PROPERTIES OF LOGARITHMIC FUNCTIONS

EXPONENTIAL FUNCTIONS

An exponential function is a function of the form $f(x) = b^x$, where b > 0 and x is any real number. (Note that $f(x) = x^2$ is NOT an exponential function.)

LOGARITHMIC FUNCTIONS

$\log_b x = y$ **means that** $x = b^y$ where $x > 0, b > 0, b \neq 1$

Think: Raise *b* to the power of *y* to obtain *x*. *y* is the exponent.
The key thing to remember about logarithms is that the logarithm is an exponent!
The rules of exponents apply to these and make simplifying logarithms easier.

Example: $\log_{10} 100 = 2$, since $100 = 10^2$.

$\log_{10} x$ is often written as just $\log x$, and is called the COMMON logarithm.
$\log_e x$ is often written as $\ln x$, and is called the NATURAL logarithm (note: $e \approx 2.718281828459...$).

PROPERTIES OF LOGARITHMS — EXAMPLES

1. $\log_b MN = \log_b M + \log_b N$ — $\log 50 + \log 2 = \log 100 = 2$

Think: Multiply two numbers with the same base, add the exponents.

2. $\log_b \frac{M}{N} = \log_b M - \log_b N$ — $\log_8 56 - \log_8 7 = \log_8\left(\frac{56}{7}\right) = \log_8 8 = 1$

Think: Divide two numbers with the same base, subtract the exponents.

3. $\log_b M^P = P \log_b M$ — $\log 100^3 = 3 \cdot \log 100 = 3 \cdot 2 = 6$

Think: Raise an exponential expression to a power and multiply the exponents together.

$\log_b b^x = x$ $\quad$ $\log_b 1 = 0$ (in exponential form, $b^0 = 1$) $\quad$ $\ln 1 = 0$

$\log_b b = 1$ $\quad$ $\log_{10} 10 = 1$ $\quad$ $\ln e = 1$

$\log_b b^x = x$ $\quad$ $\log_{10} 10^x = x$ $\quad$ $\ln e^x = x$

$b^{\log_b x} = x$ $\quad$ Notice that we could substitute $y = \log_b x$ into the expression on the left to form b^y. Simply re-write the equation $y = \log_b x$ in exponential form as $x = b^y$. Therefore, $b^{\log_b x} = b^y = x$. $\quad$ Ex: $e^{\ln 26} = 26$

CHANGE OF BASE FORMULA

$\log_b N = \frac{\log_a N}{\log_a b}$, for any positive base *a*. $\quad$ $\log_{12} 5 = \frac{\log 5}{\log 12} \approx \frac{0.698970}{1.079181} \approx 0.6476854$

This means you can use a regular scientific calculator to evaluate logs for any base.

Practice Problems contributed by Sarah Leyden, typed solutions by Scott Fallstrom

Solve for x (do not use a calculator).

1. $\log_9(x^2-10)=1$
2. $\log_3 3^{2x+1}=15$
3. $\log_x 8=3$
4. $\log_5 x=2$
5. $\log_5(x^2-7x+7)=0$
6. $\log_3 27^x=4.5$
7. $\log_x 8=-\frac{3}{2}$
8. $\log_6 x+\log_6(x-1)=1$
9. $\log_2 x^{1/2}+\log_2\left(\frac{1}{x}\right)=3$
10. $\log_2 x^2-\log_2(3x+8)=1$
11. $(1/2)\log_3 x-(1/3)\log_3 x^2=1$

Solve for x, use your calculator (if needed) for an approximation of x in decimal form.

12. $7^x=54$
13. $\log_{10} x=17$
14. $5^x=9\cdot 4^x$
15. $10^x=e$
16. $e^{-x}=1.7$
17. $\ln(\ln x)=1.013$
18. $8^x=9^x$
19. $10^{x+1}=e^4$
20. $\log_x 10=-1.54$

Solutions to the Practice Problems on Logarithms:

1. $\log_9(x^2-10)=1\Rightarrow 9^1=x^2-10\Rightarrow x^2=19\Rightarrow \boxed{x=\pm\sqrt{19}}$
2. $\log_3 3^{2x+1}=15\Rightarrow 3^{15}=3^{2x+1}\Rightarrow 2x+1=15\Rightarrow 2x=14\Rightarrow \boxed{x=7}$
3. $\log_x 8=3\Rightarrow x^3=8\Rightarrow \boxed{x=2}$
4. $\log_5 x=2\Rightarrow 5^2=x\Rightarrow \boxed{x=25}$
5. $\log_5(x^2-7x+7)=0\Rightarrow 5^0=x^2-7x+7\Rightarrow 0=x^2-7x+6\Rightarrow 0=(x-6)(x-1)\Rightarrow \boxed{x=6\text{ or }x=1}$
6. $\log_3 27^x=4.5\Rightarrow \log_3(3^3)^x=4.5\Rightarrow \log_3 3^{3x}=4.5\Rightarrow 3x=4.5\Rightarrow \boxed{x=1.5}$
7. $\log_x 8=-\frac{3}{2}\Rightarrow x^{-3/2}=8\Rightarrow x=8^{-2/3}\Rightarrow \boxed{x=\frac{1}{4}}$
8. $\log_6 x+\log_6(x-1)=1\Rightarrow \log_6(x^2-x)=1\Rightarrow x^2-x=6\Rightarrow x^2-x-6=0\Rightarrow$ $(x-3)(x+2)=0\Rightarrow \boxed{x=3}$ or $x=-2$. Note: $x=-2$ is an extraneous solution, which solves only the new equation. $x=3$ is the only solution to the original equation.
9. $\log_2 x^{1/2}+\log_2\left(\frac{1}{x}\right)=3\Rightarrow \log_2\left(\frac{x^{1/2}}{x}\right)=3\Rightarrow \log_2 x^{-1/2}=3\Rightarrow 2^3=x^{-1/2}\Rightarrow \boxed{x=(2^3)^{-2}=\frac{1}{64}}$
10. $\log_2 x^2-\log_2(3x+8)=1\Rightarrow \log_2\left(\frac{x^2}{3x+8}\right)=1\Rightarrow \frac{x^2}{3x+8}=2\Rightarrow x^2=6x+16\Rightarrow$ $x^2-6x-16=0\Rightarrow (x-8)(x+2)=0\Rightarrow \boxed{x=8\text{ or }x=-2}$
11. $(1/2)\log_3 x-(1/3)\log_3 x^2=1\Rightarrow \log_3 x^{1/2}-\log_3 x^{2/3}=1\Rightarrow \log_3\left(\frac{x^{1/2}}{x^{2/3}}\right)=1\Rightarrow x^{1/2-2/3}=3\Rightarrow$ $x^{-1/6}=3\Rightarrow \boxed{x=3^{-6}=\frac{1}{729}}$
12. $7^x=54\Rightarrow x=\log_7 54\Rightarrow \boxed{x=\frac{\log 54}{\log 7}\approx 2.0499}$
13. $\log_{10} x=17\Rightarrow \boxed{x=10^{17}}$
14. $5^x=9\cdot 4^x\Rightarrow \frac{5^x}{4^x}=9\Rightarrow \left(\frac{5}{4}\right)^x=9\Rightarrow x=\log_{\frac{5}{4}} 9\Rightarrow \boxed{x\approx 9.8467}$
15. $10^x=e\Rightarrow x=\log_{10} e\Rightarrow \boxed{x=\log e\approx 0.4343}$
16. $e^{-x}=1.7\Rightarrow -x=\ln 1.7\Rightarrow \boxed{x=-\ln 1.7\approx -0.5306}$
17. $\ln(\ln x)=1.013\Rightarrow \ln x=e^{1.013}\Rightarrow \boxed{x=e^{e^{1.013}}\approx 15.7030}$
18. $8^x=9^x\Rightarrow 1=\left(\frac{9}{8}\right)^x\Rightarrow x=\log_{\frac{9}{8}} 1\Rightarrow \boxed{x=0}$
19. $10^{x+1}=e^4\Rightarrow x+1=\log e^4\Rightarrow x=\log e^4-1=\log e^4-\log 10\Rightarrow \boxed{x=\log\left(\frac{e^4}{10}\right)\approx 0.7372}$
20. $\log_x 10=-1.54\Rightarrow x^{-1.54}=10\Rightarrow \boxed{x=10^{-1/1.54}\approx 0.2242}$