Recommended books

- Benson – University Physics
  - Chapters 18-20 and parts of 21
- Extra info in C.B.P. Finn - Thermal Physics
Thermodynamics

- Understanding the words
  - Temperature
  - Heat
  - Heat capacity
  - The 0, 1, 2 laws of thermodynamics
- (one of) Kelvin’s legacy’s

William Thompson (Lord Kelvin)
Why study thermal physics???

- Temperature affects material properties, dimensions
- Electrical or electronic components need cooling
  - Transformers
  - ICs
- Thermodynamic effects often used
  - E.g. heat pump in refrigeration
- Efficiency - why can’t I turn all my energy into useful work???
What is it all about?

- Describe a system in terms of simple \emph{State Variables}
  - E.g. Temperature, pressure, volume, amount of substance
- Independent of microscopic structure
- But consistent with microscopic understanding
- Very powerful for all sorts of systems!!
Einstein thought:

A theory is the more **impressive** the greater the **simplicity** of its premises, the more **varied** the kinds of things it relates and the more **extended** the area of its applicability. Therefore classical thermodynamics has made a **deep impression** upon me. It is the only physical theory of **universal** content which I am convinced, within the areas of the applicability of its basic concepts, will **never be overturned**.
What is Temperature?

- Perception as to hot and cold defined relative to our own body temperature, i.e. object is hotter or colder than oneself.

- Own perception misleading
  - Door and doorhandle
  - 21°C air and water
  - Hand in hot then lukewarm water
  - Hand in cold then lukewarm water
What is Temperature - 2?

- Objective measurement of temperature
  - Macroscopic, display of temperature gauge
  - Microscopic behaviour of atoms and molecules
Measuring temperature

- Properties of materials change with temperature
  - Length
  - Volume
  - Electrical Resistance
Hotter things become longer

Most solids get bigger when they get hot
- A 1 metre long bar heated by 1 degree gets bigger by
  - Steel $\approx 0.01$ mm
  - Glass $\approx 0.001$ mm
  - Zerodur $\approx 0.0001$ mm

Rails expand and may buckle on a hot summer day
A bimetallic strip

- Join two metals with different coefficient of thermal expansion

  e.g. fire alarm
Hotter things take up more volume -1

- Most materials get bigger when they get hot (but not water 0°C -> 4°C gets smaller!)
  - Thermometer relies on a thermal expansion of a liquid (e.g. mercury)

Thin tube (Gives big length change for small increase in volume)

Large volume of reservoir
Hotter things take up more volume

- Gases (as we will see) can behave near perfectly.
Hotter things change their resistance

- All hotter metals have a higher electrical resistance
  - e.g. platinum resistance thermometer
- Hotter (undoped) semiconductors have a lower electrical resistance
  - Key distinction between metals and insulators!
- Superconductors lose all electrical resistance at low T
How long do you have to leave a thermometer in your mouth?

- Hot things stay hot if you insulate them, e.g.
  - coffee in a vacuum flask (keeps things cold too)
  - an explorer in a fur coat

- The mercury in the thermometer must reach the same temperature and you
Insulation

- Example of good (thermal) insulators
  - A vacuum, polystyrene, fibreglass, plastic, wood, brick
  - (low density/foam structure, poor electrical conductors)

- Examples of poor insulators, i.e. good conductors
  - Most metals (but stainless steel better than copper)
    e.g. gold contact used within IC chips to prevent heating
  - Gases, liquids
  - (high density, “mobile”, good electrical conductors)
Ask a friend if it’s cool enough to eat

- Your friend eats the “hot” loaf and says it cool enough to eat (i.e it is “close” enough to their own temperature that it does not burn)
- Is it safe for you to eat too?
- If it is safe for them, it’s safe for you!
The 0th law of thermodynamics

- If A and B are each in thermal equilibrium with C then A and B are in thermal equilibrium with each other.

- If Alfred and the Bread are the same temperature as Cliff then Alf is the same temperature as the Bread.
So what is temperature?

- A State Variable
  - *Independent of structure, path to current state etc.*
- Describes thermal equilibrium
- Two objects in thermal contact will reach thermal equilibrium only when they have the same temperature
- Microscopically – to do with motion or vibration of atoms and molecules
Temperature and scales

- We need a linear scale for temperature
- Could use any two fixed points
- Temperature scales in historical order (melting & boiling of water)
  - Degrees Fahrenheit (MP 32°F BP 212°F)
    - 0°F – cold day, 100°F – body temperature (with a cold!!)
  - Degrees Celsius (MP 0°C 100°C)
  - Degrees Kelvin (MP 273.15 K BP 373.15 K)
Converting between scales

- Kelvin to Celsius
  - $K = C + 273.15$
  - $C = K - 273.15$

- Fahrenheit to Celsius
  - $F = C \times \frac{9}{5} + 32$
  - $C = (F - 32) \times \frac{5}{9}$
Example

- Convert the following temperatures into °F and K
- Boiling water, 100°C 212°F, 373.15K
- Freezing water, 0°C 32°F, 273.15K
- Absolute zero, -273.15°C -460°F, 0K
Types of thermometer

- Change in electrical resistance (convenient but not very linear)
- Change in length of a bar (bimetallic strip)
- Change in volume of a liquid
- Change in volume of gas (very accurate but slow and bulky)
- Thermocouple
Linear expansion

- Objects get longer when they get hotter
- Approximately linear over small range
- Their *fractional* change in length is proportional to the change in temperature
  - $\Delta L/L = \alpha \Delta T$ or $\Delta L = \alpha L \Delta T$
  - $\frac{dL}{dT} = \alpha L$
Thermal expansion ($\alpha$ [K$^{-1}$])

- Aluminium, $\alpha = 2.4 \times 10^{-5}$ K$^{-1}$
- Steel, $\alpha = 1.2 \times 10^{-5}$ K$^{-1}$
- Glass, $\alpha \approx 5 \times 10^{-6}$ K$^{-1}$
- Invar, $\alpha \approx 9 \times 10^{-7}$ K$^{-1}$
- Quartz, $\alpha \approx 4 \times 10^{-7}$ K$^{-1}$
Example

- Metre rules are calibrated at 20°C
- What is the error in a measurement of 500mm if made at 45°C?
- \( \alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ K}^{-1} \)

\[
\frac{\Delta L}{L} = \alpha \Delta T
\]

\[
\Delta L = L \alpha \Delta T = 500 \times 10^{-3} \times 1.2 \times 10^{-5} \times 25
\]

\[
\Delta L = 1.5 \times 10^{-6} \text{m} = 1.5 \mu\text{m}
\]
Thermal expansion, why?

- Every microscopic object moves due to thermal excitation - Brownian motion
- Atoms too vibrate with respect to each other
- Hotter atoms vibrate more
  - Asymmetric potential means average separation increases
Volume Expansion

- Every length goes from $L$ to $L + \Delta L = L + L\alpha \Delta T$
- Old volume $= L^3$
- New volume $= (L + \Delta L)^3$
- Ignore terms like $\Delta L^2$ and $\Delta L^3$
  - $V_{\text{new}} = (L + \Delta L)^3 \approx L^3 + 3L^2 \Delta L$
- But $\Delta L = L\alpha \Delta T$
  - $V_{\text{new}} = L^3 + 3L^2 \Delta L = L^3 + 3L^2 \alpha L \Delta T = L^3 + 3L^3 \alpha \Delta T$
  - $\Delta V = 3L^3 \alpha \Delta T$
  - $\Delta V / V = 3\alpha \Delta T$ or $\Delta L = 3\alpha V \Delta T$
- $3\alpha$ often called $\beta$
Example

- If whisky bottles are made to be exactly 1 litre at 20°C
- but, whisky is bottled at 10°C
- How much whisky do you actually get if it is served at 20°C?
  - $\beta_{\text{glass}} = 2 \times 10^{-5} \text{ K}^{-1}$
  - $\beta_{\text{whisky}} = 75 \times 10^{-5} \text{ K}^{-1}$

\[
V_{\text{bottle@10°C}} = V_{\text{bottle@20°C}} (1 + \Delta T \beta)
\]
\[
V_{\text{bottle@10°C}} = 1 (1 - 10 \times 2 \times 10^{-5})
\]
\[
V_{\text{bottle@10°C}} = 0.9998 \text{ litres}
\]

What does 0.9998 litres of whisky at 10°C occupy at 20°C?

\[
V_{\text{whisky@20°C}} = V_{\text{whisky@10°C}} (1 + \Delta T \beta)
\]
\[
V_{\text{whisky@20°C}} = 0.9998 (1 + 10 \times 2 \times 10^{-5})
\]
\[
V_{\text{whisky@20°C}} = 0.9998 (1 + 10 \times 75 \times 10^{-5})
\]
\[
V_{\text{whisky@20°C}} = 1.0073 \text{ litres}
\]
Shape change on expansion

- This can be very complex for mismatched materials
- Single material (or matched $\alpha$) much simpler

bigger diameter

bigger hole

hotter
Thermal expansion of water

- Density of ice is less than water!!!
  - Icebergs float
- Density of water maximum at 4°C
  - Nearly frozen water floats to the top of the lake and hence freezes at surface

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Density (kg/m³)</th>
</tr>
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<tbody>
<tr>
<td>1.0000</td>
<td>1.0002</td>
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<td>1.0004</td>
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</tbody>
</table>
Volume and pressure of a gas

- Gases (at constant pressure) expand with increasing temperature
  - ideal gases tend to zero volume at -273.15°C!

- Gases (at constant volume) increase pressure with increasing temperature
  - ideal gases tend to zero pressure at -273.15°C!

- In reality, gases liquefy when they get cold
Pressure

- Pressure is defined as force per unit area
  - Newtons per square metre \( N/m^2 = \text{Pa} \)
- The pressure exerted by a gas results from the atoms/molecules "bumping" into the container walls
  - More atoms gives more bumps and higher pressure
  - Higher temperature gives faster bumps and higher pressure
- At sea level and 20°C, normal atmospheric pressure is
  - \(1\text{atm} \approx 1 \times 10^5 \text{ N/m}^2 = 0.1 \text{ MPa}\)
Volume and Pressure of a Gas

- **Ideal gas law:**
  - $P V = \text{const.}$

- **Can define** $PV = NkT$
  - Where $N$ is the number of molecules
  - And $k$ is Boltzmann’s constant, $1.38 \times 10^{-23}$ J K$^{-1}$

- **Alternatively** $PV = nRT$
  - Where $n$ is the number of moles of a gas
  - $N = n \, N_A$
  - where $N_A$ is Avogadro’s number, $6.02 \times 10^{23}$ mol$^{-1}$
  - $R = k \, N_A = 8.314$ J mol$^{-1}$ K$^{-1}$
Any two temperatures defined by the ratio

\[ P_1 \ T_2 = P_2 \ T_1 \quad \text{or} \quad V_1 \ T_2 = V_2 \ T_1 \]

Can use this to calculate volume or pressure changes with temperature
Example

- A bottle of hair spray is filled to a pressure of 1 atm at 20°C
- What is the canister pressure if it is placed into boiling water?

\[ P_1 T_2 = P_2 T_1 \]
\[ 1 \times 373 = P_2 \times 293 \]
\[ P_2 = \frac{373}{293} \]
\[ P_2 = 1.27 \text{ atm} \]
Absolute zero 1

- On the Kelvin scale, the lowest possible temperature is 0 K. (zero volume and zero pressure)
- The zero point is fixed -
  - Absolute Zero (≈ -273.15°C)
- Additional point defined at triple point of water (occurs at one temp and pressure where ice, steam and liquid all coexist (≈ 0.01°C and 0.006 atm)
- $T_{\text{triple}} = 273.16K$
- $T = 273.16 \times \left( \frac{p}{p_{\text{triple}}} \right)$
Absolute zero 2

- Ideal gas has zero volume
- Resistance of metal drops to zero (superconductivity cuts in above 0K)
- Brownian motion ceases (kinetic energy due to thermal excitation $\approx 3/2 \ kT$, see later)
- But lowest temperature yet attained in lab is $\approx 10^{-9}K$
- Very difficult to get really cold!!!
Lord Kelvin

- William Thompson, born Belfast 1824
- Student in Natural Philosophy
- Professor at 22!
- Baron Kelvin of Largs in 1897
- Lived at 11 The Square
- A giant
  - Thermodynamics, Foams, Age of the Earth, Patents galore!
Thermal expansion solid-liquid-gas

- Normally, density ($\rho$) changes as...
How much energy required to heat object?

- Heat (energy) flows because of temperature difference
  - Bigger temperature difference - bigger heat flow
  - Less insulation gives more heat flow for the same temperature difference
- Heat will not flow between two bodies of the same temperature
What is heat?

- Heat flows from hot bodies to cold bodies until the temperature of the two is the same.
- Early researchers thought it was a substance called caloric.
Equilibrium

- Two objects of different temperature when placed in contact will reach the same temperature

Hot black coffee + Cold milk = Warm white coffee
Sir James Joule

- James Joule 1818-1889
- Stirring water made it warm
  - Change in temperature proportional to work done
  - Showing equivalence of heat and energy
- Also that electrical current flow through a resistor gives heating
Sir James Joule 2

- Heat and energy are the same thing
- Measured in Joules (J)
- Sometimes measured in calories
  - One cal raises one gram of water from 14.5°C to 15.5°C
  - 1 cal - 4.186J
1st Law of Thermodynamics
Heat transfer = energy transfer

- Doing work on something usually makes it hot
  - Splash in the bath and the water will get warmer!
- 1st law of thermodynamics heat and work are both forms of energy
- $\Delta U = Q - W$
- Same as conservation of energy
  - Can neither be created nor destroyed, just changes form
Thermal Physics – EE1

Lecture 3
Heat capacity and heat transfer
Ian MacLaren
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How quickly do things get hot?

- If you give heat to something
  - $T$ increases
  - How fast?
  - Depends on the substance
  - Alcohol heats quicker than water
  - Also depends on how much you have of the substance
Specific heat capacity

- $Q = mc \Delta T$
  - $Q$ is heat required
  - $m$ is the mass of substance

- $c$ is called the specific heat capacity
  - $c_{\text{water}} = 4190 \text{ J kg}^{-1} \text{ K}^{-1}$ - very difficult to heat
  - $c_{\text{ice}} = 2000 \text{ J kg}^{-1} \text{ K}^{-1}$
  - $c_{\text{ethanol}} = 2428 \text{ J kg}^{-1} \text{ K}^{-1}$ - easier to heat
  - $c_{\text{mercury}} = 138 \text{ J kg}^{-1} \text{ K}^{-1}$ - very easy to heat

- The higher $c$ is, the more energy we need for heating
Example – heat capacity

- “thrashing” around in the bath should heat up the water.
- How much will the water heat up after one minute of “thrashing”

Estimate volume of water \( \approx 0.5 \text{m}^3 \)
Estimate power of thrashing \( \approx 500 \text{W} \)

\[
\Delta T = \frac{Q}{mc_{\text{water}}}
\]
\[
\Delta T = \frac{500 \times 60}{500 \times 4190}
\]
\[
\Delta T = 0.015^\circ \text{C}
\]
Reaching thermal equilibrium

- Total energy (heat) of a closed system is constant, \( \Delta Q_{\text{coffee}} = -\Delta Q_{\text{milk}} \) i.e. \( \Sigma \Delta Q = 0 \)
- By convention heat flowing into a body \( \Delta Q \) +ve

\[
(T_H - T_w)m_{\text{coffee}}c_{\text{coffee}} = -(T_c - T_w)m_{\text{milk}}c_{\text{milk}}
\]

Hot black coffee at \( T_H \) + Cold milk at \( T_c \) = Warm white coffee at \( T_w \)
Example 2 – heat capacity

- A 2.5 kg steel bar is heated to 1000 °C
- It is then dropped into a 10 l tank of cold water at 10 °C (approx 10 kg water)
- What is the final temperature of the water?

\[
\text{c}_{\text{steel}} = 420 \text{ J kg}^{-1} \text{ K}^{-1} \\
\text{c}_{\text{water}} = 4190 \text{ J kg}^{-1} \text{ K}^{-1} \\
\]

\[
\frac{\Delta T_{\text{steel}}}{\Delta T_{\text{water}}} = -39.9 \\
\Delta T_{\text{water}} - \Delta T_{\text{steel}} = 990 \\
\Delta T_{\text{water}} + 39.9 \Delta T_{\text{water}} = 990 \\
\Delta T_{\text{water}} = 990/40.9 = 24.2 \text{ °C} \Rightarrow T_{\text{final}} = 34.2 \text{ °C} \\
\Delta T_{\text{steel}} = -990/1.025 = 965.8 \text{ °C} !!!!
Molar heat capacity

- Quote Joules per mole rather than Joules per kilogram
- i.e. \[ Q = nM_c \Delta T \]
  - \( n \) is the number of moles
  - \( M_c \) is the molar heat capacity (J mol\(^{-1}\) K\(^{-1}\))
- \( M_c \approx 25 \text{ J mol}^{-1} \text{ K}^{-1} \) for solids!
  - i.e. energy required to heat one atom of anything is about the same
  - Realised by Dulong and Petit
Phase changes (e.g. solid to liquid)

- When heating ice into water and then into steam the temperature does not go up uniformly
  - Different gradients \( c_{\text{water}} > c_{\text{ice}} \)
  - Flat bits at phase changes
  - Need heat to convert
    - Solid to liquid
    - Liquid to vapour
Energy required for phase change

- **Heat of fusion** (Q), solid -> liquid
  - \( Q = mL_f \) (\( L_f \) is latent heat of fusion)
    - \( L_f \) (water) = \( 334 \times 10^3 \) J/kg
    - \( L_f \) (mercury) = \( 11.8 \times 10^3 \) J/kg

- **Heat of vapourisation** (Q), liquid -> gas
  - \( Q = mL_v \) (\( L_v \) is latent heat of vapourisation)
    - \( L_v \) (water) = \( 2256 \times 10^3 \) J/kg
    - \( L_v \) (mercury) = \( 272 \times 10^3 \) J/kg

- **Heat of sublimation** (Q), solid -> gas
  - \( Q = mL_s \) (\( L_s \) is latent heat of sublimation)
Other phase changes

- Magnetic transitions
  - Iron is paramagnetic at high temperature
    - Can be magnetised
    - Not permanently magnetised
  - Is ferromagnetic at lower temperatures
  - Change happens at the Curie Temperature
- Changes in crystal structure, ferroelectrics etc.
- May also have latent heats associated
Using condensation to transfer energy

- Steam has two contributions to its stored thermal energy
  - The energy it took to heat it to 100°C
  - The energy it took turn it from water at 100°C to steam at 100°C

- Same idea with sweating
  - Sweat forms on the skin
  - It evaporates and this requires energy
  - Your skin gets cooler

Turning water into steam is a thermally efficient way of cooling things down.
Example

If it takes 2 mins for your kettle to begin boiling how much longer does it take to boil dry?

- Assume kettle is 3kW
- Starting temp of water 20°C

Work done by kettle = power x time
= 2 x 60 x 3000 = 360 000J

= Work to boil water of mass M
= ∆T x M x c_{water}
= 80 x M x 4190 = 335200 M

-> Mass of water = 1.07kg

Energy to boil water = M x L_v\text{(water)}
= 1.07 x 2256 x10^3 = 2420 000J

Time required = Energy /power
= 2420 000/3000 = 808 s ≈ 13mins
Transferring heat energy

- **3 mechanisms**
  - **Conduction**
    - Heat transfer through material
  - **Convection**
    - Heat transfer by movement of hot material
  - **Radiation**
    - Heat transfer by electromagnetic radiation (light, IR, etc.)
Conduction of heat

- **Conduction in solids**
  - Heat energy causes atoms to vibrate, a vibrating atom passes this vibration to the next and so on

- **Conduction in metal**
  - Have free electron “gas”
  - Conduction electrons can move where they wish
  - Heat energy causes electrons to gain energy
  - This energy is rapidly spread out through entire free electron gas
    - Metals are good conductors of both heat and electricity
Rate of heat flow

- Heat flow \((H)\) is energy transfer per unit time, depends on
  
  - Temperature difference
  - Thermal conductivity \((κ)\)
    
    \[
    H = \frac{dQ}{dt} = κA \frac{T_H - T_C}{L}
    \]
    
    - \(k_{\text{copper}} = 385 \text{ W/(m K)}\)
    - \(k_{\text{glass}} = 0.8 \text{ W/(m K)}\)
    - \(k_{\text{air}} = 0.02 \text{ W/(m K)}\)
Example

- You poke a 1.2m long, 10mm dia. copper bar into molten lead.
- How much heat energy flows through the bar to you?
- Lead melts at 600K

Temperature difference along rod
\[ \Delta T = 600 - 311 = 289 K \]

\[ H = k_{\text{copper}} A \left( \Delta T / L \right) \]

\[ A = \pi \times r^2 = 3.142 \times 0.005^2 = 0.000078 m^2 \]

\[ H = k A \left( \Delta T / L \right) = 7.3 \text{ units?} \]

Units = \{W/ (mK)\} m^2 K / m = Watts
Thermal conduction vs thermal resistance

- Can also use thermal resistance, $R$

$$H = \frac{dQ}{dt} = \kappa A \frac{T_H - T_C}{L} = A \frac{T_H - T_C}{R},$$
i.e. $R = \frac{L}{\kappa}$

- R values often quoted for household insulation (in absurd imperial units!!)

- Can make equation of heat flow more general

$$H = \frac{dQ}{dt} = \kappa A \frac{dT}{dx}$$
Convection of heat

- “Hot air rises” (and takes its heat with it!)
  - Radiators
  - Cumulus clouds
Radiation of heat

- Don’t confuse with radioactivity
- Instead realise that EM radiation (light etc.) carries heat (e.g. the sun heats the earth)
- Anything above absolute zero radiates heat
  - Heat energy emitted $\propto T^4$
Not all things emit heat the same

- Heat emission from an object of surface area A
  
  \[ H = A e \sigma T^4 \]
  
  - \( \sigma \) = Stefan’s constant = 5.6x10^{-8} W m^{-2} K^{-4}
  - \( e \) = emissivity of a body, 0 - 1
  - \( e_{\text{copper}} = 0.3 \)
  - \( e_{\text{charcoal}} \approx 1 \)
Example

- Estimate the upper limit to the heat emission of the sun
  - Sun’s surface temperature 6000k
  - Sun’s radius 7x10^8m

Emission, \( H = A\varepsilon\sigma T^4 \)

Area = \( 4\pi r^2 = 6.2 \times 10^{18} \text{ m}^2 \)

Emissivity \( \approx 1 \)

\[ H = 6.2 \times 10^{18} \times 5.6 \times 10^{-8} \times 6000^4 \]

Sun’s output = \( 4.5 \times 10^{26} \text{ W} \)
Are heat emitters also good absorbers?

- Two bodies close
  - All heat emitted from A hits B
  - All heat emitted from B hits A
  - A is a perfect absorber & emitter
  - B emissivity $e$, absorptivity $\eta$

- B in thermal equilibrium with A, i.e. heat in = heat out
  - Area $\eta_B \sigma T_A^4 = \text{Area } e_B \sigma T_B^4$
  - $T_A = T_B$ therefore $e_B = \eta_B$
The “colour” of heat

- Peak wavelength of EM radiation emitted depends on temperature
- Spectrum includes all wavelength longer than the peak but not many above
  - 20°C - peak in infrared (need thermal imaging camera to see body heat)
  - 800°C - peak in red (electric fire glows red)
  - 3000° - peak in blue (but includes green and red light hence appears white)
  - 2.7K peak in microwave (background emission in the universe left over from the Big Bang)
Equations of state

- State, identifies whether solid liquid or gas
- Key parameters or state variables
  - Volume, $V$ (m$^3$)
  - Pressure, $P$ (N/m$^2$)
  - Temperature, $T$ (K)
  - Mass, $M$ (kg) or number of moles, $n$
- Equation of state relates $V$, $P$, $T$, $M$ or $n$
Equation of state for a solid

- Increasing the temperature causes solid to expand
- Increasing the pressure causes solid to contract (0 subscript indicates initial value)

\[ V = V_0 \left[ 1 + \beta(T - T_0) - k(p - p_0) \right] \]

- \( \beta \) = thermal (volume) expansion coefficient
- \( k \) = pressure induced volume expansion coefficient
Amount of gas

- Better to describe gas in terms of number of moles (we shall see that all gases act the same!)

- Mass, \( m \) related to number of moles, \( n \)
  
  \[ m = nM \]
  
  - \( M \) = molecular mass (g/mole, 1 mole = 6x10^{23} atoms or molecules)
Equation of state for a gas

- All gases behave nearly the same
  - \( pV = nRT \)
    - \( R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1} \) for all gases (as long as they remain a gas)
    - \( T \) is in K!!!!!!

- Re-express
  - \( pV = (m/M) RT \)

- Density \( \rho = (m/V) \)
  - \( \rho = pM/RT \)
Example

What is the mass of a cubic metre of air?

- Molecular weight of air ≈ 32 g

\[ pV = nRT \]

Atmospheric pressure = 10^5 Pa
Atmospheric temp. = 300 K

For a volume of 1 m^3

\[ n = \frac{pV}{RT} = \frac{10^5}{8.3 \times 300} \]
\[ = 40 \text{ moles} \]

\[ M = 40 \times 0.032 = 1.3 \text{ kg} \]
Constant mass of gas

- For a fixed amount of gas, its mass or number of moles remains the same
  - $pV/T = nR = \text{constant}$
- Comparing the same gas under different conditions
  - $p_1V_1/T_1 = p_2V_2/T_2$
    - Hence can use pressure of a constant volume of gas to define temperature (works even if gas is impure - since all gases the same)
    - Must use $T$ in K!!!!!!
Example

- A hot air balloon has a volume of 150 m$^3$.
- If heated from 20°C to 60°C how much lighter does it get?
  - Molecular weight of air $\approx$ 32 g mol$^{-1}$

\[
pV/T = nR \\
n = \frac{pV}{RT}
\]

Balloon has constant volume and constant pressure

\[
n_{\text{cool}} = \frac{10^5 \times 150}{(8.3 \times 293)} = 6168
\]
\[
n_{\text{hot}} = \frac{10^5 \times 150}{(8.3 \times 333)} = 5427.1
\]
\[
\Delta n = 7409 \text{ moles}
\]
\[
\Delta M = 740.9 \times 0.032 = 23.7 \text{ kg}
\]
Work on and by gases

- Compress
  - $V \rightarrow V-dV$
  - Work done on the gas by the piston
  - $W$ is -ve

- Expand
  - $V \rightarrow V+dV$
  - Work done on the piston by the gas
  - $W$ is +ve
Work done on/by gas

- \( dW = F \, dx \)
- \( P = \frac{F}{A} \)
- \( dW = P \, Adx \)
- \( Adx = dV \)
- \( dW = PdV \)
- \( P = \frac{nRT}{V} \) so \( P \) depends on \( V \)

\[
W = \int PdV
\]
Change of State processes

- How to get “there” from “here”
- Isothermal
  - Same temperature
- Isobaric
  - Same pressure
- Isovolumetric
  - Same volume
- Can relate to first law of thermodynamics
  - $\Delta U=Q-W$
Isothermal

- Iso – same
- Thermal – temperature
- Pressure and volume change inversely
- PV = const
  - Boyle’s law
- For ideal gas, if T is constant, U is constant
- $\Delta U = 0 = Q - W \Rightarrow Q = W$
- Heat input = Work done
Isobaric

- Iso – same
- Baric – pressure
- $V$ increases with $T$ or vice versa

![Graph showing isobaric process]

Point A at $T_1$, point B at $T_2$. The graph illustrates the isobaric process where $P$ is constant and $V$ increases with $T$. The points A and B represent the initial and final states respectively on the $P$ vs $V$ graph.
Isovolumetric

- Iso – Same
- Volumetric – volume
- As P increases, T increases
- $V_1 = V_2$
  
  $W = \int PdV = 0$

  $\Delta U = Q$

- All heat converted to internal energy
Adiabatic

- A – not
- Dia – through
- Batic – passable
- i.e. No heat flow
- \[ Q = 0 \Rightarrow \Delta U = -W \]
- Process occurs fast
  or
- Container is well insulated
- Adiabats obey \( PV^\gamma = \text{const.} \)
Cyclic processes

- Go from one state (point) to another and return by different route
- Net work: area of cycle (shaded)

- Quasistatic process
  - Slow change in state variables $P$, $V$, $T$
Example

- Ideal gas expands isobarically at $P = 120$ kPa from A to B
- It is then compressed isothermally from B to C
- Find the work done

\[
W = \int P \, dV
\]

A to B: $P$ const

\[
W = P \int dV = P(V_2 - V_1)
\]

\[
= 120 \times 10^3 \times (50 - 30) \times 10^{-3} = 2400 \text{J}
\]
Example continued

- B-C: T constant

\[ W = \int_{V_1}^{V_2} PdV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT(\ln V_2 - \ln V_1) \]

- At B

\[ PV = 120 \times 10^3 \times 50 \times 10^{-3} = 6000J = nRT \]
\[ W = 6000(\ln V_2 - \ln V_1) = 6000 \ln \frac{V_2}{V_1} = -1339J \]
\[ W_{\text{total}} = 2400 - 1339 = 1061J \]

*Done by the gas*