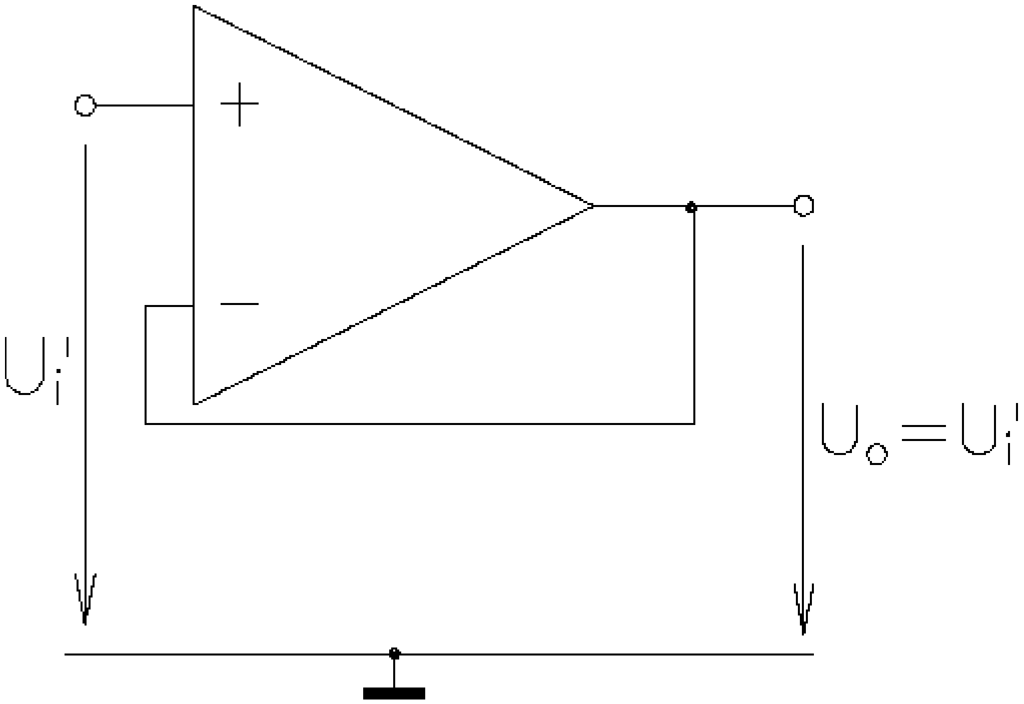


Fig. 4.2.1.

When all the output voltage is fed back to the input, the non-inverting amplifier becomes a voltage follower with unity gain.

The gain of this amplifier is one and so the output voltage is identical to the input voltage. Because of this, the circuit is called UNITY GAIN AMPLIFIER or VOLTAGE FOLLOWER.

Important characteristics of this amplifier:

Gain: $g' = 1$

Input Resistance: $R_i' = R_i * g$

Output Resistance: $R_o' = R_o/g$

Summary of important properties:

- the signals at input and output are in phase,
- the closed loop gain g' is one
- the input resistance is extremely high,
- the output resistance is extremely low.

Voltage followers are used as impedance converters in audio amplifiers and measuring amplifiers.

4.3. The Inverting Amplifier

Inverting amplifiers feed the input signal and the NFB-signal into the inverting input. The non-inverting input is connected to ground. The output signal is shifted 180° in phase to the input signal.

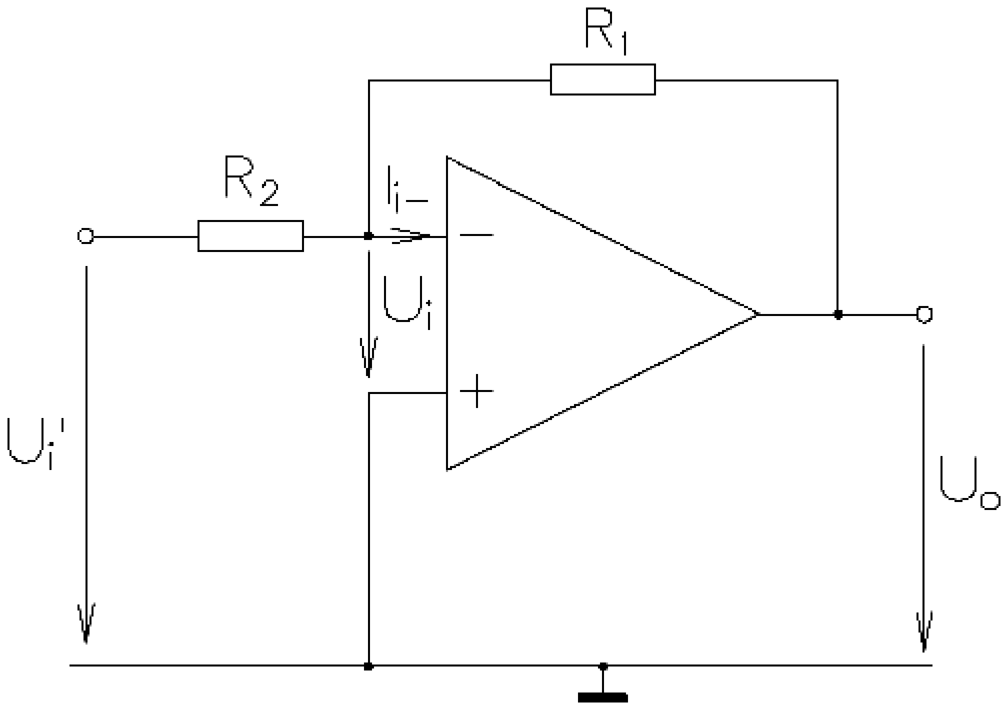


Fig. 4.3.1.

The basic configuration of the inverting amplifier.

The function of the inverting amplifier can be explained by taking two points into consideration:

1. The input voltage of the opamp U_i will be negligible compared to the input voltage of the amplifier U_i', or even compared to the output voltage U_o. The inverting input of the opamp therefore has approximately the same voltage as the non-inverting terminal, which is connected to ground. This point of the circuit is therefore called VIRTUAL GROUND. From the point of view of the signal, this point has the same properties as the ground point of the circuit.

2. The input current to the opamp I_{i-} is approximately zero.
The sum of the currents I_{R1} and I_{R2} must therefore sum up to 0. The inverting input is therefore also called the SUMMING POINT.

The main characteristics can be derived from these considerations:

Closed loop gain:

The resistor R1 and R2 are virtually connected to ground at the inverting input. The currents through the resistors R1 and R2 are equal. This requires that the input and output voltage have the same ratio as the resistors R1 and R2.

$$g' = \frac{R_1}{R_2}$$

This formula is correct if g'

Input Resistance

$$R_i' = R_2$$

The input resistance is only the resistor R2, because it is connected between input and virtual ground.

Output Resistance

$$R_o' = R_o \cdot \frac{1+g'}{g}$$

The output resistance will be reduced by the same factor as the gain. In practice, this leads to very low values (<1Ω).

Summary of properties of the inverting opamp:

- the signals at input and output are 180° out of phase,
- the closed loop gain g' is set by the ratio of R_1 to R_2
- the input resistance is set by R_2
- the output resistance is very low.
- the inverting input of the opamp can be considered as virtual ground.

If bias current compensation is required, a compensation resistor Rcomp can be used to offset current compensation. It should be selected so that the resistance in series with both inputs is approximately equal. Therefore:

$R_{comp} = R_1 // R_2$ (R_1 parallel with R_2)

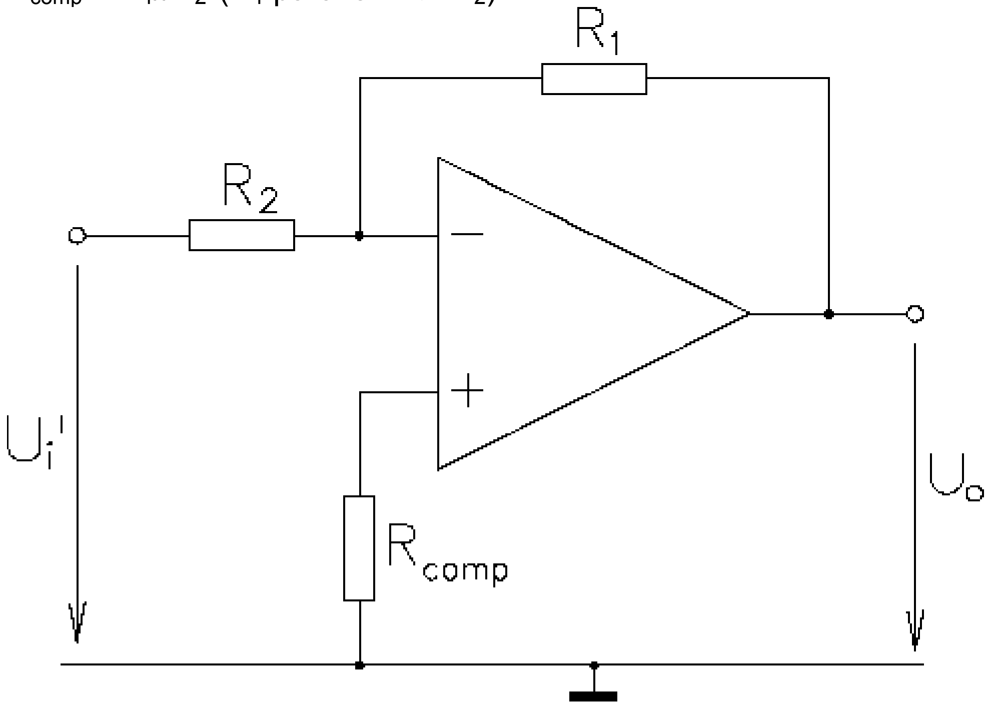


Fig. 4.3.2.

The inverting amplifier with compensation resistor for the bias current.

5. Frequency Characteristics

Opamps have a frequency range which starts at 0Hz (d.c.). At the upper end, the frequency range is limited by the BAND WIDTH and by the SLEW RATE. Both have the effect of limiting the upper operational frequencies, but have different physical causes and must be considered separately.

5.1. Band width

Opamps without NFB have only a relatively small frequency range. Some types only have an upper frequency limit (-3dB) of a few Hz or a few hundred Hz. The gain decreases with increasing frequency due to the low-pass behaviour of the internal transistor amplifier stages. Furthermore, the opamp will have several internal transistor stages in series, each forming a low-pass with its own critical frequency.

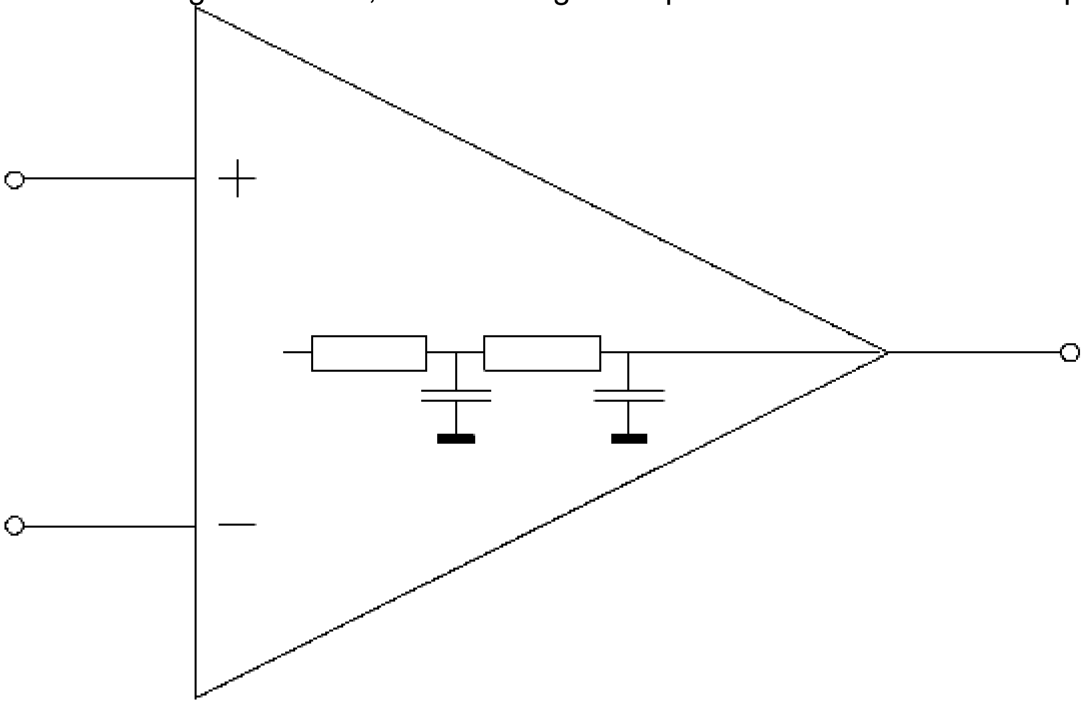


Fig. 5.1.1.

The different amplifier stages of an opamp eacg form a low-pass, which is connected in series.

The gain decreases after the first critical frequency with a slope of 20 dB/decade, after the second critical frequency with a slope of 40 dB/decade, etc. Each low pass will also produce a certain phase shift of up to 90° per low-pass. With increasing frequency, a growing phase shift will occur between input and output. The so- called "Bode-plot" shows the relations:

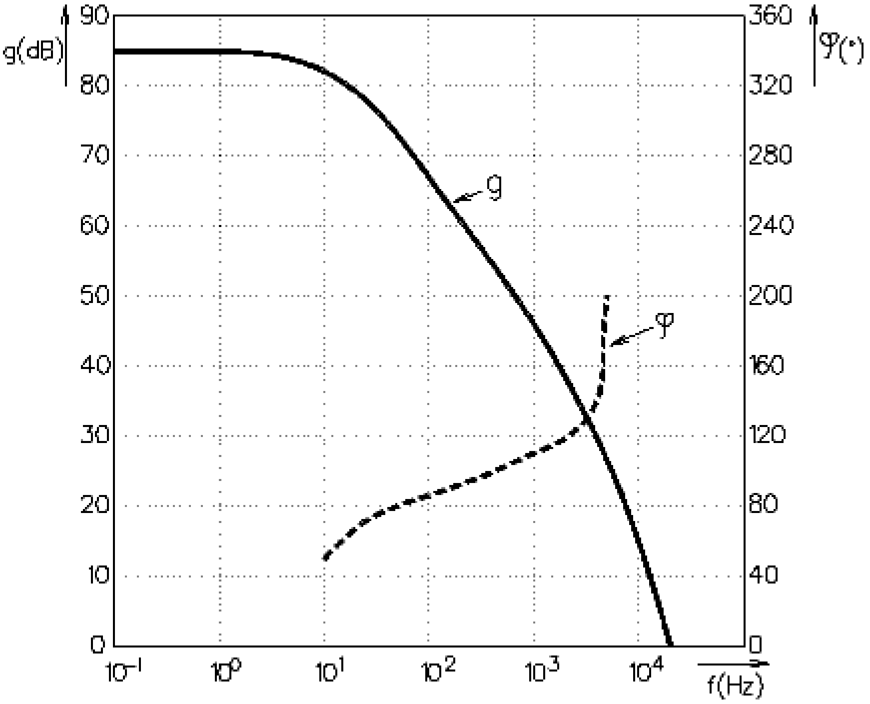


Fig. 5.1.2.

Example of the Bode plot of an opamp (TAA 861).
The critical frequency of the open loop gain (g=85dB) is about 10 Hz. Over 1kHz the gain drops with 40dB/decade due to a second internal low pass. At 5kHz the phase shift between differential input and output is more than 180°.

The limited band width makes this device unsuitable for audio applications, but introducting NFB, the band width can be increased.

Assume for the TAA 861 the gain is set by NFB to 40dB (100). Thus below 1kHz, the open loop gain will be higher than the closed loop gain, and the gain will be defined entirely by the NFB. Above 1kHz the open loop gain will be less than the desired closed loop gain, and the gain will be equal to the closed loop gain.

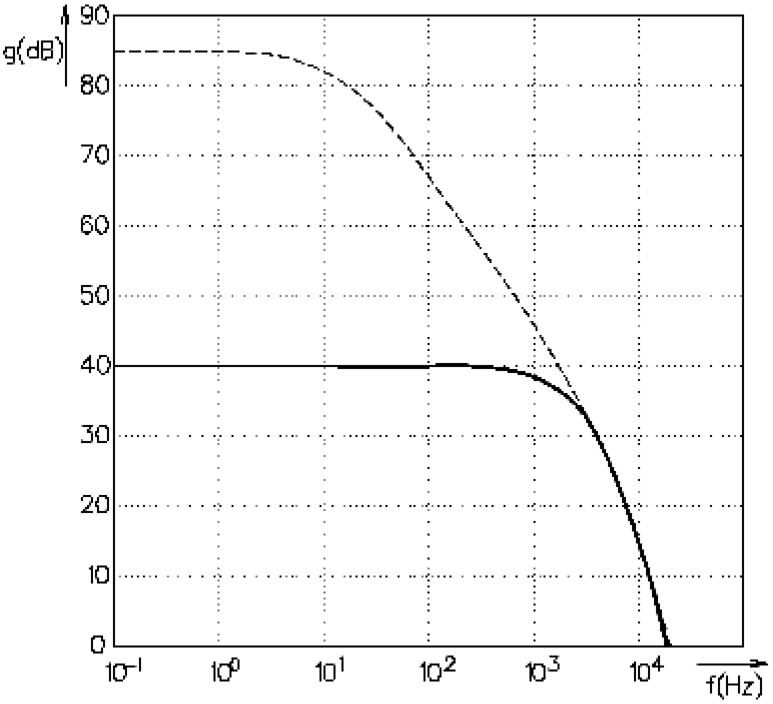


Fig. 5.1.3.

The frequency response of the same opamp with the gain set to 40dB by NFB. The upper critical frequency has been improved to 1kHz.

The band width of this amplifier could be increased to approximately 30kHz. Then the open loop gain becomes 1. But at higher frequencies only little gain is achieved. (In fact, the TAA 861 is not a suitable opamp for audio circuits!)

The lower the chosen gain, the higher the band width.

As the opamp without NFB is not used as a linear amplifier, the band width of the open loop gain plays no practical role and is thus not mentioned in the data sheets. Instead, the UNITY-GAIN BAND WIDTH is given. This is the band width of the opamp with a closed loop gain of 1.

Some examples of unity-gain band width of practical opamps:
- type TAA 861: 30kHz
- type 741: 300kHz
- type 081: 3MHz

A problem arises from the phase shift inside the opamp which increases with frequency. The NFB-signal is supplied with a nominal phase shift of 180° to the input signal (anti-phase). Additional internal phase shifts will turn the negative feed back into a positive feed back. If the gain is then still larger than 1 (0 dB), this will cause oscillation of the amplifier (instability).

In the case of the TAA 861: the lowest gain for stable conditions is 25 dB. In practice, a phase security margin of 60° is respected. This determines the lowest possible gain to 48 dB and the upper critical frequency to 900 Hz.

For an uncompensated opamp the danger of instability
increases with increasing NFB.

To allow higher band widths at smaller gains - particularly for voltage followers (g' = 0 dB) - opamps are provided with terminals for **EXTERNAL FREQUENCY COMPENSATION** by means of R and C components. The required circuit elements and their wiring depends on the type of opamp and has to be determined from the data sheets. In general, frequency compensation is achieved by a low pass function, reducing the first open loop corner frequency and providing a gain decrease of 20 dB/decade down to unity gain. Sufficient phase margin is achieved, though band width and slew rate are reduced compared to uncompensated operation. Several opamps provide internal frequency compensation (e.g. 741-types) and secure stable conditions for all gains.

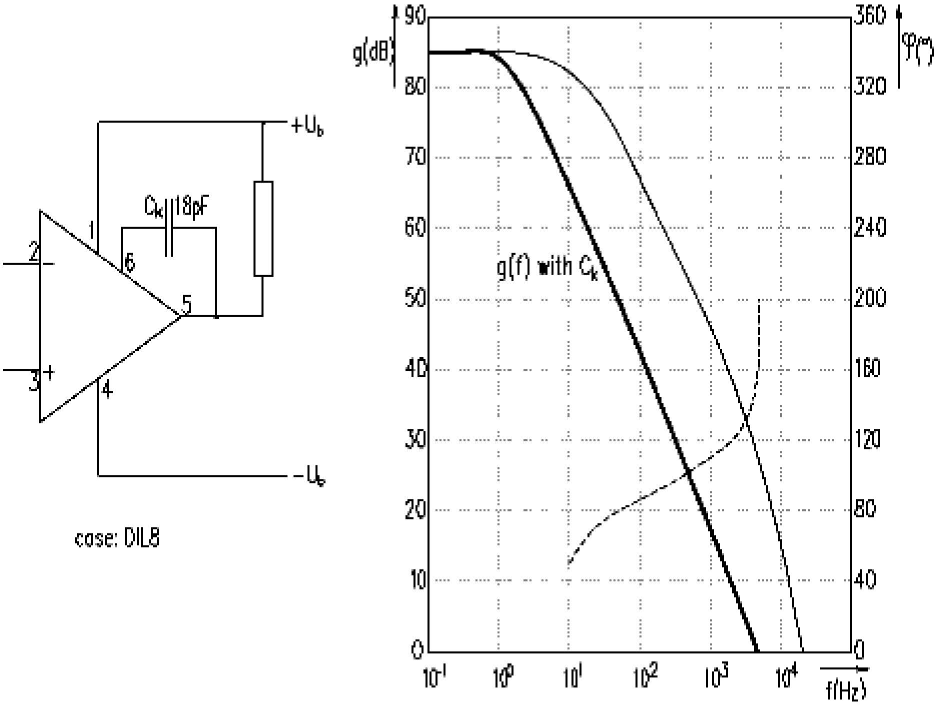


Fig. 5.1.4.

Frequency compensation of TAA 861 with C_k according to the data sheets. (This Op Amp is an open-collector device and requires the load-resistor to be connected to $+U_b$).

5.2. Slew Rate

If a step function (pulse) is applied to the input of an opamp, the output signal will not respond immediately. This is due to internal capacitances which cannot be charged instantaneously. The output will respond with a slope function, representing the highest speed in voltage change.
This is called the slew rate (or slewing rate).
It is given in volts per microseconds (V/µs).

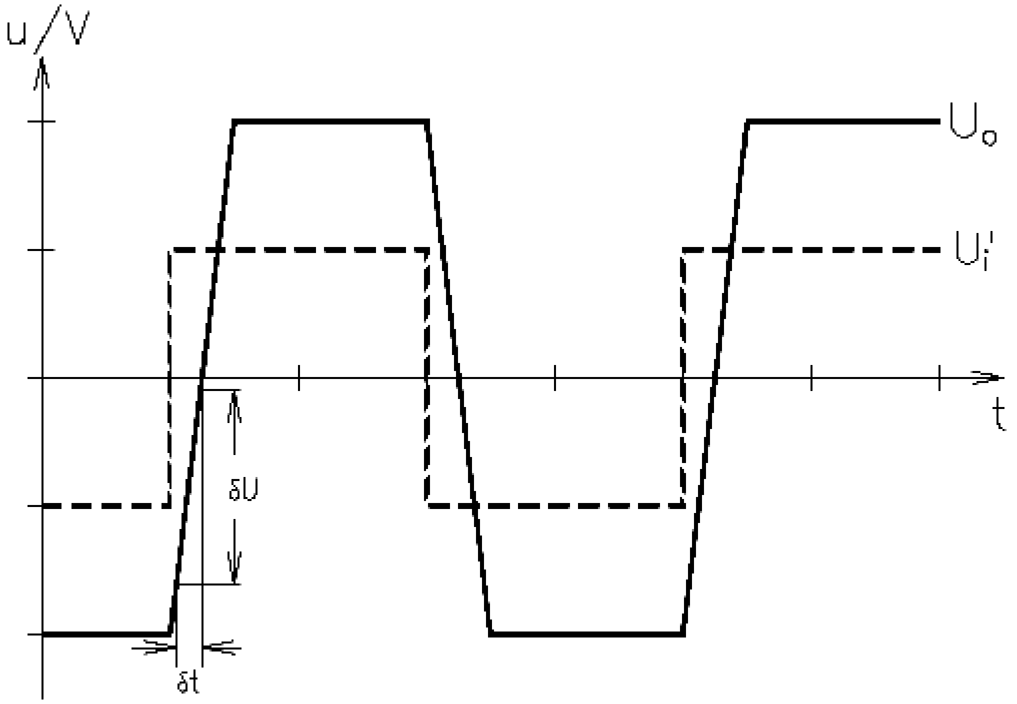


Fig. 5.2.1.

When a step function is applied to the input of an opamp, the output will respond with its maximum possible voltage rise, called the slew rate.
(The gain of this opamp is set to 2.)

In addition, when a sine wave is applied to the opamp, the output is only able to follow with its maximum slew rate. For a sine wave, the highest voltage change occurs during zero crossing and is related to frequency and magnitude. Sine waves follow the function:

$$u(t) = U_{\max} \cdot \sin(2\pi f t)$$

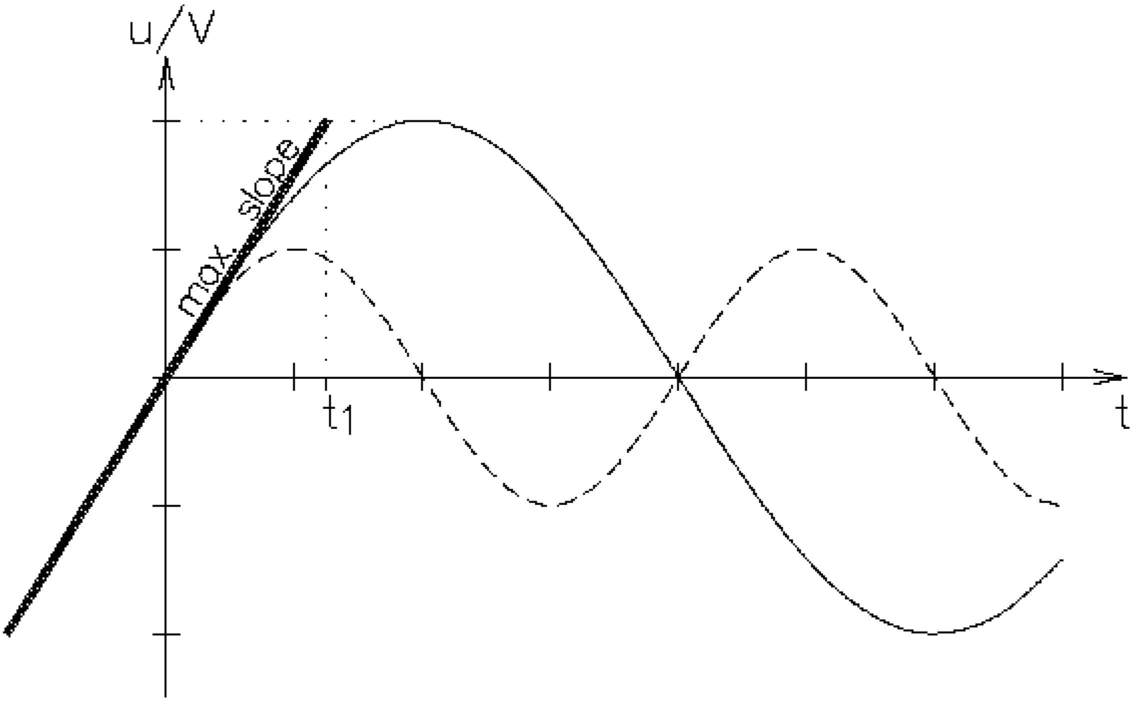


Fig. 5.2.2.

The maximum slope of a sine function occurs at the zero crossing. The slope depends on the amplitude and on the frequency.

If the voltage continues to rise with the zero-slope of the sine function, it will reach U_{max} at:

$$t_1 = \frac{1}{2\pi f} = \frac{T}{2\pi}$$

The maximum slope can therefore be expressed in terms of the amplitude and the frequency of the sine function:

$$\frac{\Delta U}{\Delta t} = \frac{U_{\mathbf{max}}}{t_1} = U_{\mathbf{max}} \cdot 2\pi f$$

This means for a given slew rate: the higher the output voltage, the smaller the maximum frequency, resp. band width;
and vice versa: the larger the required band width, the smaller the maximum amplitude.

The slew rate relates the maximum amplitude and
the maximum frequency of the output signal.

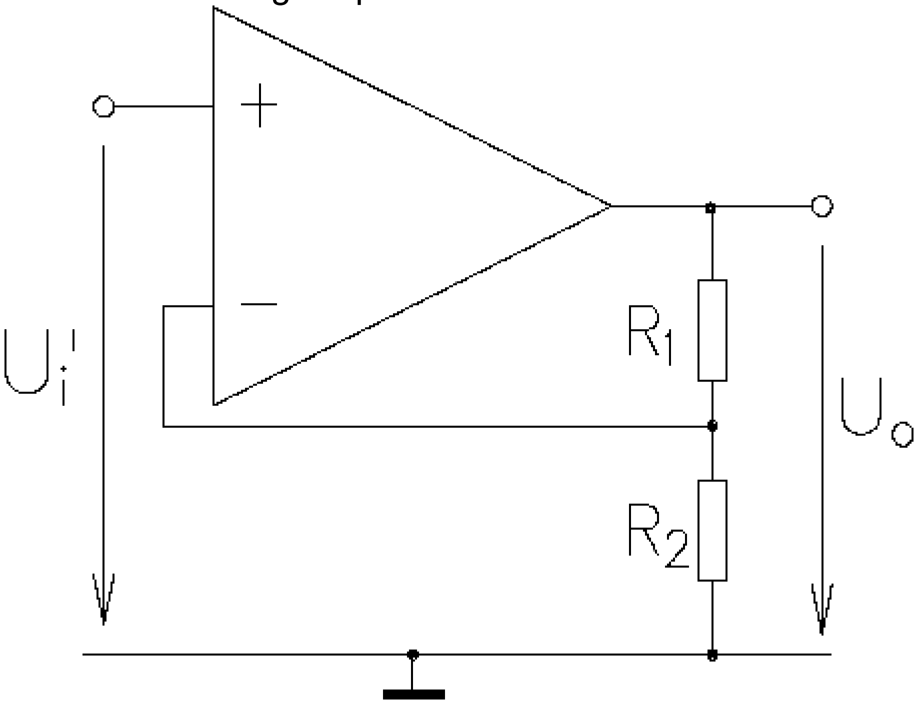
The slew rate cannot be influenced by NFB.

Examples of the slew rate of some practical opamps:
- type 741: O.3V/µs
- type 081: 13V/µs

6. Applications

This chapter sums up some of the most important opamp applications and gives their main characteristics and design rules.

6.1. Non-Inverting Amplifier

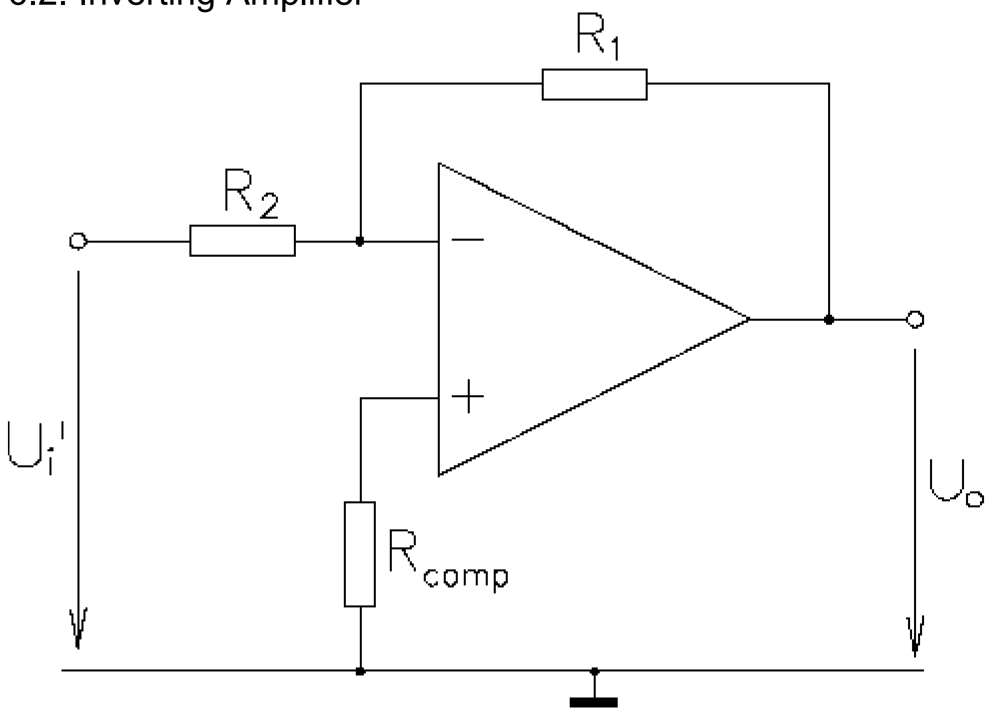


$$g' = \frac{R_1}{R_2} + 1$$

$$R_i' = R_i \cdot \frac{g}{g'}$$ (very high)

$$R_o' = R_o \cdot \frac{g'}{g}$$ (very low)

6.2. Inverting Amplifier



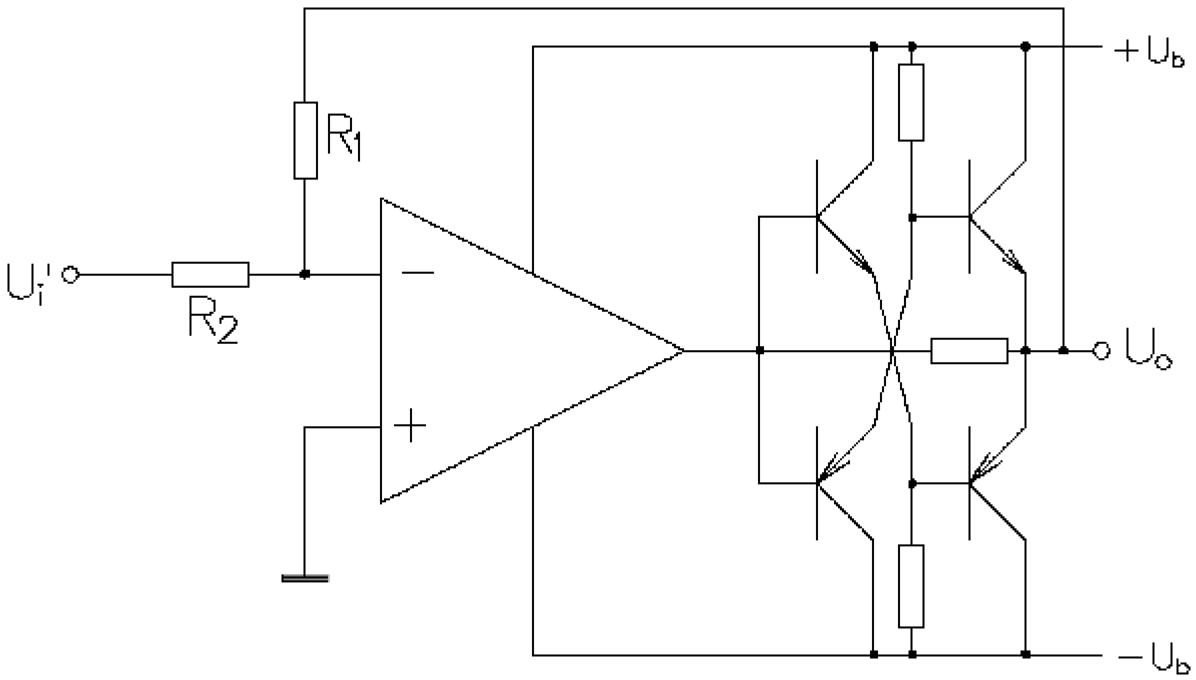
$$g' = \frac{R_1}{R_2}$$

$$R_i' = R_2$$

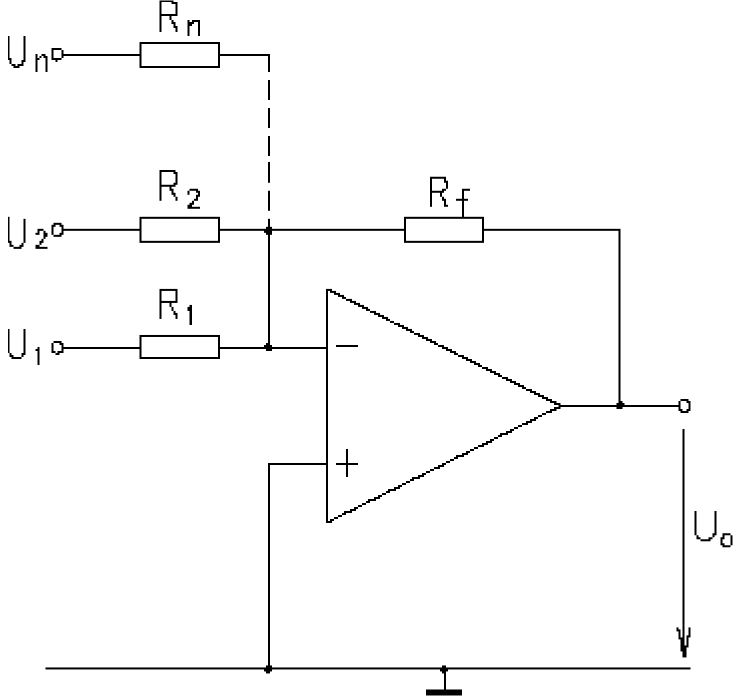
$$R_o' = R_o \cdot \frac{1+g'}{g}$$ (very low)

6.3. With push-pull output

The complementary push-pull stage boosts the output current.
If it is included in the NFB-loop, the take-over distortions are compensated.



6.4. Summing Amplifier

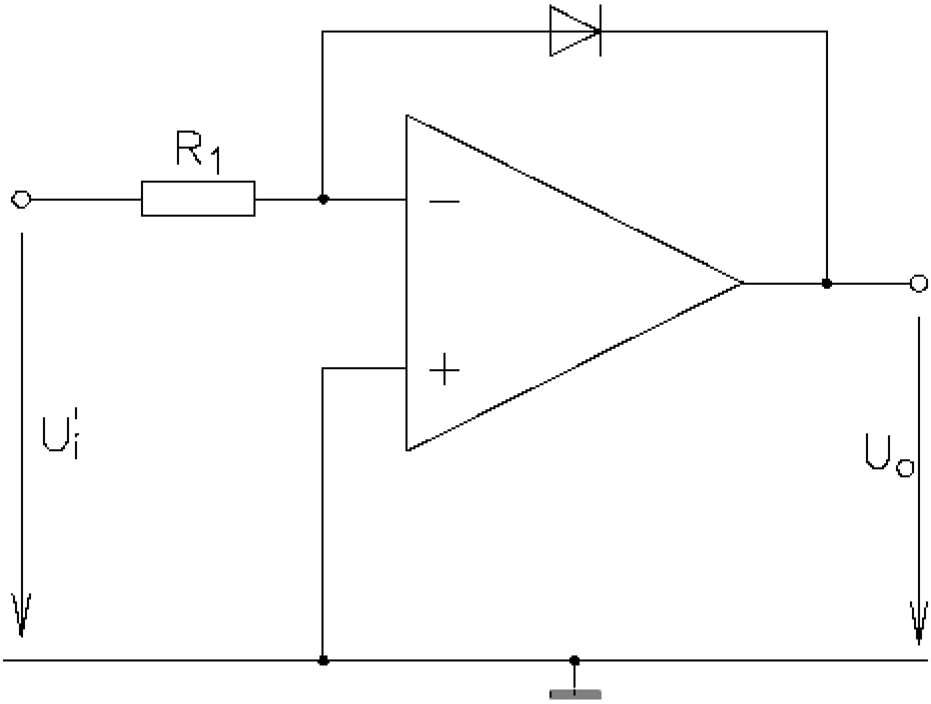


The input signals U_1, U_2, etc. are added up and amplified. As the summing point is the virtual ground (Zero-Ohms-Circuit), the inputs are fully decoupled from each other.

$$U_o = \frac{1}{R_f} \cdot (U_1 R_1 + U_2 R_2 + \ldots)$$

6.5. Logarithmizing Amplifier

A non-linear NFB-circuit will result in an non-linear characteristic of the amplifier. The exponential U-I-characteristic of the diode produces a logarithmic U_{in}-U_{out}-relationship.



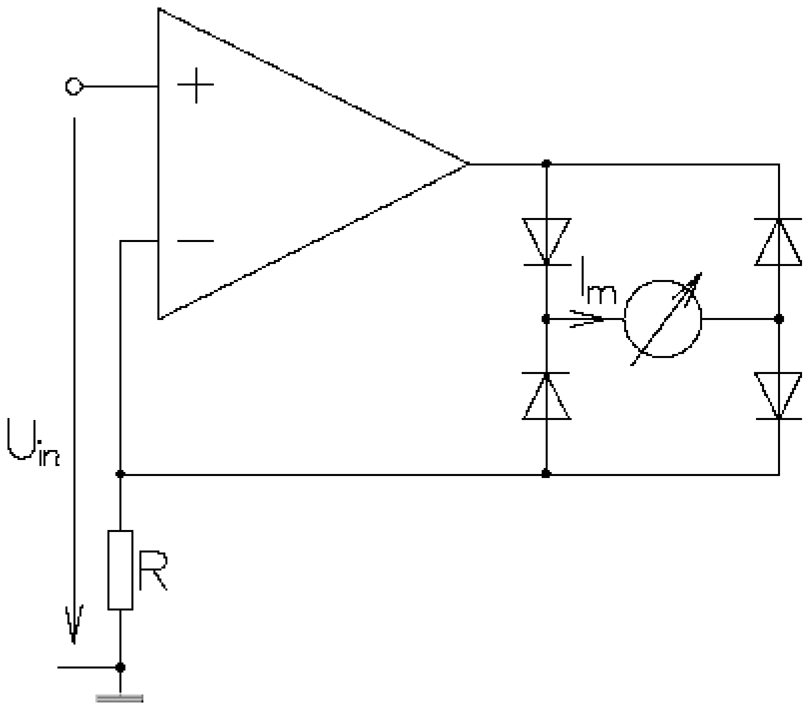
$$U_o = -U_T \cdot \ln \frac{U_i'}{I_o R_1}$$

(U_T is the inherent temperature voltage of the diode which, for silicon diodes, is approx. 40mV at 25°C. I_o is the minority current of the diode at 0V, which is appr. 10nA at 25°C)

6.6. Signal Rectification

The threshold voltage of rectifier diodes produce incorrect indications when small signal voltages have to be rectified for indication. Putting the rectifier into the NFB-loop of an opamp will produce a linear indication of the meter.

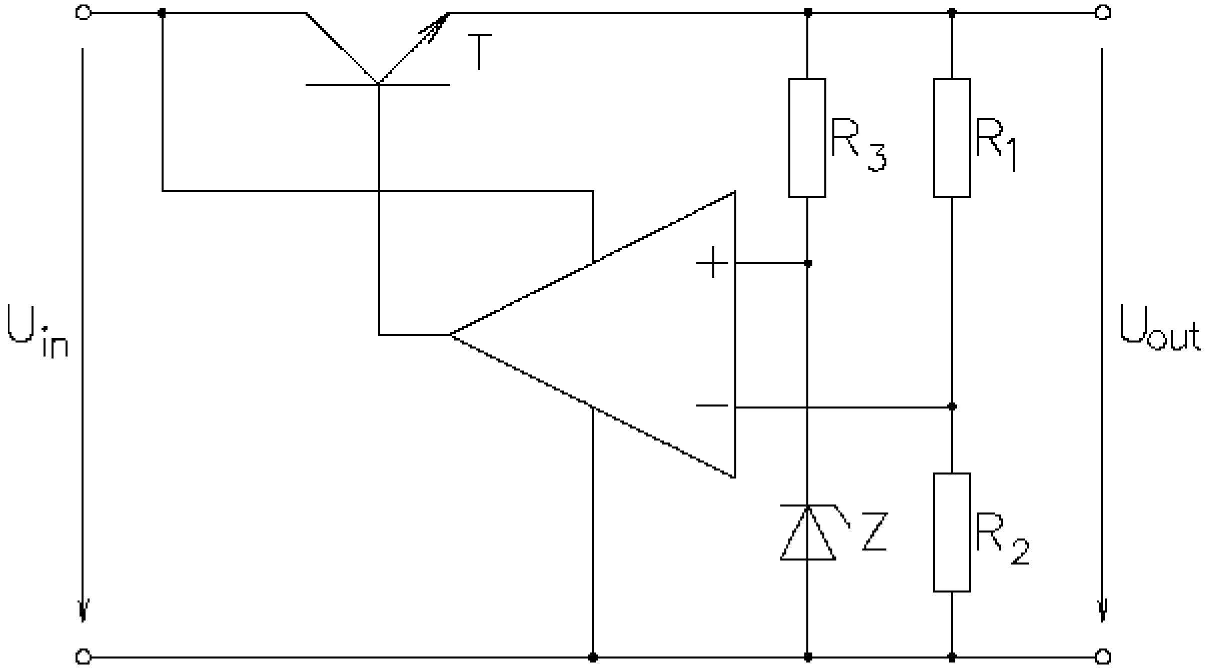
It is a disadvantage of this circuit that the meter cannot be grounded on one side.



$$I_m = \frac{U_i'}{R_1}$$

6.7. Voltage Regulator

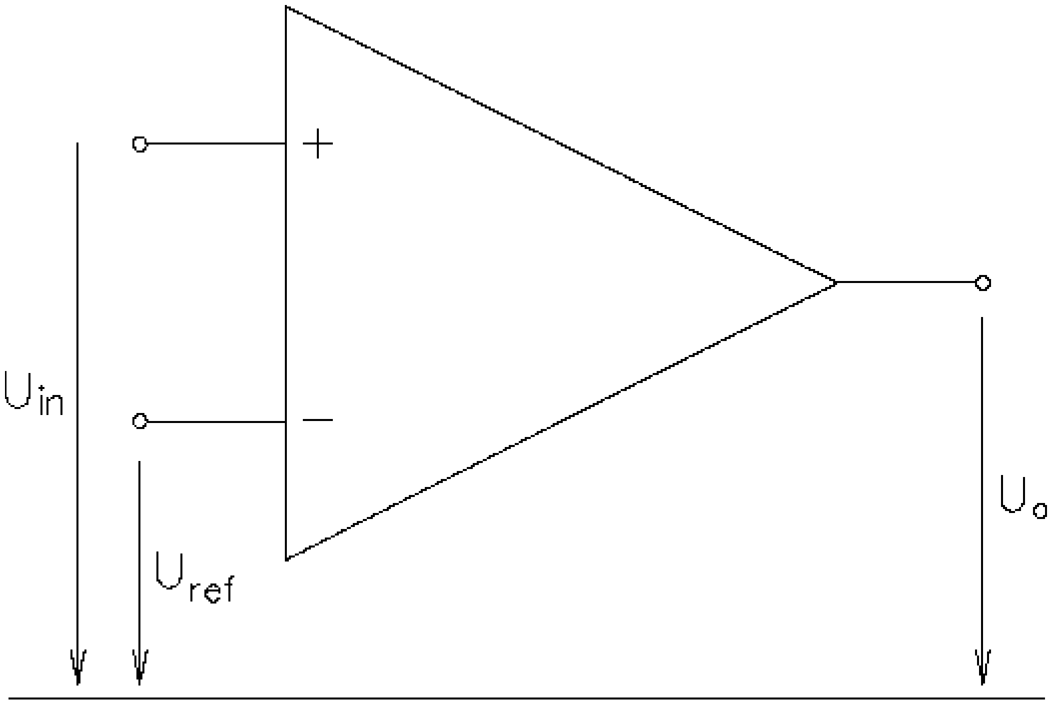
The opamp is used as an error amplifier, comparing the reference voltage with the actual output voltage. Depending on how much output current is required, several current amplifier transistor stages are required.



$$U_{out} = U_{ref}\left(1 + \frac{R_1}{R_2}\right)$$

6.8. Comparator

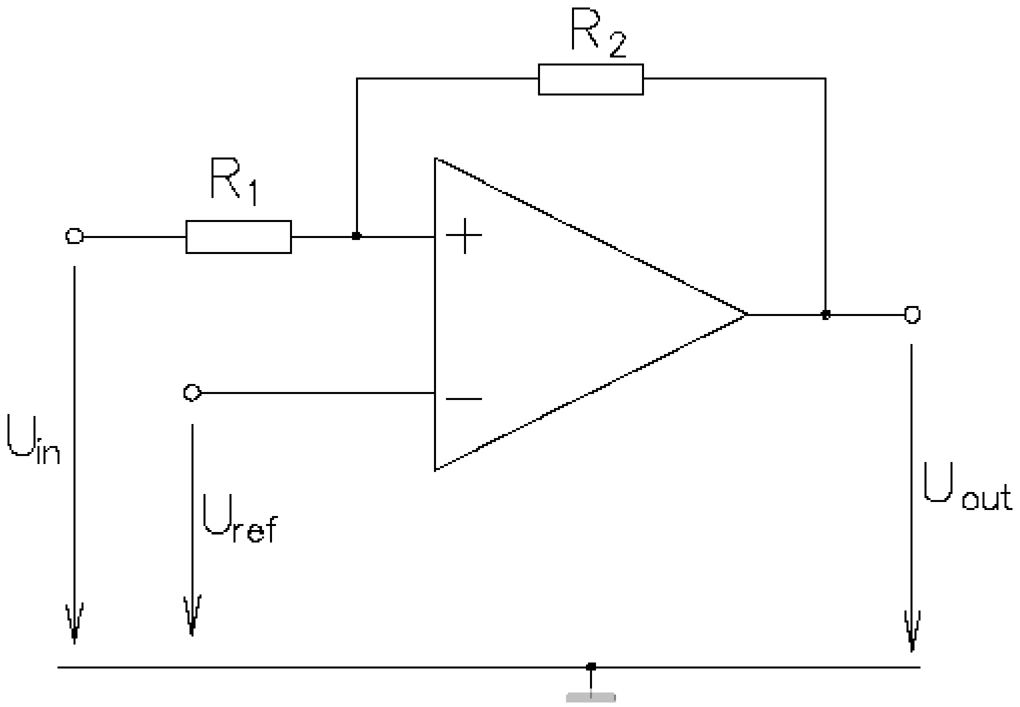
The comparator is an analog-digital converter. The output signal is high or low, depending on whether the input voltage is higher or lower than the reference voltage. If the reference voltage is applied to the non-inverting input, it will be an inverting comparator.



$$U_{out} = +U_{sat} \qquad \text{if } U_{in} > U_{ref}$$
$$U_{out} = -U_{sat} \qquad \text{if } U_{in} < U_{ref}$$

6.9. Schmitt Trigger

The Schmitt Trigger can be considered a comparator with hysteresis. By applying positive feedback, the output is always saturated. The threshold voltages for changing the output from positive to negative is different from the voltage which will change it from negative to positive.



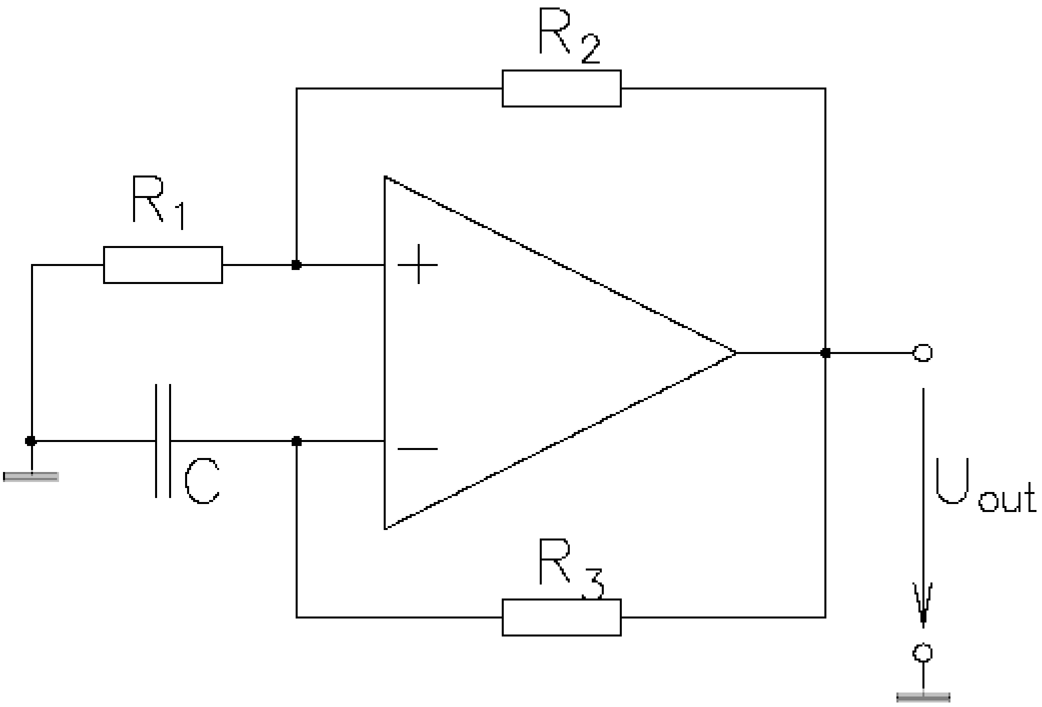
$$U_{inup} = U_{ref} - \left(\frac{R_1}{R_2}\right) \cdot (-U_{sat})$$

$$U_{indn} = U_{ref} - \left(\frac{R_1}{R_2}\right) \cdot (+U_{sat})$$

$$U_{inup} - U_{indn} = \left(\frac{R_1}{R_2}\right) \cdot (2 \cdot U_{sat})$$

6.10. Astable Multivibrator

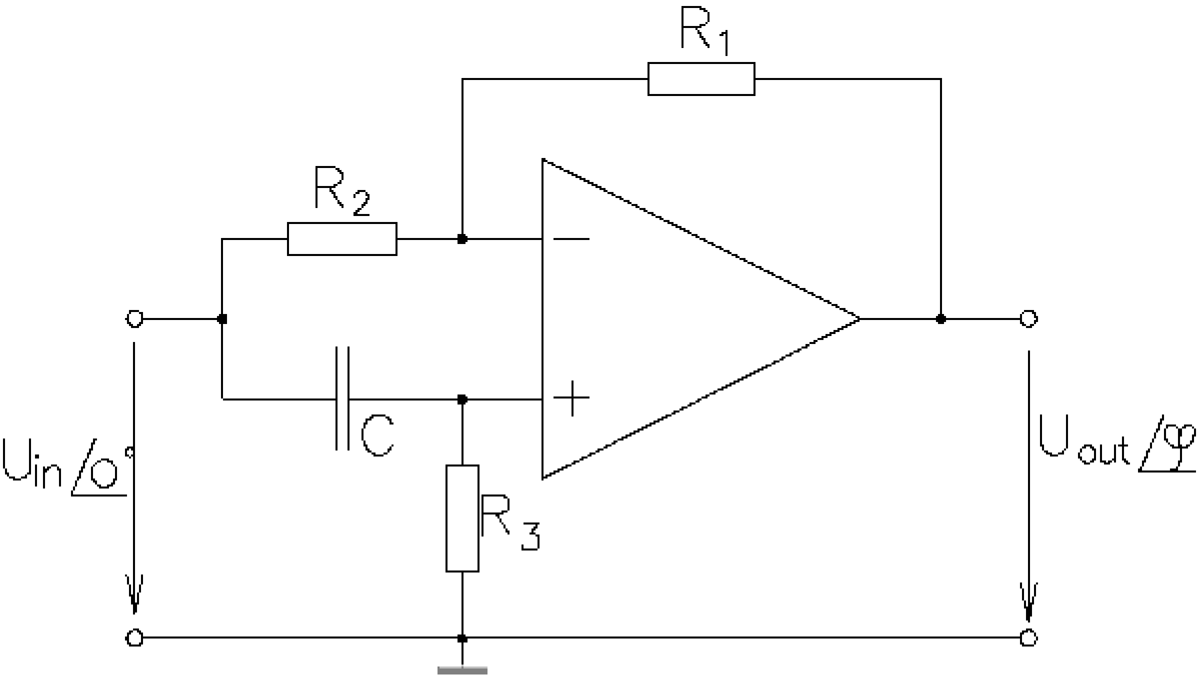
This circuit produces a symmetrical square wave at the output of the opamp. The amplitude is given by the saturation voltage of the opamp. The steepness of the flanks is limited by the slew rate.



$$f = \frac{1}{2 \cdot R_3 C \cdot \ln(1 + 2\frac{R_1}{R_2})}$$

6.11. Phase Shifter

This circuit provides a frequency depending phase shift between the input and output signal, but has a linear amplitude response. It is therefore also called an ALL PASS FILTER. The phase shift will vary between 0 and 180°. The gain is defined by the negative feedback of R_1 and R_2. Normally, the gain is set to 1 (R_1=R_2).



$$\varphi(f) = 180^{\circ} - 2 \cdot arctg(2\pi f R_3 C)$$