TECHNICAL MEMORANDUM – DENNIS KIRK

Subject: Flow through Orifice Plates in Compressible Fluid Service at high dP

The calculation of compressible flow through orifice plates at high dP (critical flow) appears to be carried out incorrectly in most instances. This flow condition is often encountered on gas plants, compressor stations and pipelines where orifice plates are commonly used as a cheap and convenient way to regulate blowdown and pressurising rates. Blowdown must be achieved within the time period required to make the plant safe but at the same time without an excessive rate of pressure drop that could damage equipment. A more accurate calculation method is proposed below and should be used to improve the engineering predictions.

Orifice flow calculations typically use the following equations or some variant of them;

$$q_1\left[Am^3/\sec\right]=Y.C.A_o.\sqrt{\frac{2.(P_1-P_2).1000}{\rho_1}}$$ (Crane Eqn 2-24)

$$w\left[kg/\sec\right]=Y.C.A_0.\sqrt{2.\rho_1.(P_1-P_2).1000}$$

Where $_1$ indicates upstream conditions, $_2$ indicates downstream conditions and A_o is the orifice cross section area

For metering applications the ASME equation is often used to determine the Expansion Factor Y;

$$Y=1-\left(0.41+0.35.\beta^4\right).\frac{\left(1-\frac{P_2}{P_1}\right)}{k}$$ - for flange taps, where P and T are in absolute units.

For high accuracy metering (e.g. AGA 3) use of these calculations is often limited to $P_2 \geq 0.8 \times P_1$.

For density use; $\rho_1=\frac{MW.P_1}{Z_1.8.3145.T_1}$, where P and T are in absolute units.

It is then often assumed that the orifice flow goes critical (i.e. sonic velocity) at $P_2 = 0.528 \times P_1$ (for air) and the choked flow equation is then applied;

$$q_1\left[Am^3/\sec\right]=Y.C.A.\sqrt{\frac{2.(P_1-P_{critical}).1000}{\rho_1}}$$ (Crane Pg 2-15), where $P_{cr} = 0.528 \times P_1$

$$w\left[kg/\sec\right]=Y.C.A.\sqrt{2.\rho_1.(P_1-P_{cr}).1000}$$

For P_2 less than P_{cr} this method assumes that the flow does not increase further.

Experiments carried out by RG Cunningham and published by ASME in July 1951 clearly demonstrated that the assumption of a fixed limit to critical flow through thin square edged orifice plates is not correct. The flow continued to increase as P_2 was reduced below the expected critical condition. Limiting flow was not evident even with P_2 as low as $0.1 \times P_1$.

The fluid does achieve sonic flow but in the vena contracta not in the orifice. The vena contracta occurs downstream of the plate and has a smaller area than the orifice. As the downstream pressure is further reduced the vena contracta moves closer to the orifice plate and increases in diameter.

Cunningham's work included tests with air and steam with the results and conclusions presented as tables, charts and formulas. Limited information is provided for the tests with steam.

The results demonstrated that <u>with suitable corrections to the Expansion Factor Y, the formula for non-critical flow should be used in all cases</u> for thin square edge orifice plates. Critical flow can, however, be expected for thick orifice plates with $t \geq 6 \times$ the orifice diameter.

Cunningham's paper also includes an equation for the Flow Coefficient $C=0.608+0.415.\beta^4$, though this appears to provide only a rough approximation and other methods may be preferable (e.g. AGA 3).

The ASME formula for Y was shown to be appropriate only down to P_2 = 0.63 x P_1; (not the normally expected 0.528 from thermodynamic analysis) at which point there is a distinct discontinuity in the flow to lower discharge pressures. Continued use of the ASME formula for Y produces errors of up to 12% if used for lower discharge pressures. Alternative methods reviewed involved errors of up to 40%.

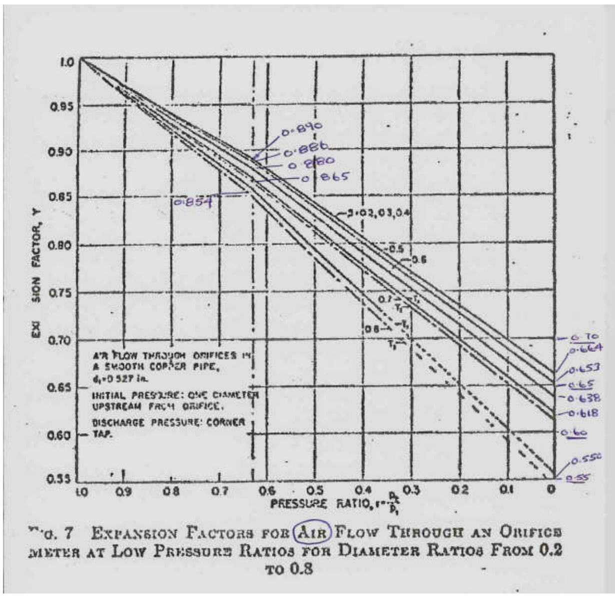
Analysis of the Cunningham data suggests the following formula may be used to determine Y for flange taps at discharge pressures below 0.63 x P_1;

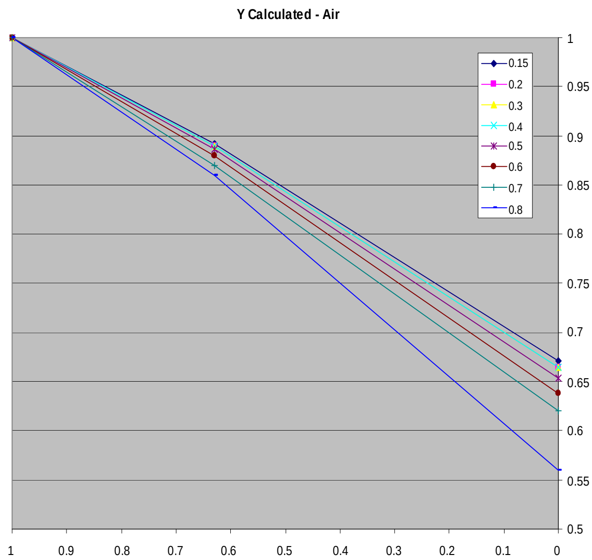
$$Y = Y_{0.63} - \left(0.49 + 0.45.\beta^4\right).\frac{\left(0.63 - \frac{P_2}{P_1}\right)}{k}$$

- where $Y_{0.63}$ is Y from the ASME formula at P_2 = 0.63 x P_1

The use of a formula similar to the form of the ASME equation is based on an expectation that there is a reasonable probability that the flow to lower pressures will be similarly sensitive to the same geometric and process parameters. The use of β to the 4th power provides a reasonable fit to the experimental data. Since the relationship between Y and the pressure ratio is linear the (0.63-P_2/P_1) component is clearly appropriate. The inclusion of k as a direct divisor in the equation is less obvious and difficult to confirm from the limited data available.

The chart on the left is extracted directly from the Cunningham report and clearly shows the discontinuity at a pressure ratio of 0.63 and the potential for error if the ASME formula is used beyond this point. The chart on the right is generated using the proposed method.





The information on the following page demonstrates the derivation of the proposed formula from the published Cunningham experimental results. Plots on the last page show a comparison of some alternative calculation methods. These indicated a good match of the proposed method to that of Perry and another derived from the Grace-Lapple tests, but only for Beta ratios less than 0.6 where significant discontinuities appear with these alternative methods.

References: Technical Paper 410M, Crane, 1983
Orifice Meters with Supercritical Compressible Flow, RG Cunningham – ASME, July 1951
Perry's Chemical Engineering Handbook – 7th Edn – RH Perry, DW Green
Flow Measurement Engineering Handbook, 3rd Edn – RW Miller
(Note: formula 9.58 is correct only for air with β = 0.15 and could be replaced with the method proposed above.)

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Orifice Plate Coefficient of Expansion for High dP Compressible Fluid Flow - refer Cunningham "Orifice Meters with Supercritical Compressible Flow - Transactions of the ASME - July 1951"

Air k **1.4**

Experimental (Cunningham)

			KY Exp			Y Exp			Y Fig 7		
Beta Nom	Beta Act	K Calc	1	0.63	0	1	0.63	0	1	0.63	0
0.15	0.1503	0.608212	0.5993			1	0.8916	0.671	1	0.8916	0.671
0.2	0.2035	0.608712	0.616	0.548	0.407	1.011973	0.900262	0.668625	1	0.89	0.664
0.3	0.3027	0.611484	0.616	0.548	0.407	1.007385	0.89618	0.665594	1	0.89	0.664
0.4	0.4047	0.619132	0.616	0.548	0.407	0.994941	0.88511	0.657372	1	0.89	0.664
0.5	0.5037	0.634714	0.634	0.562	0.413	0.998875	0.885438	0.650687	1	0.886	0.653
0.6	0.6119	0.666179	0.665	0.585	0.423	0.998229	0.878142	0.634964	1	0.88	0.638
0.7	0.7049	0.710461	0.719	0.624	0.445	1.012019	0.878303	0.626354	1	0.87	0.62
0.8	0.8071	0.784099	0.82	0.703	0.452	1.045786	0.89657	0.576458	1	0.86	0.56

using K = 0.608 + 0.415 x B^4

Formula (Cunningham)

Beta Nom	Beta Act		1	0.63	0	
0.15	0.1503	Y = 1.0 - (0.41 + 0.35 x B^4) x (1 - r) / k	1	0.891596	0.671033	Y = $Y_{0.63}$ - 0.3501 x (0.63 - r)
0.2			1	0.891495	0.661545	Y = $Y_{0.63}$ - 0.3650 x (0.63 - r)
0.3			1	0.890894	0.660944	Y = $Y_{0.63}$ - 0.3650 x (0.63 - r)
0.4			1	0.889275	0.659325	Y = $Y_{0.63}$ - 0.3650 x (0.63 - r)

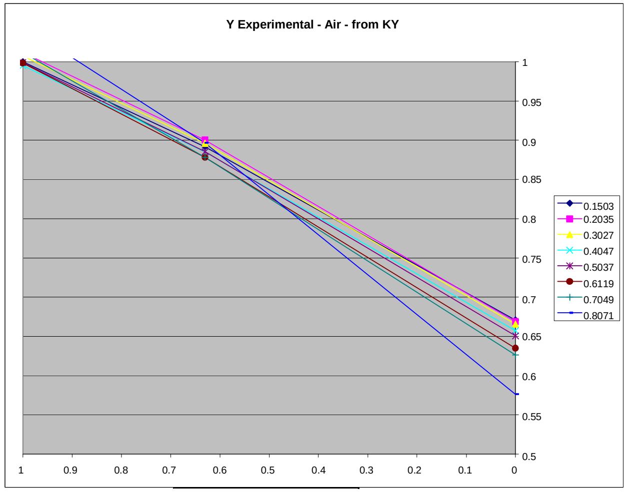
k **1.4**

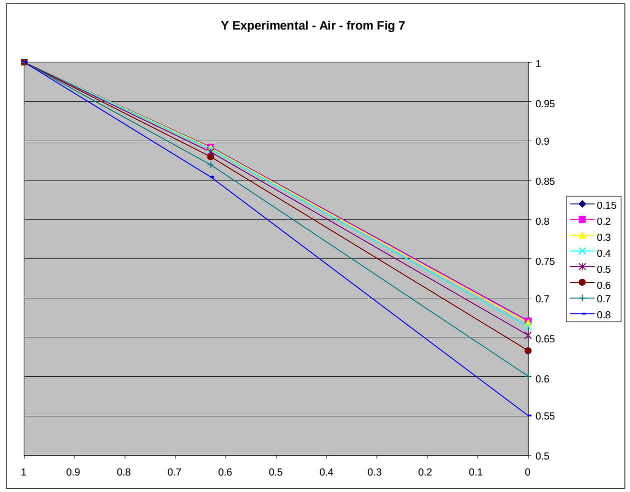
Calculated (D Kirk)

			0.41 / 0.35	0.49 / 0.45	Y ASME		Y Kirk
Beta Nom	Beta Act	1	0.63	0	1	0.63	0
0.15	0.1503	1	0.891596	0.670992	1	0.891596	0.670994
0.2	0.2035	1	0.891484	0.670637	1	0.891495	0.670671
0.3	0.3027	1	0.890866	0.668666	1	0.890894	0.668753
0.4	0.4047	1	0.889162	0.66323	1	0.889275	0.663591
0.5	0.5037	1	0.885689	0.652154	1	0.885862	0.652705
0.6	0.6119	1	0.878675	0.629786	1	0.879655	0.632911
0.7	0.7049	1	0.868805	0.598309	1	0.869434	0.600313
0.8	0.8071	1	0.852392	0.545964	1	0.853755	0.550311

Comparison of calculated and experimental values for Y

	0.999995	0.999989	0.999996	0.99999	0.999996	0.99999
	0.99025	1.003009	0.990262	1.003059	1.00168	1.010046
	0.99407	1.004616	0.994101	1.004747	1.001004	1.007159
	1.004578	1.008911	1.004706	1.009461	0.999185	0.999384
	1.000283	1.002254	1.000478	1.003102	0.999844	0.999549
	1.000607	0.991846	1.001723	0.996767	0.999608	0.992023
	0.989186	0.955225	0.989902	0.958425	0.999349	0.968247
	0.950725	0.947101	0.952245	0.954642	0.992738	0.982698
Average	99.12%	98.91%	99.17%	99.13%	99.92%	99.49%





Steam k **1.3**

Experimental (Cunningham)

Beta Nom	Beta Act	Y Fig 4: 1	0.63	0
0.15	0.1514	1	0.884	0.664

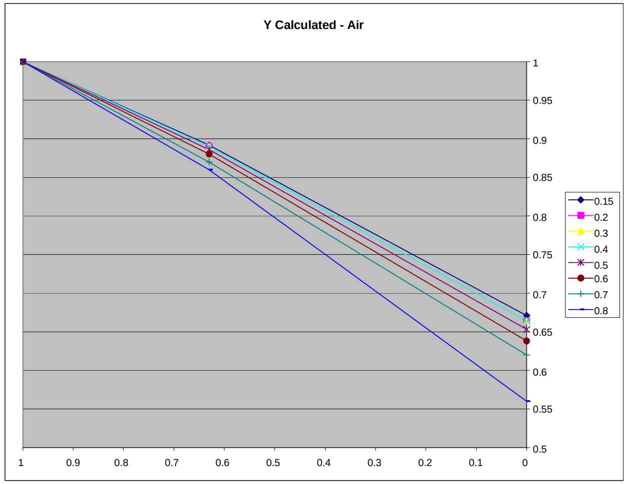
Formula (Cunningham)

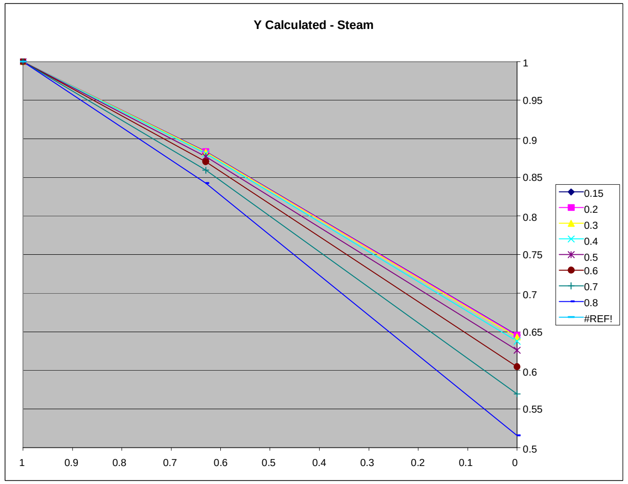
Beta Nom	Beta Act		1	0.63	0	
0.15	0.1514	Y = 1.0 - (0.41 + 0.35 x B^4) x (1 - r) / k	1	0.891594	0.672354	Y = $Y_{0.63}$ - 0.3480 x (0.63 - r)

k **1.3**

Calculated (D Kirk)

			0.41 / 0.35	0.49 / 0.45	Y ASME		Y Kirk
Beta Nom	Beta Act	1	0.63	0	1	0.63	0
0.15	0.1514	1	0.883255	0.645679	1	0.883257	0.645685
0.2	0.2035	1	0.883137	0.645301	1	0.883148	0.645338
0.3	0.3027	1	0.882471	0.643179	1	0.882501	0.643273
0.4	0.4047	1	0.880636	0.637324	1	0.880758	0.637713
0.5	0.5037	1	0.876895	0.625396	1	0.877082	0.62599
0.6	0.6119	1	0.869342	0.601308	1	0.870398	0.604673
0.7	0.7049	1	0.858713	0.56741	1	0.85939	0.569568
0.8	0.8071	1	0.841037	0.511038	1	0.842505	0.515719





$$Y = 1 - \left(0.41 + 0.35.\beta^4\right).\frac{\left(1 - \frac{P2}{P1}\right)}{k}$$

- the ASME Formula is valid down to P2/P1 = 0.63

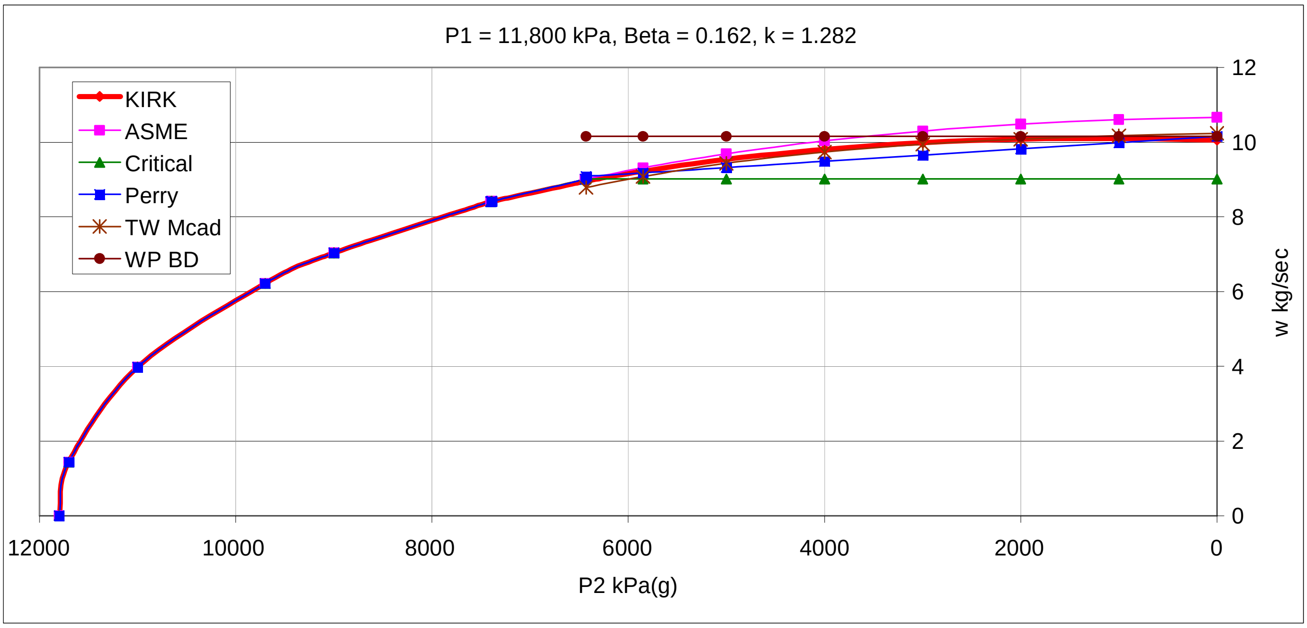
$$- for 1 > \frac{P2}{P1} > 0.63$$

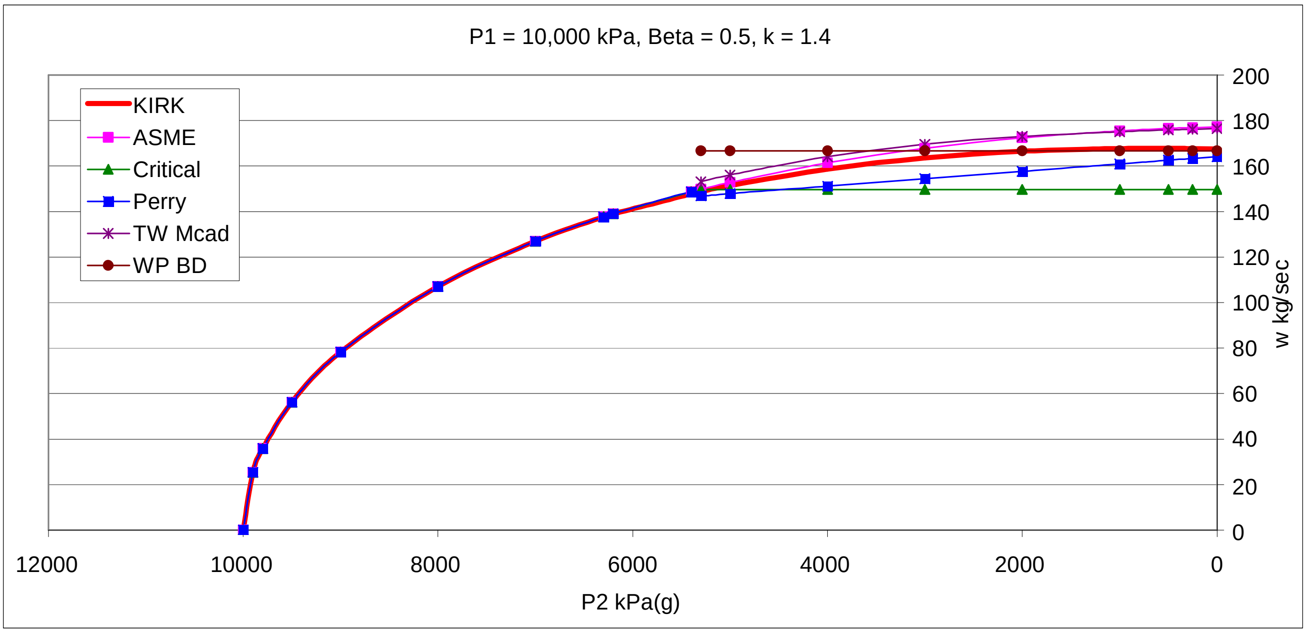
$$Y = Y_{0.63} - \left(0.49 + 0.45.\beta^4\right).\frac{\left(0.63 - \frac{P2}{P1}\right)}{k}$$

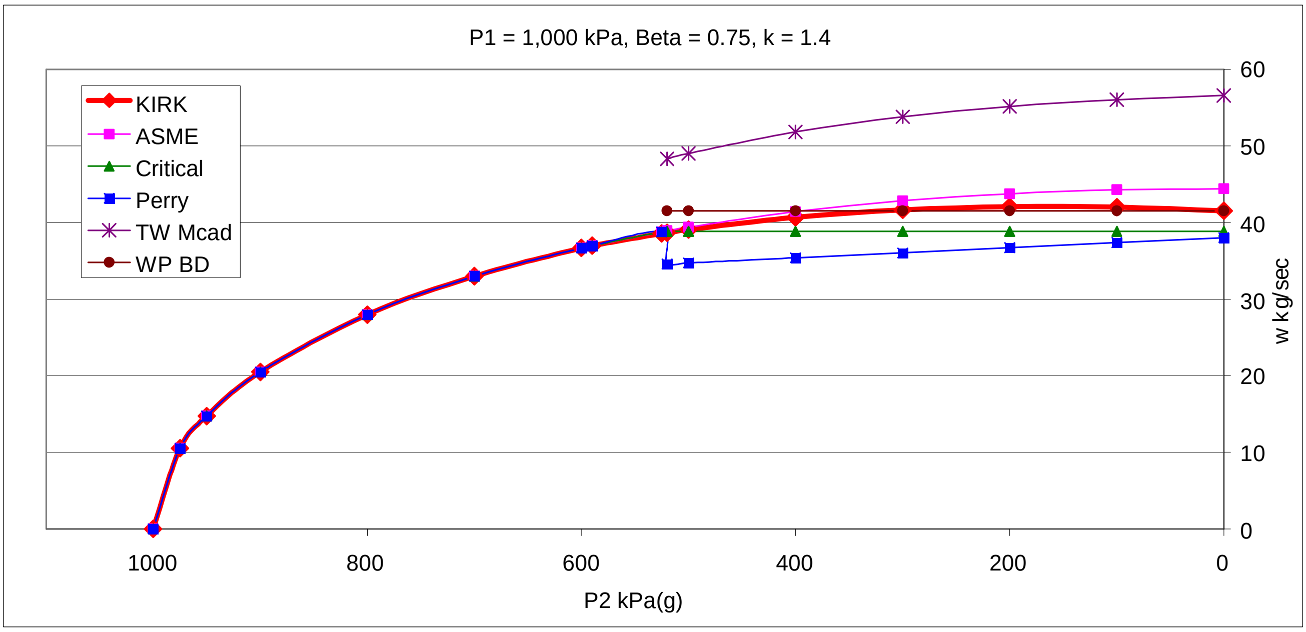
- $Y_{0.63}$ is Y from the ASME formula at P2/P1 = 0.63

$$- for 0.63 > \frac{P2}{P1} > 0$$

- use this formula for P2/P1 < 0.63


P1 = 11,800 kPa, Beta = 0.162, k = 1.282
KIRK
ASME
Critical
Perry
TW Mcad
WP BD
w kg/sec
P2 kPa(g)


P1 = 10,000 kPa, Beta = 0.5, k = 1.4
KIRK
ASME
Critical
Perry
TW Mcad
WP BD
w kg/sec
P2 kPa(g)


P1 = 1,000 kPa, Beta = 0.75, k = 1.4
KIRK
ASME
Critical
Perry
TW Mcad
WP BD
w kg/sec
P2 kPa(g)