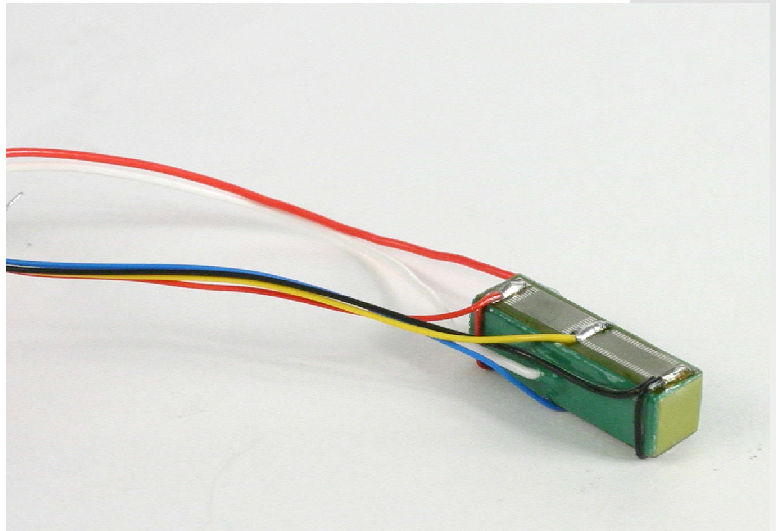




Definition of Strain

Tutorial



Part 1. Definition of Strain

Strain is the parameter used to quantify the deformation of an object. In Figure 1B and 1C, opposing forces are applied to each end of a rod with an original length of L_0 . The applied forces will cause the rod to deform so that

$$L_f = L_0 + \Delta L, \quad (1)$$

where L_f is the length of the rod when the opposing forces are applied and

$$\Delta L = L_f - L_0. \quad (2)$$

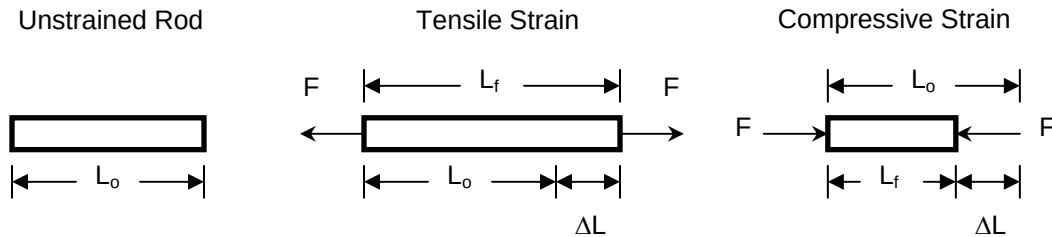


Figure 1: Schematic drawings of a rod with no strain (A), tensile strain (B), and compressive strain (C).

The strain parameter ε is then defined as the ratio of the rod's change in length to the rod's original length.

$$\varepsilon \equiv \frac{\Delta L}{L_0}. \quad (3)$$

When ΔL is positive, the rod is undergoing tensile strain, which is also referred to as positive strain. When ΔL is negative, the rod is undergoing compressive strain, which is also referred to as negative strain. Although the strain parameter is a dimensionless quantity, it is often expressed as a ratio of length dimensions such as in/in or mm/mm. Since most rigid objects do not macroscopically compress or stretch when subjected to forces, the magnitude of ε is usually small, and as a result, strain is typically reported as microstrain ($\mu\varepsilon = \varepsilon \times 10^{-6}$).

Part 2. The Metallic Strain Gauge

Metallic strain gauges are one of many devices, along with piezo resistors and devices based on interferometric techniques, that have been developed to measure microstrain. Invented by Edward E. Simmons in 1938, the metallic strain gauge consists of a fine wire or metallic foil with an electrical resistance (R_o) adhered to a flat rigid substrate. R_o typically varies from tens to thousands of ohms and the substrate is often referred to as the carrier. The wire or metallic foil usually has a serpentine conducting path in order to maximize its length parallel to the axis that strain is being measured, while minimizing the surface area of the metallic strain gauge, see Figure 2.

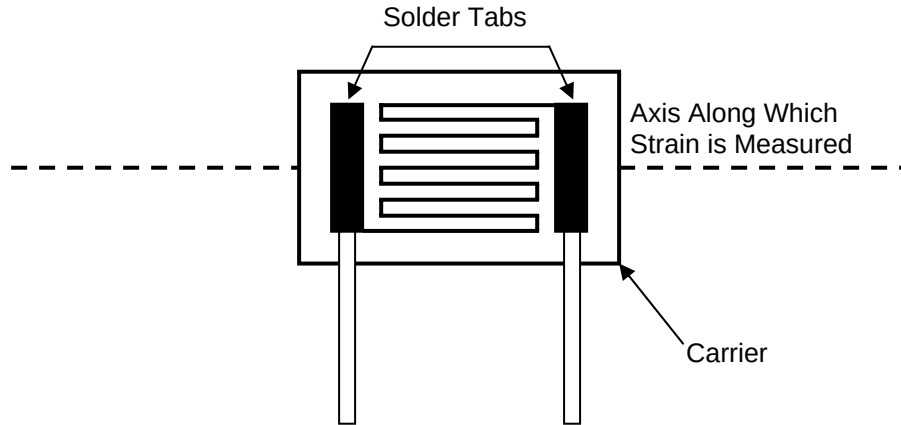


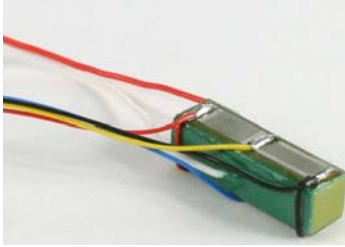
Figure 2: Schematic drawing of a metallic strain gauge

The strain gauge is then bonded to the object being measured. Any expansion or contraction of the object along the axis of the strain gauge will also occur in the conducting path, and as a result, will cause a proportional change in the resistance of the strain gauge. The proportionality factor, or gauge factor (G_F), for metallic strain gauges is usually two.

$$GF \equiv \frac{\frac{\Delta R}{R_o}}{\frac{\Delta L}{L_o}} = \frac{\frac{\Delta R}{R_o}}{\varepsilon}, \quad (4)$$

where ΔR is the change in the resistance of the strain gauge when it is strained.

Part 3. Measuring Strain



The PZS001 actuator has an L_0 of 20 mm. The full bridge strain gauge bonded to the surface is made from four individual strain gauges, each with a G_F of two, and an R_0 of 350 Ω . For a single strain gauge of the type used in the PZS001, a 5 nm change of length in the actuator corresponds to a strain of 2.5×10^{-7} . As a result, the length of the actuator can only be controlled if a change of resistance in the strain gauge equal to $1.75 \times 10^{-4} \Omega$ can be measured.

Trying to measure the absolute resistance of the strain gauge to less than a milliohm is difficult; however, making a precise measurement is not the only problem. Changes in the temperature of the gauge will cause significant, on the milliohm scale, changes in the resistance of the gauge. Both of these issues can be addressed by placing two active and two dummy strain gauges, see Figure 3, in a Wheatstone bridge configuration as shown in Figure 4.

A dummy strain gauge is oriented in order to minimize the length of conducting wire parallel to the axis along which the strain is being measured.

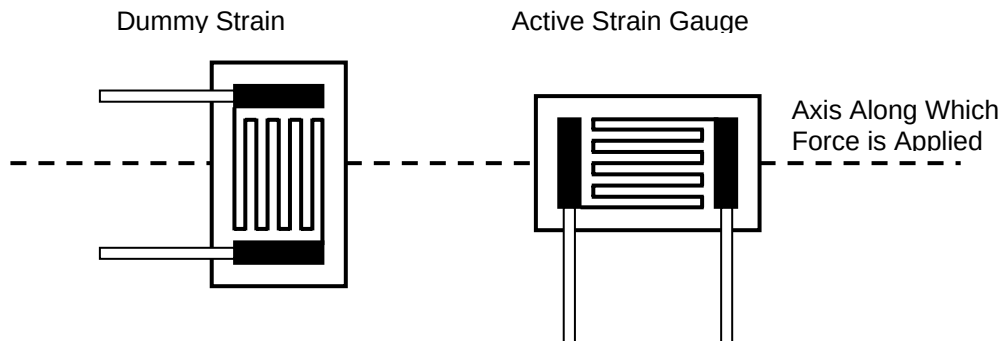


Figure 3: Dummy strain gauge versus active strain gauge orientation

In this orientation, the dummy strain gauge is unaffected by the applied strain; however, the resistance of the strain gauge is still dependent on the temperature in the same way as the active strain gauge.

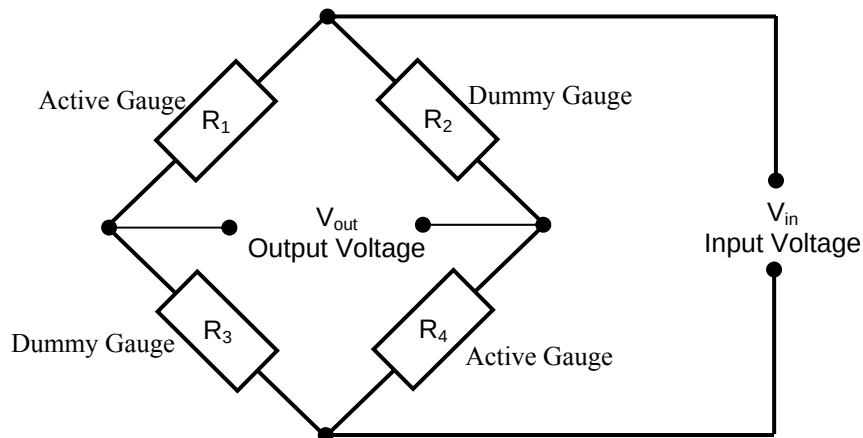


Figure 4: Wheatstone bridge circuit with two active and two dummy strain gauges

The Wheatstone bridge, or full bridge configuration, of strain gauges shown in Figure 4 allows for the precise determination of the strain gauge resistance by measuring the change in the output voltage (V_{out}), while a fixed input voltage (V_{in}) is applied to the bridge. Ideally, the bridge should be built so that $R_1=R_3$ and $R_2=R_4$, which makes the V_{out} equal to zero, independent of the V_{in} , for an unstrained object. R_1 , R_2 , R_3 , and R_4 are used to symbolize the unstrained resistance of each strain gauge. Since there is a dummy strain gauge in each branch of the Wheatstone bridge, the V_{out} will be independent of temperature, but dependent on the strain experienced by the object. In general, the ratio of the V_{out} to the V_{in} can be written as:

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + \Delta R}{R_1 + \Delta R + R_3} - \frac{R_4 + \Delta R}{R_4 + \Delta R + R_2} \quad (5)$$

Note that if $\Delta R=0$, no strain, then $V_{out}=0$ even if the values of R_1 and R_4 change due to temperature since R_2 and R_3 will change by the same amount since they are dummy strain gauges. Since each branch of the Wheatstone bridge is a voltage divider, typically, the full bridge strain gauge is built so that the unstrained resistance of each strain gauge is equal:

$$R_o = R_1 = R_2 = R_3 = R_4 \quad (6)$$

Replacing R_1 , R_2 , R_3 , and R_4 in equation 5 with R_o yields:

$$\frac{V_{out}}{V_{in}} = \frac{\Delta R}{2R_o + \Delta R} \quad (7)$$

Using equation 4, ΔR in equation 7 can be replaced to create equation 8:

$$\frac{V_{out}}{V_{in}} = \frac{R_o G_F \epsilon}{2R_o + R_o G_F \epsilon} = \frac{G_F \epsilon}{2 + G_F \epsilon} \quad (8)$$

Solving equation 8 for ϵ yields:

$$\epsilon = \frac{2V_{out}}{G_F (V_{in} - V_{out})} \quad (9)$$

If the V_{in} is 10 volts, the V_{out} must be measured with microvolt precision in order to control the length of the actuator on the nanometer scale for the PZS001.

Part 4. Practical Considerations When Measuring Strain

When measuring the V_{out} of a full bridge strain gauge on a microvolt scale, several factors must be controlled in order to get an accurate measurement of the strain.

The V_{in} must be a low-noise AC or DC voltage source. Obviously, if the V_{in} is increased, the V_{out} will increase proportionally. However, there is a practical limit to the V_{in} magnitude, because as that increases, the amount of heat generated by the circuit increases as well, which can lead to temperature-related errors. Typically a V_{in} of ten of volts or less is used.

Because it is difficult to build a circuit in which the resistance of each strain gauge is exactly the same, an installed full bridge strain gauge is unlikely to have a zero output voltage reading when there is no strain. An inequality in the resistance of the leads, which should be kept as similar as possible, to the circuit will also create a non-zero V_{out} in the unstrained circuit. To compensate for small non-zero voltages for unstrained circuits, an external offset circuit, or the [APT Software System](#) that comes with Thorlabs' [piezo-electric drivers](#), can be used. Compensating for larger voltages with this software, however, reduces the dynamic range of the measurement due to limits in the amplifier gain that can be applied to V_{out} .

The strain gauge circuit should be located as close as possible to the driver, because the length of the leads is proportional to their resistance. The resistance of the leads adds to the total resistance of the circuit. This increases the denominator of the right side of equation 7, which reduces V_{out} for a given amount of strain. The leads to the strain gauge circuit should also be kept short to minimize the effects of transmission line noise. A low-pass filter can be added to the circuit to remove high-frequency noise prevalent in most work environments.

The strain gauge circuit can be calibrated by monitoring the V_{out} for known amounts of strain, or by simulating strain. Strain can be simulated by placing a large known resistor in parallel to each active resistor in the full bridge strain gauge. The large resistor creates a known theoretical ΔR , used to calculate a value for V_{out} that can be used to calibrate the measured V_{out} .