Synchronous Machines

1.0 Introduction

One might easily argue that the synchronous generator is the most important component in the power system, since synchronous generators

- Are the source of 99% of the MW in most power systems;
- Provide frequency regulation and load following;
- Are the main source of voltage control;
- Are an important source of oscillation damping.

For that reason, we will spend the remainder of the course studying this component.

EE 303 contains a chapter on synchronous generators (Module G1). Sections 2 and 3 of these notes will be basically a review of this module, except that we will more rigorously develop the smooth rotor model used there.

Sections 4 and 5 will extend your knowledge of synchronous generators to account for salient pole machines,

2.0 Synchronous Generator Construction

The synchronous generator converts mechanical energy from the turbine into electrical energy.

The turbine converts some kind of energy (steam, water, wind) into mechanical energy, as illustrated in Fig. 1 [1].

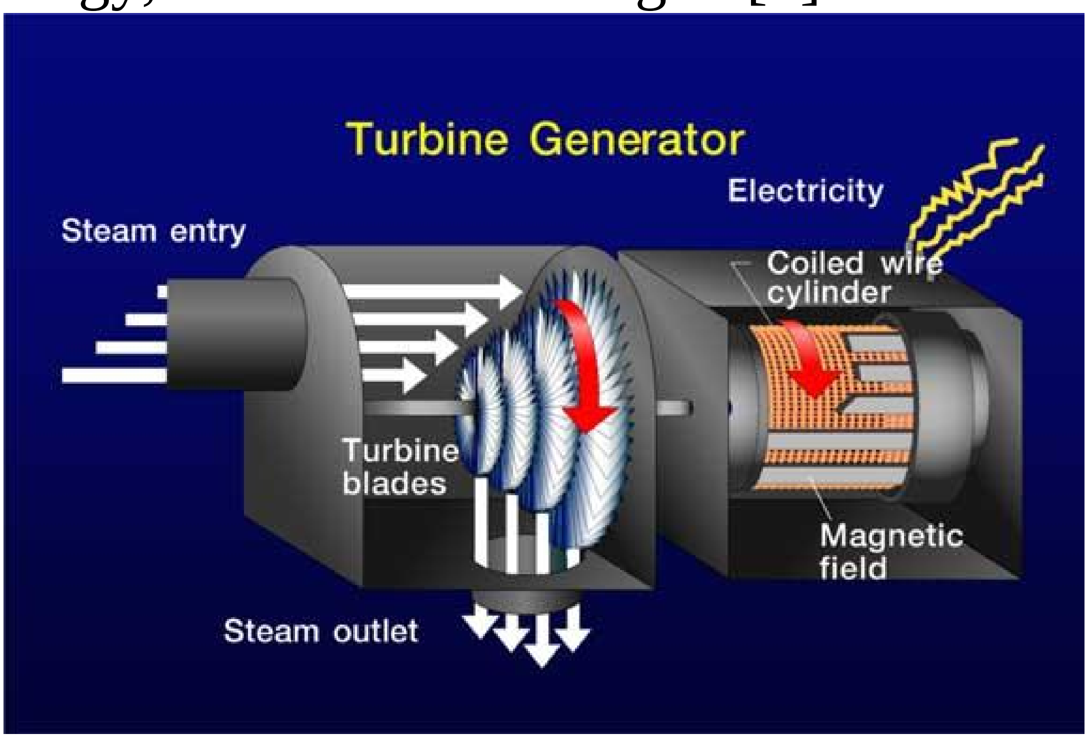

Fig. 1 [1]

The synchronous generator has two parts:

- Stator: carries 3 (3-phase) armature windings, AC, physically displaced from each other by 120 degrees
- Rotor: carries field windings, connected to an external DC source via slip rings and brushes or to a revolving DC source via a special *brushless* configuration.

Fig. 2 shows a simplified diagram illustrating the slip-ring connection to the field winding.

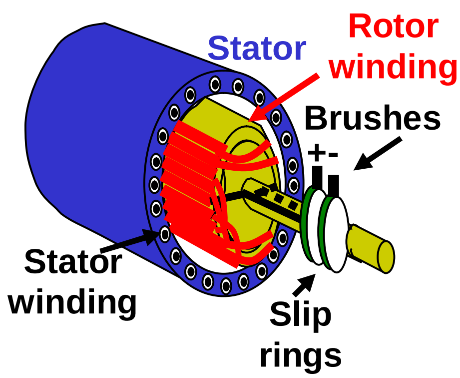

Fig. 2

Fig. 3 shows the rotor from a 200 MW steam generator. This is a smooth rotor.

Repairs to the overhand insulation of this 200MW generator rotor were carried out by experienced tradesmen working on shift to ensure the earliest possible return to service.

Fig. 3

Fig. 4 shows the rotor and stator of a hydro-generator, which uses a salient pole rotor.

Fig. 4

Fig. 5 illustrates the synchronous generator construction for a salient pole machine, with 2 poles. Note that Fig. 5 only represents one "side" of each phase, so as to not crowd the picture too much. In other words, we should also draw the Phase A return conductor 180° away from the Phase A conductor shown in the picture. Likewise for Phases B and C.

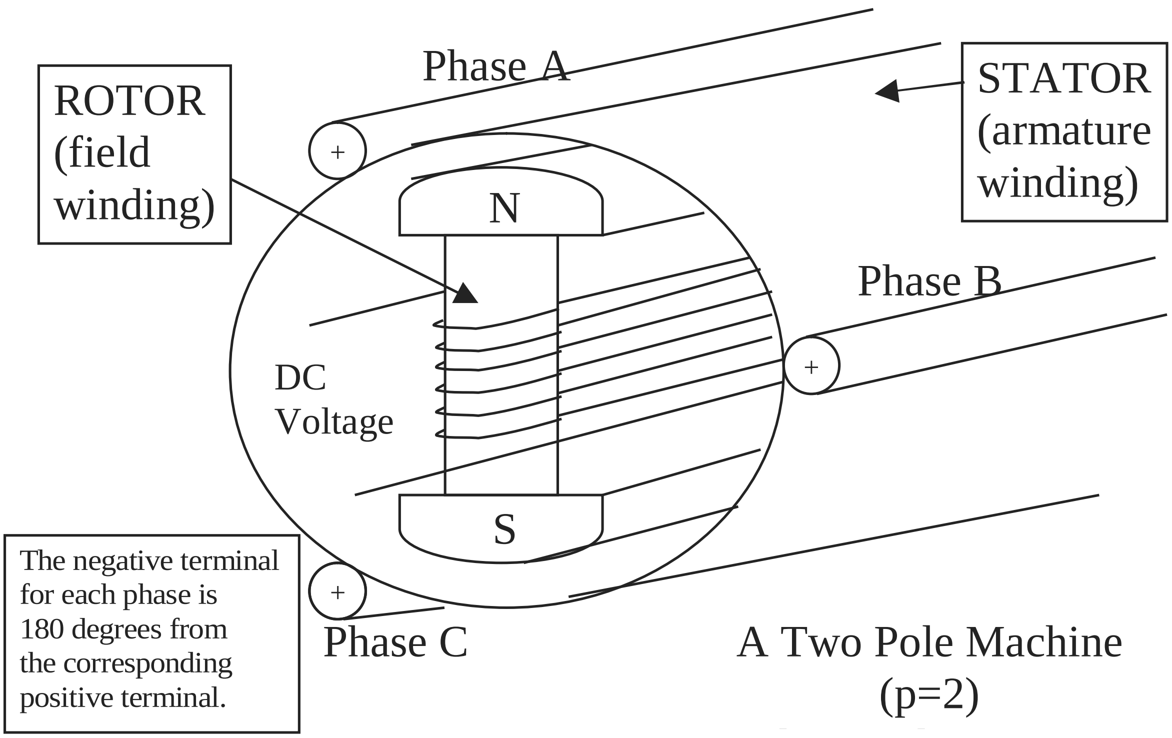

Fig. 5

Fig. 6 shows just the rotor and stator (but without stator winding) for a salient pole machine with 4 poles.

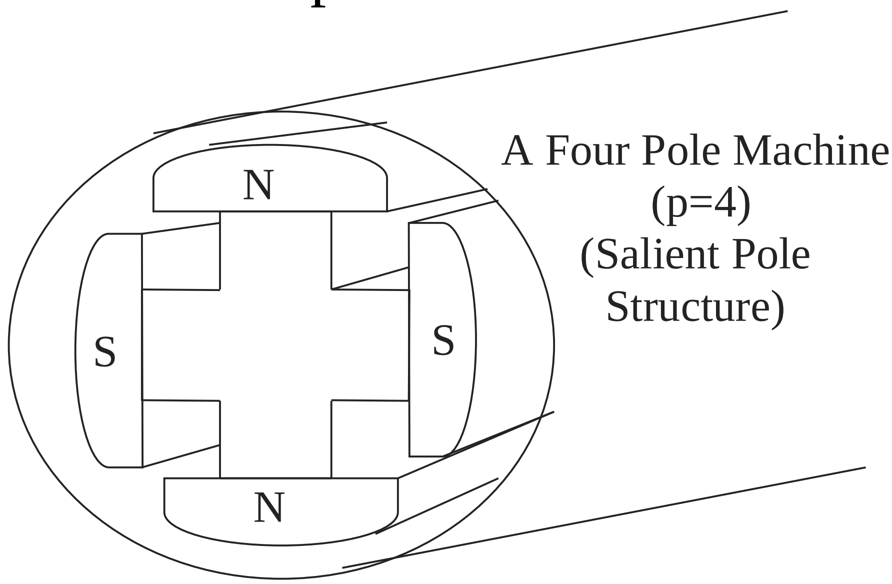

Fig. 6

The difference between smooth rotor construction and salient pole rotor construction is illustrated in Fig. 7. Note the air-gap in Fig. 7.

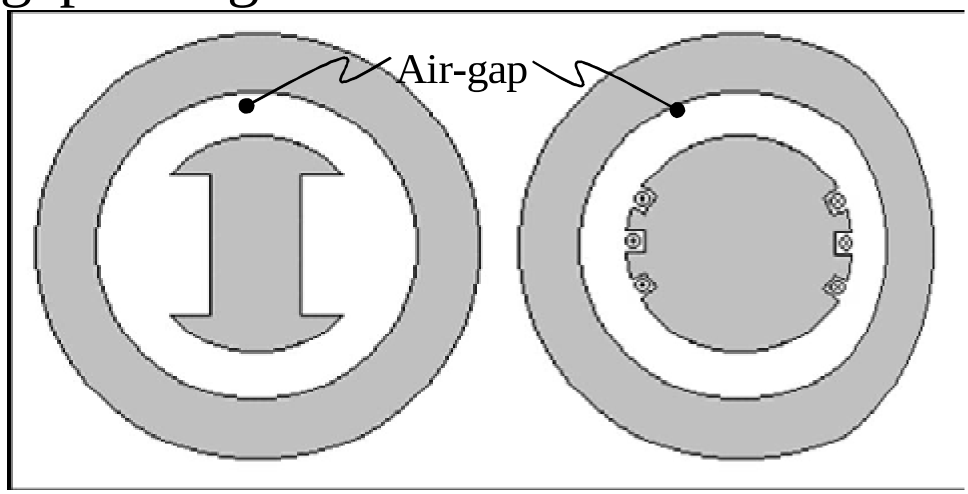

Fig. 7

The synchronous generator is so-named because it is only at synchronous speed that it functions properly. We will see why later. For now, we define synchronous speed as the speed for which the induced voltage in the armature (stator) windings is synchronized with (has same frequency as) the network voltage. Denote this as ω_e.

In North America,

$$\omega_e=2\pi(60)=376.9911\approx 377\text{rad/sec}$$

In Europe,

$$\omega_e=2\pi(50)=314.1593\approx 314\text{rad/sec}$$

On an airplane,

$$\omega_e=2\pi(400)=2513.3\approx 2513\text{rad/sec}$$

The mechanical speed of the rotor is related to the synchronous speed through:

$$w_m = \frac{2}{p}(w_e) \tag{1}$$

where both ω_m and ω_e are given in rad/sec. This may be easier to think of if we write

$$w_e = \frac{p}{2}(w_m) \tag{2}$$

Thus we see that, when p=2, we get one electric cycle for every one mechanical cycle. When p=4, we get two electrical cycles for every one mechanical cycle.

If we consider that ω_e must be constant from one machine to another, then machines with more poles must rotate more slowly than machines with less.

It is common to express ω_m in RPM, denoted by N; we may easily derive the conversion from analysis of units:

$N_m=(\omega_m \text{ rad/sec})*(1 \text{ rev}/2\pi \text{ rad})*(60\text{sec/min})$
$= (30/\pi)\omega_m$

Substitution of $\omega_m=(2/p)\ \omega_e=(2/p)2\pi f=4\pi f/p$

$$N_m= (30/\pi)(4\pi f/p)=120f/p \qquad (3)$$

Using (3), we can see variation of N_m with p for f=60 Hz, in Table 1.

Table 1

No. of Poles (p)	Synchronous speed (Ns)
2	3600
4	1800
6	1200
8	900
10	720
12	600
14	514
16	450
18	400
20	360
24	300
32	225
40	180

Because steam-turbines are able to achieve high speeds, and because operation is more efficient at those speeds, most steam turbines are 2 pole, operating at 3600 RPM. At this rotational speed, the surface speed of a 3.5 ft diameter rotor is about 450 mile/hour. Salient poles incur very high mechanical stress and windage losses at this speed and therefore cannot be used. All steam-turbines use smooth rotor construction.

Because hydro-turbines cannot achieve high speeds, they must use a higher number of poles, e.g., 24 and 32 pole hydro-machines are common. But because salient pole construction is less expensive, all hydro-machines use salient pole construction.

Fig. 8 illustrates several different constructions for smooth and salient-pole rotors. The red arrows indicate the direction of the flux produced by the field windings.

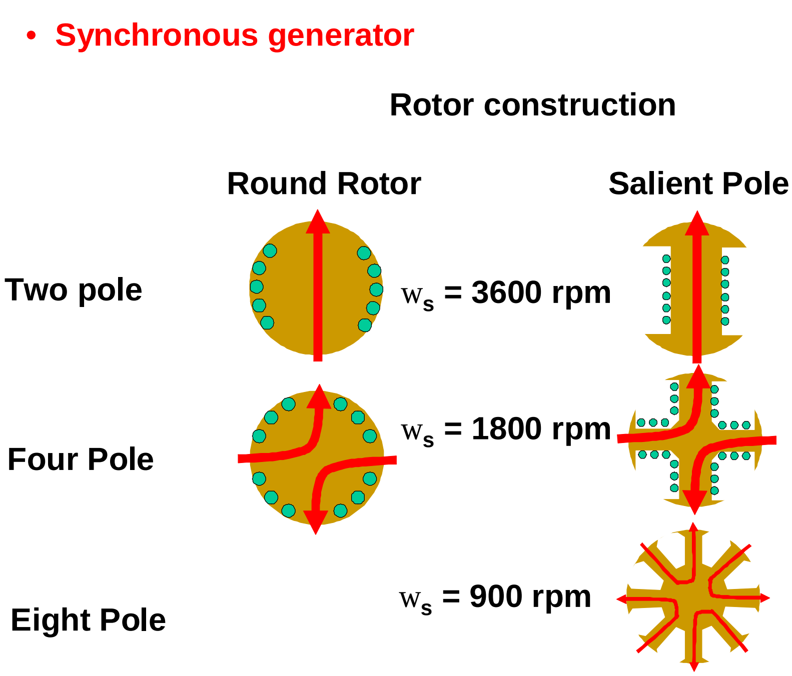

Fig. 8

3.0 Conceptual description

The following outlines the conceptual steps associated with production of power in a synchronous generator.

1. DC is supplied to the field winding.
2. If the rotor is stationary, the field winding produces magnetic flux which is strongest radiating outwards from the center of the pole face and diminishes with distance along the air-gap away from the pole face center. Figure 9 illustrates. The left-hand-figure plots flux density plotted as a function of angle from the main axis. The right-hand plot shows the main axis and the lines of flux. The angle θ measures the point on the stator from the main axis. In this particular case, we have aligned the main axis with the direct-axis of the rotor.

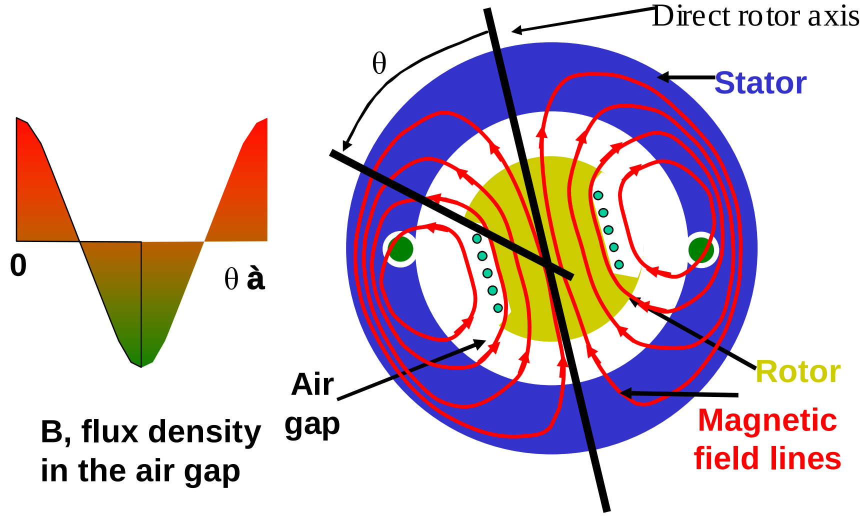

Fig. 9

3.The turbine rotates the rotor. This produces a **<u>rotating magnetic field</u>** (text calls it sinusoidal traveling wave) in the air gap, i.e., the plot on the left of Fig. 9 "moves" with time. Figure 10 illustrates, where we see that, *for fixed time (just one of the plots), there is sinusoidal variation of flux density with space.* Also, if we stand on a single point on the stator (e.g., θ=90°) and measure B as a function of time, we see that *for fixed space (the vertical dotted line at 90°, and the red eye on the pictures to the right), there is sinusoidal variation of flux density w/time.*

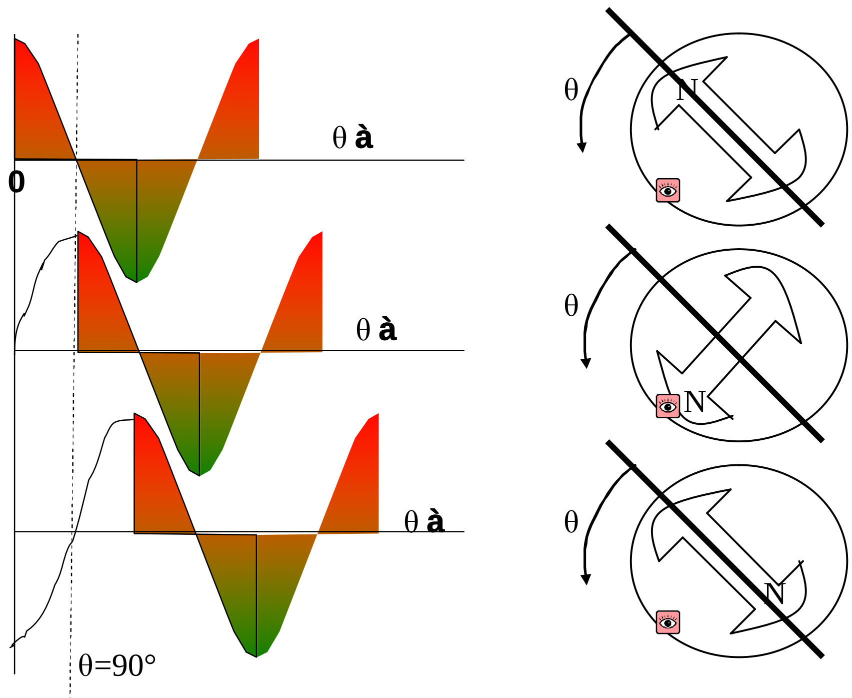

Fig. 10

4. Given that the stator windings, which run down the stator sides parallel to the length of the generator (see Fig. 5) are fixed on the stator (like the eye of Fig. 10), those conductors will see a time varying flux. Thus, by Faraday's law, a voltage will be induced in those conductors.
   a. Because the phase windings are spatially displaced by 120°, then we will get voltages that are time-displaced by 120°.

b. If the generator terminals are open-circuited, then the amplitude of the voltages are proportional to
   - Speed
   - Magnetic field strength

   And our story ends here if generator terminals are open-circuited.

5. If, however, the phase (armature) windings are connected across a load, then current will flow in each one of them. Each one of these currents will in turn produce a magnetic field. So there will be 4 magnetic fields in the air gap. One from the rotating DC field winding, and one each from the three stationary AC phase windings.
6. The three magnetic fields from the armature windings will each produce flux densities, and the composition of these three flux densities result in a single **rotating magnetic field** in the air gap.

7. This rotating magnetic field from the armature will have the same speed as the rotating magnetic field from the rotor, i.e., these two rotating magnetic fields are *in synchronism*.
8. The two rotating magnetic fields, that from the rotor and the composite field from the armature, are "locked in," and as long as they rotate in synchronism, a torque (Torque=P/ω_m=Force×radius, where Force is tangential to the rotor surface), is developed. This torque is identical to that which would be developed if two magnetic bars were fixed on the same pivot [2, pg. 171] as shown in Fig 11. In the case of synchronous generator operation, we can think of bar A (the rotor field) as pushing bar B (the armature field), as in Fig. 11a. In the case of synchronous motor operation, we can think of bar B (the armature field) as pulling bar A (the rotor field), as in Fig. 11b.

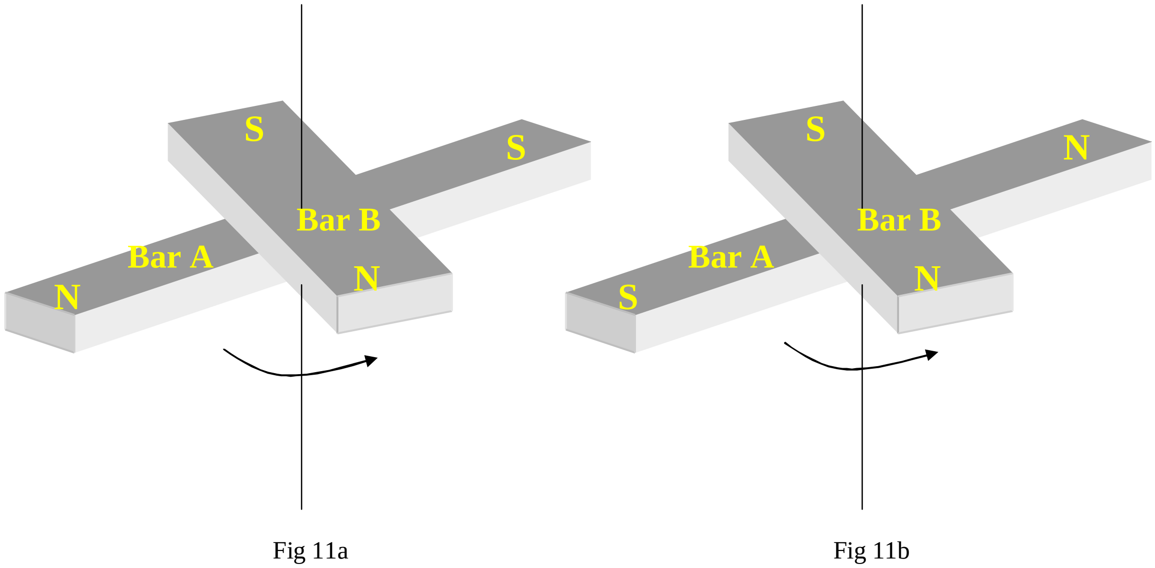

Fig. 11

4.0 Analytical model: open circuit voltage

We will now associate an analytical model with the previous conceptual description. We begin with Fig. 11 (Fig. 6.1 from text).

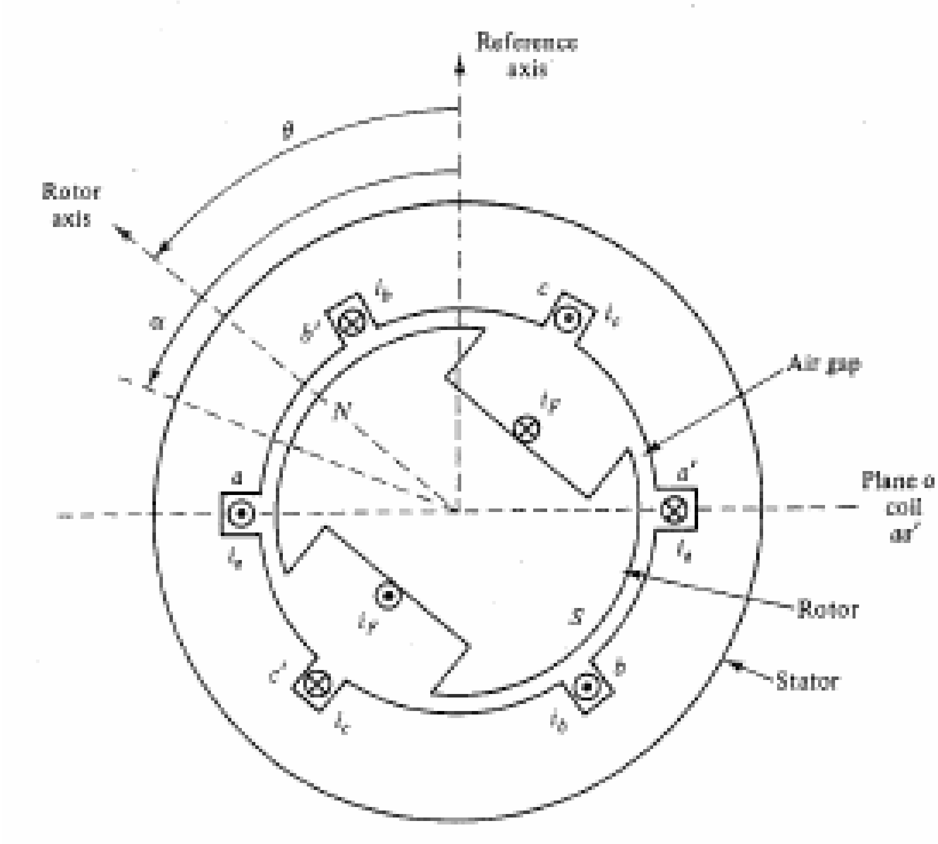

Figure 6.1 Generator cross section.

Fig. 11

In this figure, note the following definitions:

- θ: the absolute angle between a reference axis (i.e., fixed point on stator) and the center line of the rotor north pole (direct rotor axis); it is the same as θ of Fig. 9 if, in Fig. 9, we assume that the direct rotor axis is aligned with the reference axis.
- α: the angle made between the reference axis and some point of interest along the air gap circumference.

Thus we see that, for any pair of angles θ and α, α-θ gives the angular difference between the centerline of the rotor north pole and the point of interest.

We are using two angular measurements in this way in order to address

- variation with time as the rotor moves; we will do this using θ (which gives the rotational position of the centerline of the rotor north pole)
- variation with space for a given θ; we will do this using α (which gives the rotational position of any point on the stator *with respect to θ*)

We want to describe the flux density, B, in the air gap, due to field current i_F only.

Assume that maximum air gap flux density, which occurs at the pole center line (α=θ), is B_{max}. Assume also that flux density B varies sinusoidally around the air gap (as illustrated in Figs. 9 and 10). Then, for a given θ,

$$B(a) = B_{\max} \cos(a - q) \qquad (4)$$

Keep in mind that the flux density expressed by eq. (4) represents only the magnetic field from the winding on the rotor.

But, you might say, this is a fictitious situation because the currents in the armature windings will also produce a magnetic field in the air gap, and so we cannot really talk about the magnetic field from the rotor winding alone.

We may deal with this issue in an effective and forceful way: assume, for the moment, that the phase A, B, and C armature windings are open, i.e., not connected to the grid or to anything else. Then, currents through them must be zero, and if currents through them are zero, they cannot produce a magnetic field.

So we assume that $i_a=i_b=i_c=0$.

So what does this leave us to investigate? Even though currents in the phases are zero, voltages are induced in them. So it is these voltages that we want to describe. These voltages are called *the open circuit voltages*.

Consider obtaining the voltage induced in just one wire-turn of the a-phase armature winding. Such a turn is illustrated in Fig. 12 (Fig. 6.2 of the text). We have also drawn a half-cylinder having radius equal to the distance of the air-gap from the rotor center.

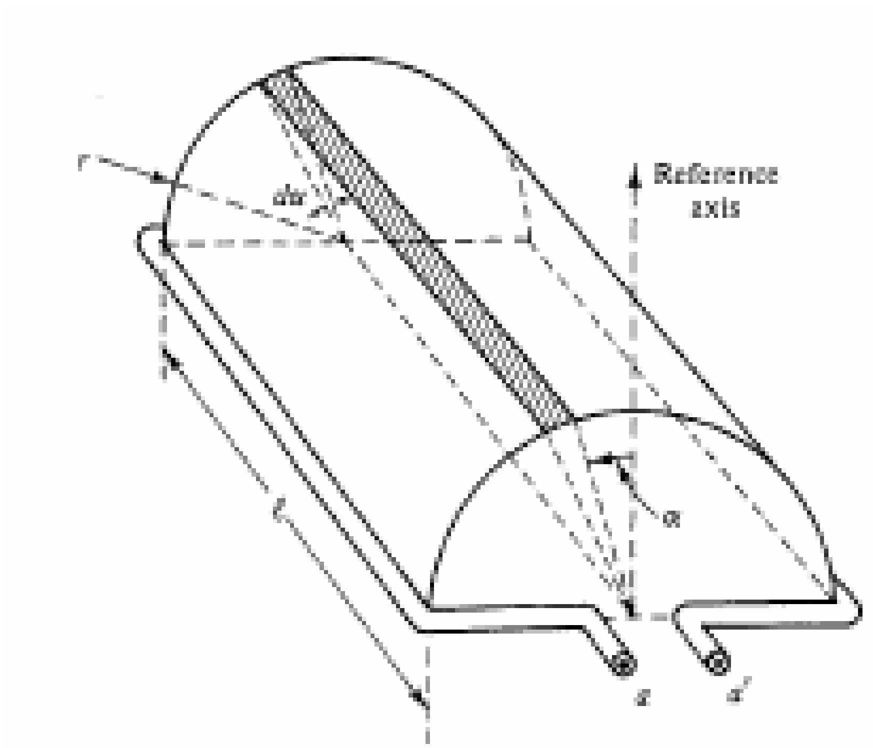

Fig. 12

Note in Fig. 12 that the current direction in the coil is assumed to be from the X-terminal (on the right) to the dot-terminal (on the left).

With this current direction, a positive flux direction is established using the right-hand-rule to be upwards. We denote a-phase flux linkages associated with such a directed flux to be $\lambda_{aa'}$. Our goal, which is to find the voltage induced in this coil of wire, $e_{aa'}$, can be achieved using Faraday's Law, which is:

$$e_{aa'} = -\frac{d\lambda_{aa'}}{dt} \tag{5}$$

So our job at this point is to express the flux linking the a-phase λ_{aa}, which comes entirely from the magnetic field produced by the rotor, as a function of time.

An aside: The minus sign of eq. (5) expresses Lenz's Law [3, pp. 27-28], which states that the direction of the voltage in the coil is such that, *assuming the coil is the source* (as it is when operating as a generator), and the ends are shorted, it will produce current that will cause a flux opposing the original flux change that produced that voltage. Therefore

- if flux linkage $\lambda_{aa'}$ is increasing (originally positive, meaning upwards through the coil a-a', and then becoming larger),
- then the current produced by the induced voltage needs to be set up to provide flux linkage in the downward direction of the coil,
- this means the current needs to flow from the terminal a to the terminal a'
- to make this happen across a shorted terminal, the coil would need to be positive at the a' terminal and negative at the a terminal, as shown in Fig. 13.

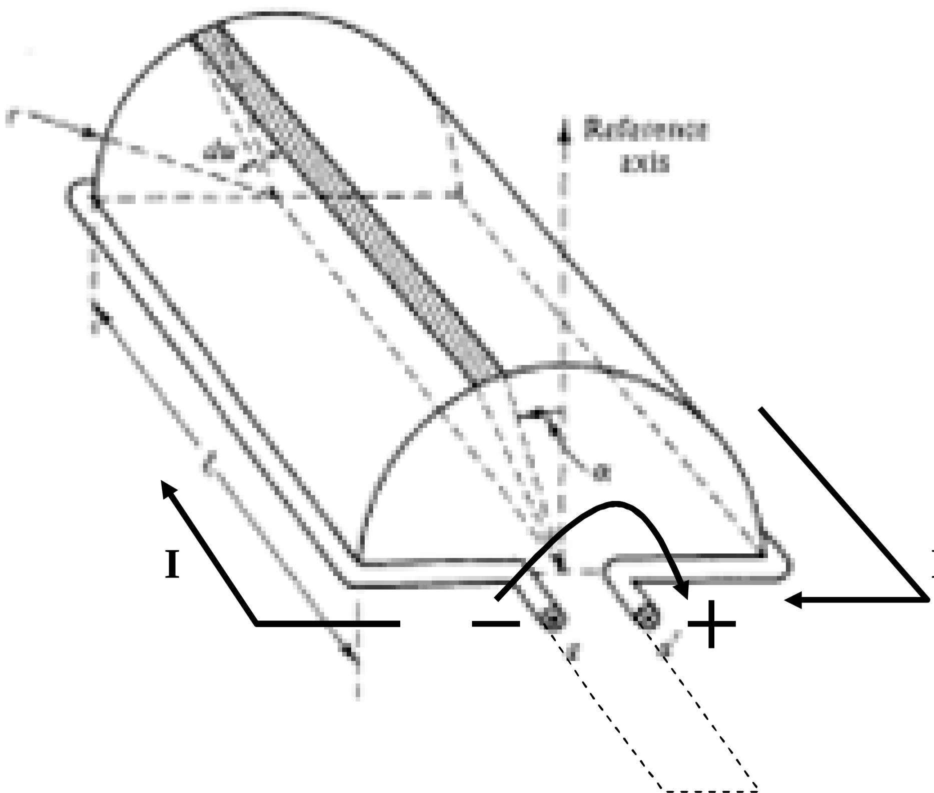

Fig. 13

To compute the flux linking with the coil of wire a-a’, we begin by considering the flux passing through the small slice of the cylinder, $d\alpha$. The amount of flux through this slice, denoted by $d\varphi_{aa'}$, will be the flux density at the slice, as given by eq. (4), multiplied by the area of that slice, which is (length) × (width) = (l) × (r $d\alpha$), that is:

$$
\begin{aligned}
d\phi_{aa'} &= B_{\max}\cos(\alpha-\theta)lrd\alpha \\
&= lrB_{\max}\cos(\alpha-\theta)d\alpha
\end{aligned} \tag{6}
$$

We can now integrate eq. (6) about the half-cylinder to obtain the flux passing through it (integrating about a full cylinder will give 0, since we would then pick up flux entering and exiting the cylinder).

$$
\begin{aligned}
\phi_{aa'} &= \int_{-\pi/2}^{\pi/2} lrB_{\max}\cos(\alpha-\theta)d\alpha \\
&= lrB_{\max}\sin(\alpha-\theta)\Big|_{-\pi/2}^{\pi/2} \\
&= lrB_{\max}\left(\underbrace{\sin\left(\frac{\pi}{2}-\theta\right)}_{\cos\theta} - \underbrace{\sin\left(-\frac{\pi}{2}-\theta\right)}_{-\cos\theta}\right) \\
&= lrB_{\max}\left(\cos(\theta) - -\cos(\theta)\right) \\
&= 2lrB_{\max}\cos\theta
\end{aligned} \tag{7}
$$

Define $\varphi_{max}=2lrB_{max}$, and we get

$$
\phi_{aa'} = \varphi_{\max}\cos\theta \tag{8}
$$

which is the same as eq. (6.2) in the text.

Equation (8) indicates that the flux passing through the coil of wire a-a’ depends only on θ. That is,

- given the coil of wire is fixed on the stator, and
- given that we know the flux density occurring in the air gap as a result of the rotor winding,
- we can determine how much of the flux is actually linking with the coil of wire by simply knowing the rotational position of the centerline of the rotor north pole (θ).

But eq. (8) gives us flux, and we need flux linkage. We can get that by just multiplying flux $\varphi_{aa'}$ by the number of coils of wire N. In the particular case at hand, N=1, but in general, N will be something much higher. Then we obtain:

$$\lambda_{aa'} = N\phi_{aa'} = N\varphi_{\max}\cos\theta \qquad (9)$$

Now we need to understand clearly what θ is. It is the centerline of the rotor north pole, BUT, the rotor north pole is rotating!

Let's assume that when the rotor started rotating, it was at $\theta=\theta_0$, and it is moving at a rotational speed of ω_0, then

$$\theta = \omega_0 t + \theta_0 \qquad (10)$$

Substitution of eq. (10) into eq. (9) yields:

$$\lambda_{aa'} = N\phi_{\max}\cos(\omega_0 t + \theta_0) \qquad (11)$$

Now, from eq. (5), we have

$$e_{aa'} = -\frac{d\lambda_{aa'}}{dt} = \frac{-d}{dt}\left(N\phi_{\max}\cos(\omega_0 t + \theta_0)\right) \qquad (12)$$

We get a –sin from differentiating the cos, and thus we get two negatives, resulting in:

$$e_{aa'} = N\phi_{\max}\omega_0 \sin(\omega_0 t + \theta_0) \qquad (13)$$

Define

$$E_{\max} = N\phi_{\max}\omega_0 \qquad (14)$$

Then

$$e_{aa'} = E_{\max}\sin(\omega_0 t + \theta_0) \qquad (15)$$

We can also define the RMS value of $e_{aa'}$ as

$$\left|E_{aa'}\right| = \frac{E_{\max}}{\sqrt{2}} \tag{16}$$

which is the magnitude of the generator *internal voltage*.

We have seen internal voltage before, in EE 303, where we denoted it as $|E_f|$. In EE 303, we found it in the circuit model we used to analyze synchronous machines, which appeared as in Fig. 14.

Note that internal voltage is the same as terminal voltage on the condition that I_a=0, i.e., when the terminals are open-circuited. This is the reason why internal voltage is also referred to as open-circuit voltage.

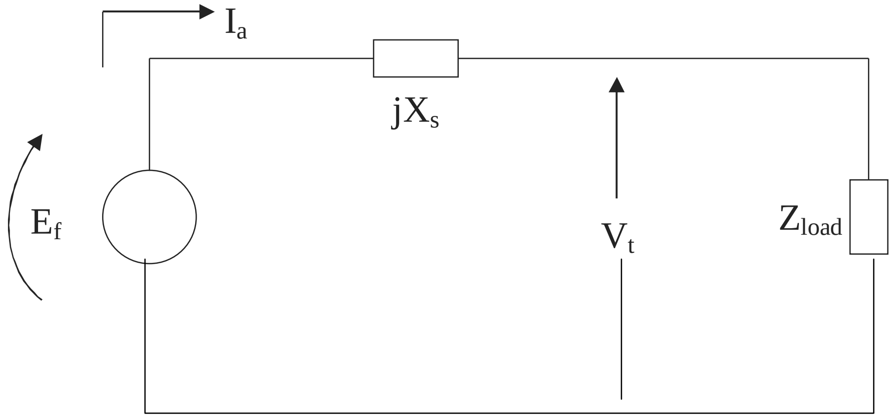

Fig. 14

We learned in EE 303 that internal voltage magnitude is proportional to the field current i_f. This makes sense here, since by eqs. (14) and (16), we see that

$$\left|E_{aa'}\right| = \frac{E_{\max}}{\sqrt{2}} = \frac{N\varphi_{\max}\omega_0}{\sqrt{2}} \qquad (17)$$

and with N and ω_0 being machine design parameters (and not parameters that can be adjusted once the machine is built), the only parameter affecting internal voltage is φ_{max}, which is entirely controlled by the current in the field winding, i_f.

One last point here: it is useful at times to have an understanding of the phase relationship between the internal voltage and the flux linkages that produced it. Recall eqs. (11) and (13):

$$\lambda_{aa'} = N\varphi_{\max}\cos(\omega_0 t + \theta_0) \qquad (11)$$

$$e_{aa'} = N\varphi_{\max}\omega_0 \sin(\omega_0 t + \theta_0) \qquad (13)$$

Using sin(x)=cos(x-π/2), we write (13) as:

$$e_{aa'} = N\varphi_{\max}\omega_0 \cos(\omega_0 t + \theta_0 - \pi/2) \qquad (18)$$

Comparing eqs. (11) and (18), we see that the internal voltage lags the flux linkages that produced it by π/2=90° (1/4 turn).

This is illustrated by Fig. 15 (same as Fig. E6.1 of Example 6.1). In Fig. 15, the flux linkage phasor is in phase with the direct axis of the rotor.

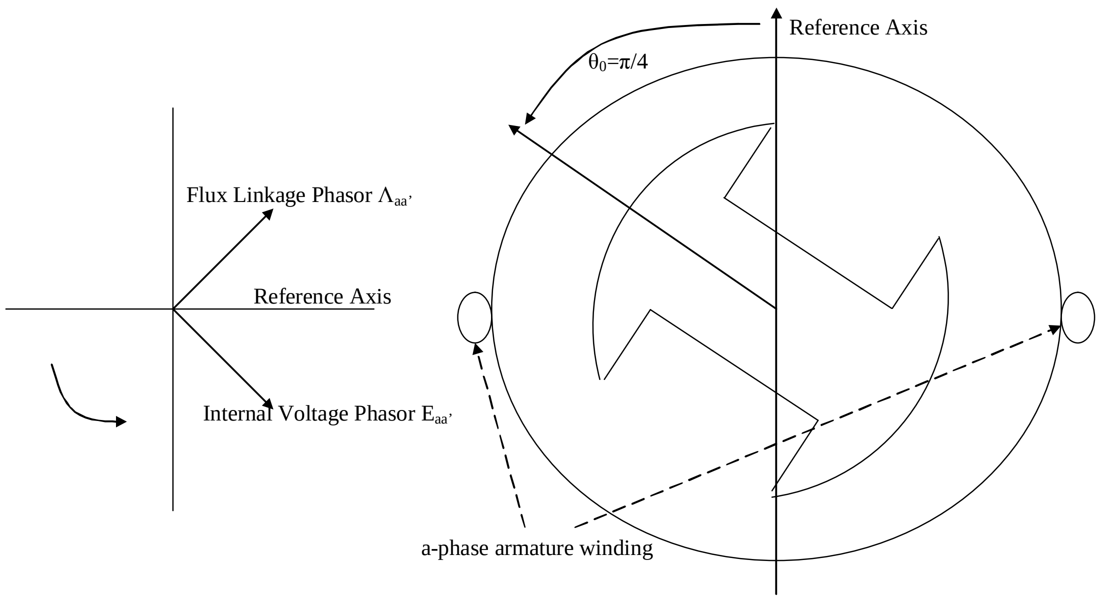

Fig. 15

Therefore, the flux linkages phasor is represented by

$$\Lambda_{aa'} = \frac{N\phi_{\max}}{\sqrt{2}} e^{j\theta_0} = \left|\Lambda_{aa'}\right| e^{j\theta_0} \qquad (19)$$

and then the internal voltage phasor will be

$$E_{aa'} = \frac{N f_{\max} w_0}{\sqrt{2}} e^{j(q_0 - p/2)} = \left|E_{aa'}\right| e^{j(q_0 - p/2)} \qquad (20)$$

Let's drop the a' subscript notation from $E_{aa'}$, just leaving E_a, so that:

$$E_a = \frac{N f_{\max} w_0}{\sqrt{2}} e^{j(q_0 - p/2)} = \left|E_a\right| e^{j(q_0 - p/2)} \qquad (21)$$

Likewise, we will get similar expressions for the b- and c-phase internal voltages, according to:

$$E_b = \left|E_a\right| e^{j(q_0 - p/2 - 2p/3)} \qquad (22)$$

$$E_c = \left|E_a\right| e^{j(q_0 - p/2 + 2p/3)} \qquad (23)$$

5.0 Armature reaction: one phase winding

Armature reaction refers to the influence on the magnetic field in the air gap when the phase windings a, b, and c on the stator are connected across a load.

Let’s consider a smooth rotor, as given in Fig. 16 (Fig. 6.3 in text). For a smooth rotor, the flux sees constant (& high) permeability path throughout the rotor and stator, with the only exception being the air gap.

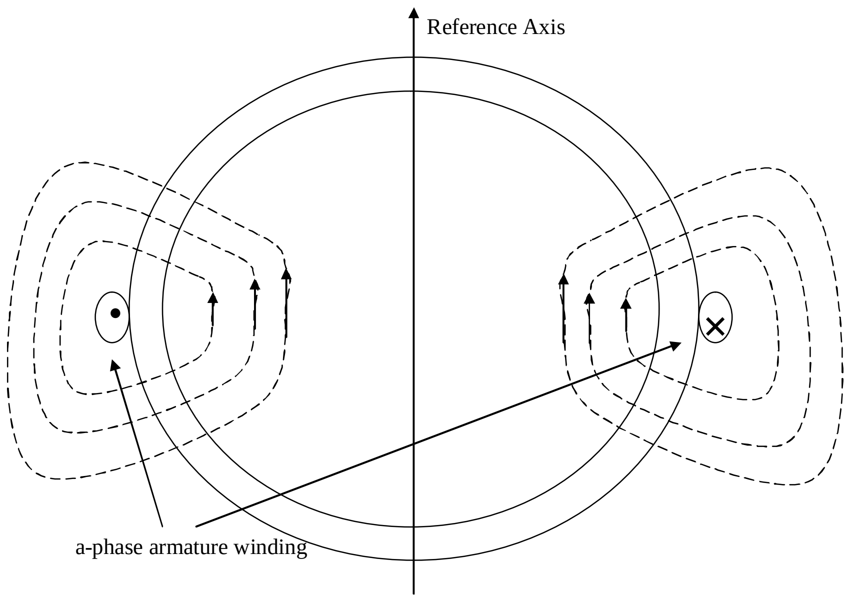

Fig. 16

Two important observations from Fig. 16:

1. Because the armature windings are stationary, and since the rotor is round (and therefore flux path is constant), the way the picture characterizes lines of flux is representative for all time. The only temporal variation will be the magnetic field *strength* as it pulsates with current.

2. The lines of flux made by each side of the a-phase winding will combine in the rotor in such a way so that its maximum strength is along the reference axis, and it varies sinusoidally along the air-gap.

Let's assume that the current in the a-phase winding is given by:

$$\boldsymbol{i_a(t)} = \sqrt{\mathbf{2}}\left|\boldsymbol{I_a}\right|\mathbf{cos}\left(\omega_{\mathbf{0}}\boldsymbol{t} + \angle \boldsymbol{I_a}\right) \tag{24}$$

where $|I_a|$ is the RMS current magnitude and $\angle \boldsymbol{I_a}$ is the angle made by the current phasor enabling proper phase relation with its corresponding voltage phasor, eq. (18),

$$\boldsymbol{e_a(t)} = \boldsymbol{N}\varphi_{\mathbf{max}}\omega_{\mathbf{0}}\,\mathbf{cos}\left(\omega_{\mathbf{0}}\boldsymbol{t} + \theta_{\mathbf{0}} - \pi / \mathbf{2}\right) \tag{18}$$

where $\angle \boldsymbol{E_a} = \theta_{\mathbf{0}} - \pi / \mathbf{2}$.

Then the air-gap flux density at the reference axis, where the flux density is maximum, is given by

$$\boldsymbol{B_{a,\mathbf{max}}(t)} = \boldsymbol{K i_a(t)} \tag{25}$$

We emphasize eq. (22) gives flux density at the reference axis only, a fixed point in the air gap corresponding to α=0°. Fig. 17 uses the orange to illustrate what eq. (25) is capturing. The sequence of the 4 pictures, numbered 1,2,3,4, correspond to the sequence-in-time that characterizes flux density seen at the α=0° fixed stator point.

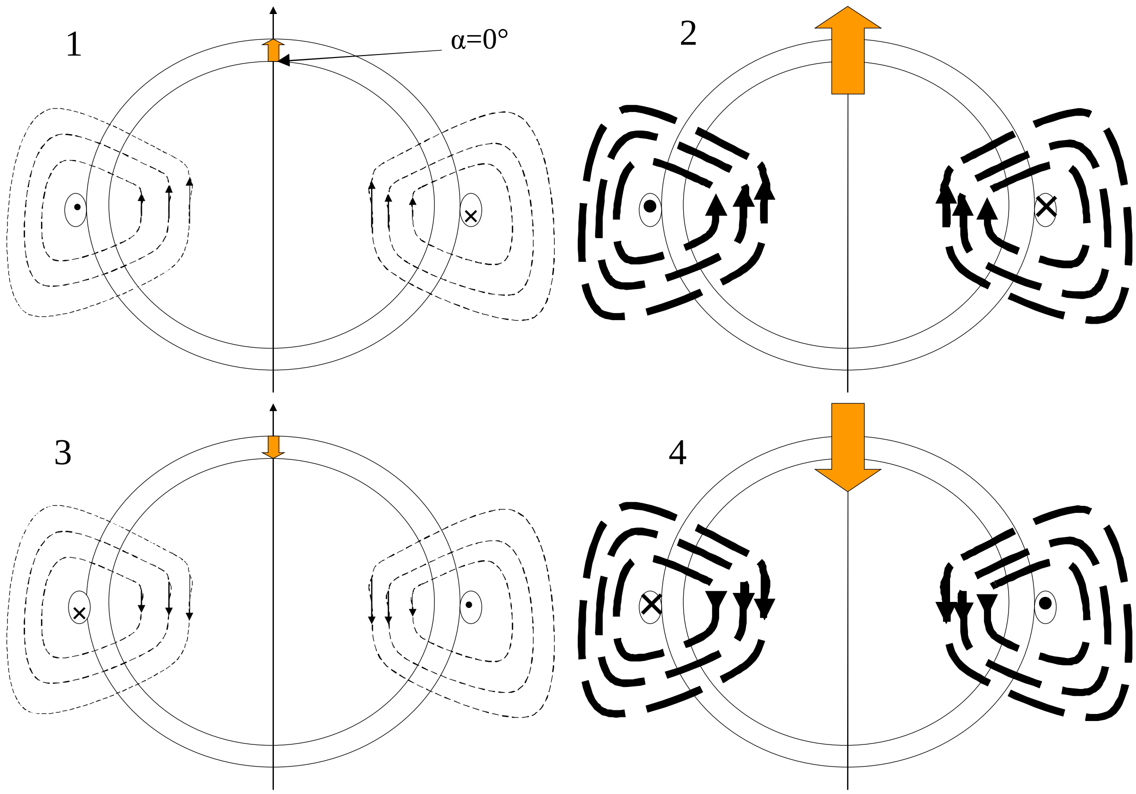

Fig. 17

Because flux minimizes travel in high-reluctance paths, all flux lines are directed radially outwards in the airgap.

Substitution of eq. (24) in eq. (25) results in:

$$\boldsymbol{B}_{a,\max}(\boldsymbol{t}) = \boldsymbol{K}\sqrt{2}\left|\boldsymbol{I}_a\right|\cos\left(w_0 \boldsymbol{t} + \text{Ð}\boldsymbol{I}_a\right) \quad (26)$$

Studying the pictures of Fig. 17, we observe that if the flux density is maximum at α=0°, then it will be minimum (i.e., a maximum but negative, or radially directed inwards) at α=180°, and it therefore must be zero halfway between these two points, at α=90° and α=270°. Moving from the maximum to the zero-point in either direction, the flux density will decrease, and a similar thing will occur in moving from the minimum to the zero-point in either direction. Figure 18 illustrates.

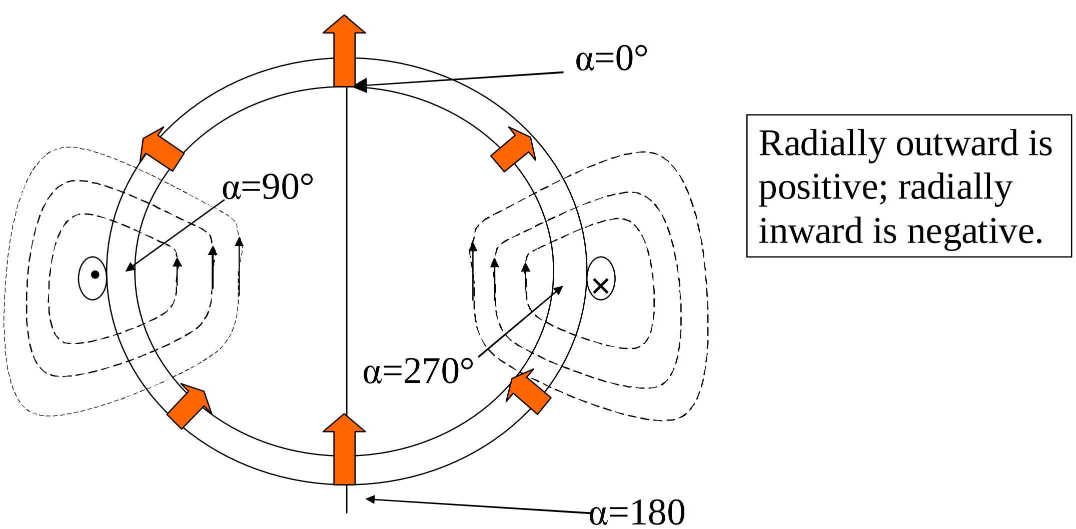

Fig. 18

And so, for a particular time t, we can express the spatial variation of the flux density from the a-phase winding as a sinusoidal function of α, having a value of $B_{a,max}(t)$ at α=0°. Therefore:

$$\boldsymbol{B_a(\alpha, t) = B_{a,\max}(t)\cos\alpha} \qquad (27)$$

Substituting eq. (26) into (27), we obtain:

$$\boldsymbol{B_a(\alpha, t) = K\sqrt{2}\left|I_a\right|\cos\left(\omega_0 t + \angle I_a\right)\cos\alpha} \qquad (28)$$

Now define

$$\boldsymbol{B'_{\max} = K\sqrt{2}\left|I_a\right|} \qquad (29)$$

(This B'_{max} is a little different from that implied in the text, page 195, because we have assumed a perfect sinusoidal distribution of flux density in the air gap, whereas the book assumed a square wave distribution and then took the fundamental of that square wave).

Substituting (29) into (28) results in:

$$\boldsymbol{B_a(\alpha, t) = B'_{\max}\cos\left(\omega_0 t + \angle I_a\right)\cos\alpha} \qquad (30)$$

Question: Does eq. (30) characterize a rotating magnetic field?

To answer this question, we need to recall precisely what we mean by a rotating magnetic field. Let's re-examine the rotating magnetic field developed by the rotating rotor winding. We expressed this in eq. (4):

$$\boldsymbol{B}(\alpha) = \boldsymbol{B}_{\mathbf{max}} \cos(\alpha - \theta) \tag{4}$$

Inserting explicitly the dependence of θ on t:

$$\boldsymbol{B}(\alpha, t) = \boldsymbol{B}_{\mathbf{max}} \cos(\alpha - \theta(t)) \tag{31}$$

where $\theta(t)=\omega_0 t+\theta_0$. So there are three attributes:

1. Constant amplitude
2. One sinusoid
3. Argument of sinusoid a function of space and time

Equation (30) has two sinusoids with one being a function of time (first one) and the other being a function of space (second one). And employing trig identities can not result in satisfying the above criteria.

Physically, a rotating magnetic field

- maintains a constant amplitude waveform that
- continuously moves around the air gap.

Equation (30), on the other hand, characterizes a field that

- has a time-varying amplitude
- which is stationary in the air gap.

If you stand at a particular point on the air gap and observe only the field at that point, you will not be able to tell the difference between the two.

However, if you cut the stator and spread out the air gap linearly, and observe the entire waveform of both fields, you will find:

- the rotating magnetic field moves.
- the other one pulsates.

A good way to think about this is to consider that in the case of the rotating magnetic field, the flux density is **never 0 everywhere in the air gap**.WS

In contrast, every time the current in the a-phase winding goes to zero, the flux density of the eq. (30) pulsating field **goes to zero everywhere in the air gap**.

Cool.

6.0 Armature reaction: all phase windings

We can go through a similar thought process to obtain the flux density produced by the b- and c-phase armature windings. The result will be identical to eq. (30), except that

- the time-dependent term will be phase-shifted consistent with the phase-shifts of the current, and
- the space-dependent term will be phase-shifted consistent with the phase-shifts of the physical windings.

Therefore, the flux densities from all phases will be:

$$B_a(\alpha,t) = B'_{\max}\cos\left(\omega_0 t + \angle I_a\right)\cos\alpha$$

$$B_b(\alpha,t) = B'_{\max}\cos\left(\omega_0 t + \angle I_a - \frac{2\pi}{3}\right)\cos\left(\alpha - \frac{2\pi}{3}\right)$$

$$B_c(\alpha,t) = B'_{\max}\cos\left(\omega_0 t + \angle I_a + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{2\pi}{3}\right)$$

What will the composition of these three flux densities look like? Let's add them up!

$$\begin{aligned}B_{abc}(\alpha,t) &= B'_{\max}\cos\left(\omega_0 t + \angle I_a\right)\cos\alpha \\ &+ B'_{\max}\cos\left(\omega_0 t + \angle I_a - \frac{2\pi}{3}\right)\cos\left(\alpha - \frac{2\pi}{3}\right) \\ &+ B'_{\max}\cos\left(\omega_0 t + \angle I_a + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{2\pi}{3}\right)\end{aligned}$$

Factor out the $B'_{\max}$:

$$\begin{aligned}B_{abc}(\alpha,t) = B'_{\max}\Bigg\{&\cos\left(\omega_0 t + \angle I_a\right)\cos\alpha \\ &+ \cos\left(\omega_0 t + \angle I_a - \frac{2\pi}{3}\right)\cos\left(\alpha - \frac{2\pi}{3}\right) \\ &+ \cos\left(\omega_0 t + \angle I_a + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{2\pi}{3}\right)\Bigg\}\end{aligned}$$

Now deploy the trig identity

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

to all three terms.

$$\begin{aligned}
B_{abc}(\alpha,t) = \frac{B'_{\max}}{2}\Big\{ & \cos(\omega_0 t + \angle I_a - \alpha) + \cos(\omega_0 t + \angle I_a + \alpha) \\
& + \cos\left(\omega_0 t + \angle I_a - \frac{2\pi}{3} - \alpha + \frac{2\pi}{3}\right) + \cos\left(\omega_0 t + \angle I_a - \frac{2\pi}{3} + \alpha - \frac{2\pi}{3}\right) \\
& + \cos\left(\omega_0 t + \angle I_a + \frac{2\pi}{3} - \alpha - \frac{2\pi}{3}\right) + \cos\left(\omega_0 t + \angle I_a + \frac{2\pi}{3} + \alpha + \frac{2\pi}{3}\right)\Big\}
\end{aligned}$$

Combine all of the terms with π.

$$\begin{aligned}
B_{abc}(\alpha,t) = \frac{B'_{\max}}{2}\Big\{ & \cos(\omega_0 t + \angle I_a - \alpha) + \cos(\omega_0 t + \angle I_a + \alpha) \\
& + \cos(\omega_0 t + \angle I_a - \alpha) + \cos\left(\omega_0 t + \angle I_a + \alpha - \frac{4\pi}{3}\right) \\
& + \cos(\omega_0 t + \angle I_a - \alpha) + \cos\left(\omega_0 t + \angle I_a + \alpha + \frac{4\pi}{3}\right)\Big\}
\end{aligned}$$

Now we notice a very interesting thing. The three cos terms on the right side of each row constitute a balanced set (equal-magnitude terms 120° out of phase). They therefore add to zero! So we have

$$\begin{aligned}
B_{abc}(\alpha,t) = \frac{B'_{\max}}{2}\Big\{ & \cos(\omega_0 t + \angle I_a - \alpha) \\
& + \cos(\omega_0 t + \angle I_a - \alpha) \\
& + \cos(\omega_0 t + \angle I_a - \alpha)\Big\}
\end{aligned}$$

But now we can see that all three of the remaining cos terms are identical! So we have:

$$\boldsymbol{B}_{abc}(\alpha,\boldsymbol{t})=\frac{\mathbf{3}\boldsymbol{B}'_{\max}}{\mathbf{2}}\cos\left(\omega_0\boldsymbol{t}+\angle\boldsymbol{I}_a-\alpha\right) \qquad (32)$$

And this is an expression for a rotating magnetic field because it has

1. Constant amplitude
2. One sinusoid
3. Argument of sinusoid a function of space and time

Note that the text, in eq. (6.15) gives:

$$\boldsymbol{B}_{abc}(\alpha,\boldsymbol{t})=\frac{\mathbf{3}\boldsymbol{B}'_{\max}}{\mathbf{2}}\cos\left(\alpha-\omega_0\boldsymbol{t}-\angle\boldsymbol{I}_a\right) \qquad (33)$$

But eq. (32) and (33) are the same, since cos(x)=cos(-x).

<u>Comment on angular relations</u>: Consider eq. (33) when *t=0*:

$$\boldsymbol{B}_{abc}(\alpha,\boldsymbol{t}=\mathbf{0})=\frac{\mathbf{3}\boldsymbol{B}'_{\max}}{\mathbf{2}}\cos\left(\alpha-\angle\boldsymbol{I}_a\right) \qquad (34)$$

The maximum of this rotating magnetic field occurs when the argument of the cosine function is 0, and this occurs when $\alpha = \angle I_a$. Another way of saying this is, at t=0, the centerline (or north pole) of this rotating magnetic field (from armature reaction) is at an angle $\angle I_a$ with respect to the reference axis.

Now consider the following equations that we have previously used:

$$\theta = \omega_0 t + \theta_0 \tag{10}$$

$$\lambda_{aa'} = N\varphi_{\max} \cos(\omega_0 t + \theta_0) \tag{11}$$

$$e_{aa'} = N\varphi_{\max}\omega_0 \cos(\omega_0 t + \theta_0 - \pi/2) \tag{18}$$

These equations say the following for angular relations at t=0.

- From eq. (10), $\theta = \theta_0$ implies that the north pole of the rotor is at θ_0.
- From eq. (11), the a-phase flux linkage phasor is also at θ_0, aligned with the north pole of the rotor.

- From eq. (18), the a-phase induced voltage is at θ_0-$\pi/2$, 90° behind the rotor and the flux linkage phasor.
- And from eq. (34), as mentioned, the north pole of the armature reaction rotating magnetic field is at and angle Đ$\boldsymbol{I_a}$.

Figure 19 illustrates.

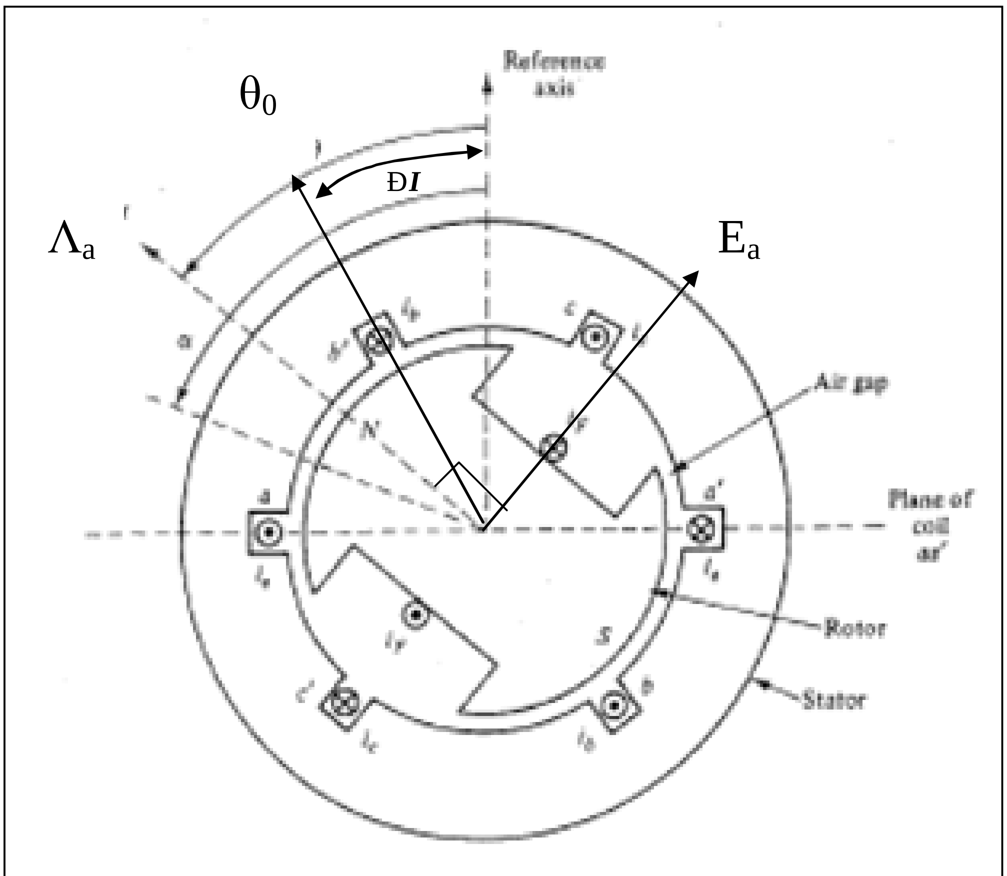

Figure 6.1 Generator cross section.

Fig. 19

7.0 Reactances and terminal voltage

You will recall that when we were developing the induced voltage in the armature windings (the open circuit voltage) from the rotating magnetic field produced by the rotor, we expressed the flux density from the rotor as

$$\boldsymbol{B}(\alpha) = \boldsymbol{B}_{\max} \cos(\alpha - \theta) \qquad (4)$$

and then we integrated to get total flux about the half-cylinder surrounding the rotor via:

$$\begin{aligned}
\lambda_{aa'} &= N \int_{-\pi/2}^{\pi/2} lrB_{\max} \cos(\alpha - \theta)\, d\alpha \\
&= NlrB_{\max} \sin(\alpha - \theta)\Big|_{-\pi/2}^{\pi/2} \\
&= NlrB_{\max} \left[\underbrace{\sin\left(\frac{\pi}{2} - \theta\right)}_{\cos\theta} - \underbrace{\sin\left(-\frac{\pi}{2} - \theta\right)}_{-\cos\theta} \right] \\
&= NlrB_{\max} \left(\cos(\theta) - -\cos(\theta)\right) \\
&= 2NlrB_{\max} \cos\theta = \phi_{\max} \cos\theta = N\phi_{\max} \cos(\omega_0 t + \theta_0)
\end{aligned} \qquad (7)$$

Finally, we used Faraday's law to get

$$e_{aa'} = -\frac{d\,l_{aa'}}{dt} = \frac{-d}{dt}\left(Nj_{\max}\cos\left(w_0 t + q_0\right)\right) \qquad (12)$$

$$e_{aa'} = E_{\max}\sin\left(w_0 t + q_0\right) \qquad (15)$$

The whole point of the above exercise was that the a-phase windings were experiencing flux linkages that varied with time, and this produced a voltage in which we were interested.

Now the a-phase windings are seeing additional fields from armature reaction, and as we have shown, the composite of the fields from all three phases is a rotating magnetic field. Therefore the a-phase is experiencing flux linkages that vary with time (in addition to those from the rotor field) that are caused by the rotating magnetic field of armature reaction.

So let's define λ_{ag} as the total air-gap flux linkages seen by coil aa'. Then, as eq. (6.18) states in the text:

$$\lambda_{ag} = \lambda_{aa'} + \lambda_{ar} \qquad (35)$$

where

- $\lambda_{aa'}$ is the flux linkages from the rotating magnetic field of the rotor
- λ_{ar} is the flux linkages from the rotating magnetic field of armature reaction

We want to obtain the voltage induced by the time variation in λ_{ag}. This will be:

$$\boldsymbol{v}_{ag} = \frac{-\boldsymbol{d}\lambda_{ag}}{\boldsymbol{dt}} = \frac{-\boldsymbol{d}\lambda_{aa'}}{\boldsymbol{dt}} + \frac{-\boldsymbol{d}\lambda_{ar}}{\boldsymbol{dt}} \qquad (36)$$

But we have already obtained the first term in eq. (36); it was $e_{aa'}$ as given by eq. (15). You will recall that we renamed this e_a, and that we showed it had a phasor representation, from eq. (21), of

$$\boldsymbol{E}_a = \left|\boldsymbol{E}_a\right| \boldsymbol{e}^{j(\theta_0 - \pi/2)} \qquad (21)$$

So our job is only to obtain the second term in eq. (36).

I will spare you the details of this effort. Suffice it to say that the steps are virtually identical to the steps we took to get eqs. (7), (12), (15), and (21), and are provided in the text on pp. 196-197.

But I do want to impress upon you four important ideas that are at the heart of this development. The first two and the last one are articulated by the text, the 3rd one is not.

- Although the flux linkages λ_{ar} are due to all three phase currents, the voltages induced by $d\lambda_{ar}/dt$ are given by

$$v_{ar} = \frac{-d\lambda_{ar}}{dt} = -L_{s1}\frac{di_a}{dt} \qquad (37)$$

  where the influence of the other two phases is absorbed into the inductance L_{s1}.
- The expression (37), in the time-domain, can also be expressed in the phasor domain as:

$$V_{ar} = -j\omega_0 L_{s1} I_a \qquad (38)$$

- Because of Lenz's law, the voltage induced by the armature reaction flux linkages will be in opposition to the voltage which produced the time varying currents responsible for these time varying flux linkages. This means that this induced voltage will subtract from E_a:

$$\boldsymbol{V}_{ag} = \boldsymbol{E}_a - \boldsymbol{V}_{ar} = \boldsymbol{E}_a - jw_0 L_{s1} \boldsymbol{I}_a \qquad (39)$$

Eq. (39) is the same as (6.23) in the text:

- In the ideal world, V_{ag} would be the terminal voltage, but in reality, the machine will also experience two other effects that need to be included:
    - o Flux leakage
    - o Conductor resistance

To account for these two effects, we subtract from eq. (39) a drop across an impedance $r+jX_l$, where r is the resistance of the a-phase armature winding, and X_l is a small reactance that accounts for the drop in induced voltage caused by some of the flux leaking. Therefore:

$$\boldsymbol{V_a = V_{ag} - (r + jX_l)I_a} \qquad (40)$$

Substitution of eq. (39) into eq. (40) results in

$$\boldsymbol{V_a = E_a - jw_0 L_{s1} I_a - (r + jX_l)I_a} \qquad (41)$$

Define $X_{s1}=j\omega_0 L_{s1}$, so that eq. (41) becomes:

$$\boldsymbol{V_a = E_a - jX_{s1} I_a - (r + jX_l)I_a} \qquad (42)$$

Now rearrange eq. (42) to obtain:

$$\boldsymbol{V_a = E_a - j(X_{s1} + X_l)I_a - rI_a} \qquad (43)$$

Now define the *synchronous reactance* $X_s=X_{s1}+X_l$, so we obtain:

$$\boldsymbol{V_a = E_a - jX_s I_a - rI_a} \qquad (44)$$

Eq. (44) is the same as eq. (6.24) in the text.

The text mentions (page 197) that the armature reaction term X_{s1} is the dominant term; the leakage reactance X_l is typically only about 10% of X_{s1}, and r is only 1%.

Figure 20 illustrates per-phase circuit of a synchronous generator, and shows the relation between eqs. (43) and (44):

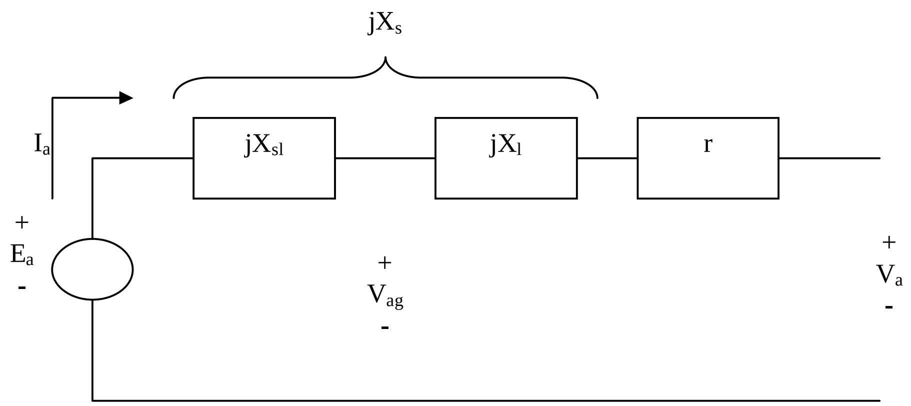

Fig. 20

One last thing: the angle by which the phasor E_a leads the phasor V_a is typically referred to as the *power angle* and represented by δ_m, i.e.,

$$\delta_m = \angle \mathbf{E}_a - \angle \mathbf{V}_a \tag{45}$$

If V_a is the reference (has 0 degree angle), then δ_m is the angle of the internal voltage.

Eq. (44) provides the instruction manual for drawing a phasor diagram of a synchronous machine. Usually, however, we begin assuming that we know V_a and I_a. We assume that V_a is the reference (i.e., has an angle of 0 degrees).

To obtain E_a from V_a and I_a, we re-write eq. (44) like this:

$$\boldsymbol{E_a = V_a + jX_s I_a + rI_a} \tag{46}$$

Now draw the E_a vector for the lagging condition (this means that I_a is lagging V_a):

Now draw the E_a vector for the leading condition (this means that I_a is leading V_a):

What can you say about the relative magnitude of the field current in the two cases above?

Lagging:

Leading:

What can you say about the strength of the magnetic field produced by the rotor winding, and the var supply, in the two cases above?

Lagging:

Leading:

8.0 Power for the smooth rotor case

The per-phase equivalent circuit of Fig. 20 is simplified to that of Fig. 21:

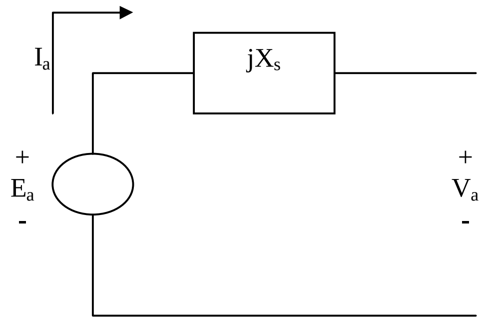

Fig. 21

The complex power will be

$$S = V_a I_a^* = V_a \left(\frac{E_a - V_a}{jX_s} \right)^* \tag{47}$$

Assuming that V_a is the reference, and with $E_a = |E_a|(\cos\delta_m + j\sin\delta_m)$, we obtain:

$$S = V_a \left(\frac{|E_a|\cos\delta_m + j|E_a|\sin\delta_m - V_a}{jX_s} \right)^* \tag{48}$$

Multiplying top and bottom by j we get:

$$S = V_a \left(\frac{j|E_a|\cos\delta_m - |E_a|\sin\delta_m - jV_a}{-X_s} \right)^*$$

$$= V_a \left(\frac{-j|E_a|\cos\delta_m + |E_a|\sin\delta_m + jV_a}{+X_s} \right)^*$$

Rearranging, we obtain:

$$S = V_a \left(\frac{|E_a|\sin\delta_m - j|E_a|\cos\delta_m + jV_a}{X_s} \right)^*$$

Taking the conjugate of what is inside the parenthesis:

$$S = V_a \left(\frac{|E_a|\sin\delta_m + j|E_a|\cos\delta_m - jV_a}{X_s} \right)$$

Recalling V_a has 0 degree angle, we distribute it through:

$$S = \frac{|V_a||E_a|\sin\delta_m + j|V_a||E_a|\cos\delta_m - j|V_a|^2}{X_s}$$

Recalling $S=P_g+jQ_g$, we can separate the above expression into its real and imaginary parts to obtain:

$$P_g = \frac{|V_a||E_a|\sin\delta_m}{X_s} \tag{49}$$

$$Q_g = \frac{|V_a||E_a|\cos\delta_m - |V_a|^2}{X_s} \tag{50}$$

Equation (50) provides us with an important way to understand the relation between leading and lagging generator operation.

The difference between leading and lagging will occur when Q_g=0.

$$Q_g = \frac{|V_a||E_a|\cos d_m - |V_a|^2}{X_s} = 0$$

$$|V_a||E_a|\cos d_m - |V_a|^2 = 0$$

$$|V_a||E_a|\cos d_m = |V_a|^2$$

$$|E_a|\cos d_m = |V_a| \qquad (51)$$

So, eq. (51) provides the condition corresponding to the boundary between leading and lagging.

Draw the corresponding phasor diagram for this boundary, and from that, identify the condition for lagging generator operation and the condition for leading generator operation.

9.0 Excitation control

Moving between lagging and leading condition is performed via control of the generator field current, which produces the field flux f_f. Field current control can be done manually, but it is also done automatically via the *excitation control system.*

The excitation control system is an automatic feedback control having the primary function of maintaining a predetermined terminal voltage by modifying the field current of the synchronous generator based on changes in the terminal voltage.

Without excitation control, terminal voltage would fluctuate as a result of changes in P_g or external network conditions. The control is referred to as “negative feedback” because when terminal voltage increases, field current is decreased, and when terminal voltage decreases, field current is increased. A simplified block diagram of an excitation control system is shown in Figure G1.9.

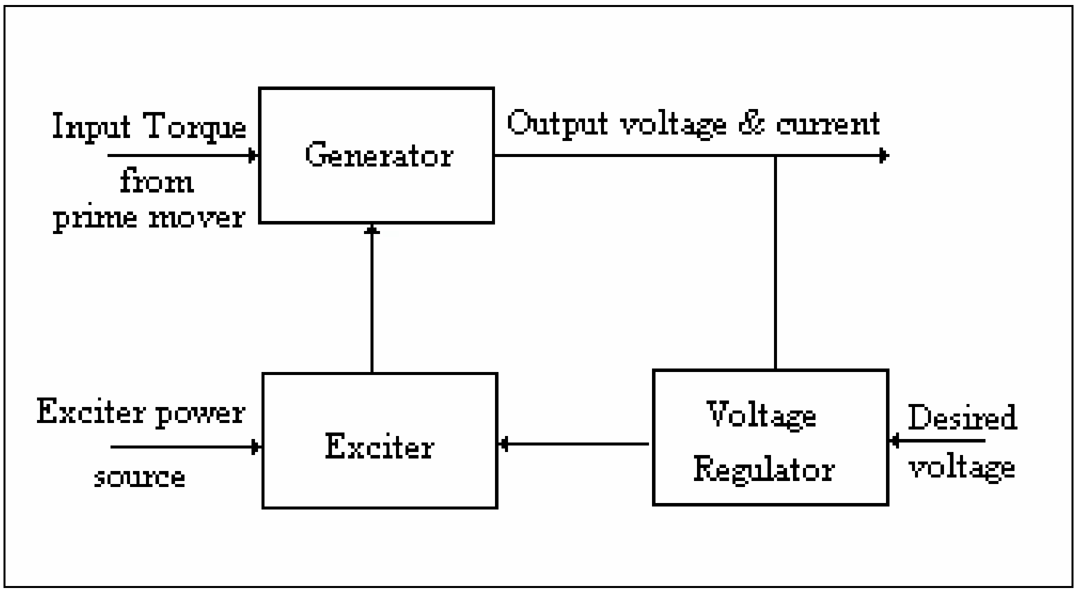

Fig. 22
Block Diagram of Excitation Control System

There are three fundamental components to any excitation system. The *main exciter*, or more simply, the *exciter*, is the device that provides the field current for the synchronous generator. The *automatic* *voltage regulator* (AVR) couples the terminal voltage to the input of the main exciter. The *amplifier* increases the power of the regulating signal to that required by the exciter. If the amplifier is electromechanical, it is called the *pilot exciter* or the *rotating amplifier*. If the amplifier is solid state, it is usually considered as part of the AVR.

There are three basic types of excitation systems. These are:

- rotating DC commutator
- rotating AC alternator
- static

These are illustrated in Fig. 23, 24, and 25.

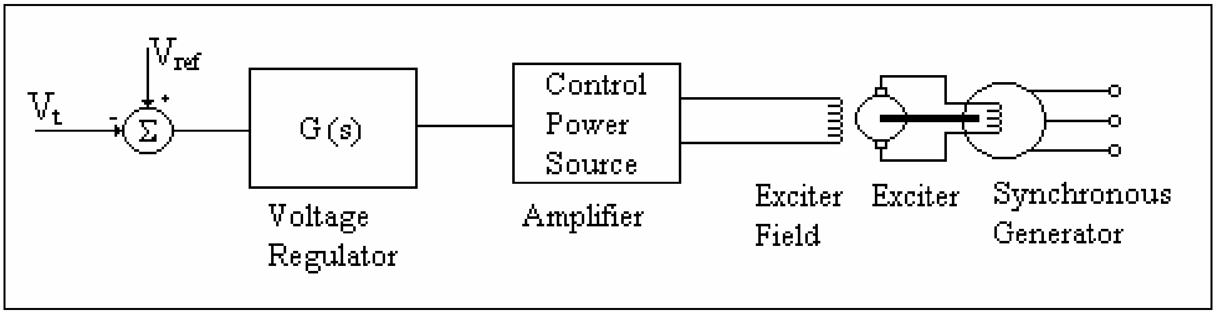

Fig. 23
Rotating DC Commutator Type Excitation System

The DC commutator excitation system utilizes a DC generator mounted on the shaft of the synchronous generator to supply the field current. This type of system is no longer used in new facilities because it is slow in response, and because it requires high maintenance slip rings and brushes to couple the exciter output to the field windings.

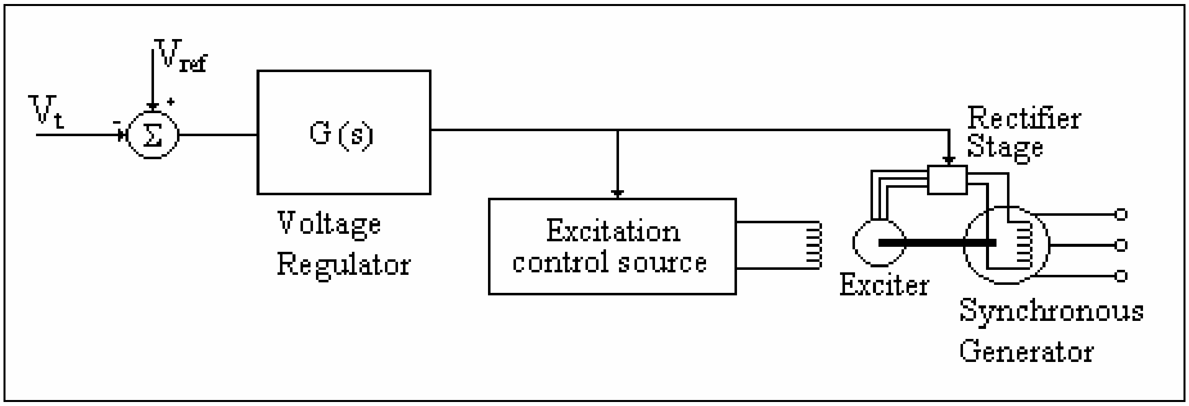

Fig. 24
Rotating AC Alternator Type Excitation System

The AC alternator excitation system uses an AC alternator with AC to DC rectification to supply the field winding of the synchronous generator. An important advantage over DC commutator systems is that AC alternator systems may be brushless, i.e., they do not use slip rings to couple the exciter to the rotor-mounted field winding. For example, the General Electric Althyrex© uses an "inverted" alternator to supply the field voltage through a rectifier. The alternator is inverted in that, unlike the power generator, the field winding is on the stator and the armature windings are on the rotor. Therefore the alternator field can be fed directly without the need for slip rings and brushes. Rectification to DC, required by the synchronous generator field, takes place by feeding the alternator three-phase output to a thyristor controlled bridge. The thyristor or silicon controlled rectifier (SCR) is similar to a diode, except that it remains “off” until a control signal is applied to the gate. The device will then conduct until current drops below a certain value or until the voltage across it reverses.

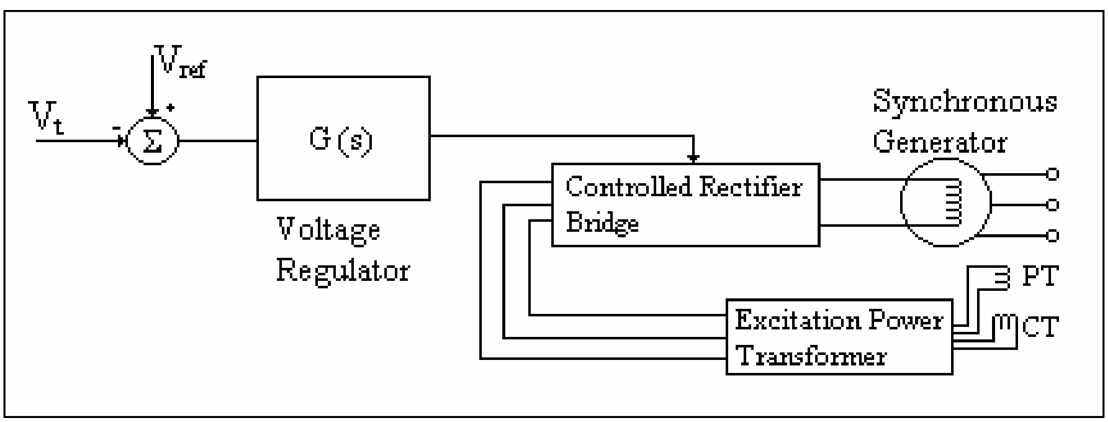

Figure 25:
Static Type Excitation System

The third type of excitation system is called a *static* system because it is composed entirely of solid state circuitry, i.e., it contains no rotating device. The power source for this type of system is a potential and/or a current transformer supplied by the synchronous generator terminals. Three-phase power is fed to a rectifier, and the rectified DC output is applied to the synchronous generator field via slip rings and brushes. Static excitation systems are usually less expensive than AC alternator types, and the additional maintenance required by the slip rings and brushes is outweighed by the fact that static excitation systems have no rotating device.

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[1] http://geothermal.marin.org/GEOpresentation/

[2] A. Fitzgerald, C. Kingsley, and A. Kusko, "Electric Machinery, Processes, Devices, and Systems of Electromechanical Energy Conversion," 3rd edition, 1971, McGraw Hill.

[3] S. Chapman, "Electric Machinery Fundamentals," 1985, McGraw-Hill.