# Thermal Physics EE1

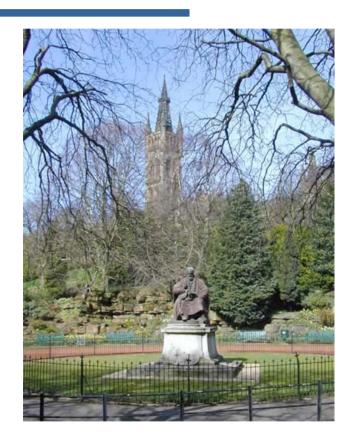
Lecture 1
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#### Recommended books

- Benson University Physics
  - Chapters 18-20 and parts of 21
- Extra info in C.B.P. Finn Thermal Physics

### Thermodynamics

- Understanding the words
  - Temperature
  - # Heat
  - Heat capacity
  - The 0, 1, 2 laws of thermodynamics
- (one of) Kelvin's legacy's



WilliamThompson (Lord Kelvin)

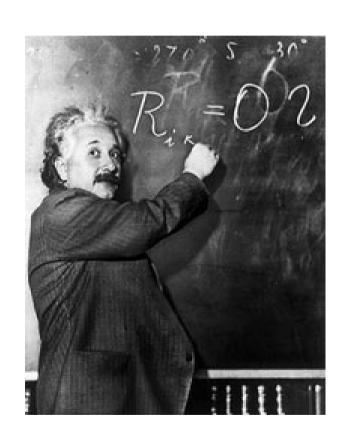
### Why study thermal physics???

- Temperature affects material properties, dimensions
- Electrical or electronic components need cooling
  - Transformers
  - ICs
- Thermodynamic effects often used
  - E.g. heat pump in refrigeration
- Efficiency why can't I turn all my energy into useful work???

#### What is it all about?

- Describe a system in terms of simple State Variables
  - E.g. Temperature, pressure, volume, amount of substance
- Independent of microscopic structure
- But consistent with microscopic understanding
- Very powerful for all sorts of systems!!

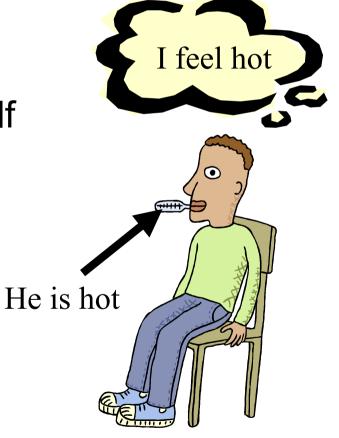
#### Einstein thought:



A theory is the more **impressive** the greater the **simplicity** of its premises, the more varied the kinds of things it relates and the more extended the area of its applicability. Therefore classical thermodynamics has made a deep impression upon me. It is the only physical theory of universal content which I am convinced, within the areas of the applicability of its basic concepts, will never be overthrown.

### What is Temperature?

- Perception as to hot and cold defined relative to our own body temperature, i.e. object is hotter or colder than oneself
- Own perception misleading
  - Door and doorhandle
  - 21°C air and water
  - Hand in hot then lukewarm water
  - Hand in cold then lukewarm water



### What is Temperature - 2?

- Objective measurement of temperature
  - Macroscopic, display of temperature gauge
  - Microscopic behaviour of atoms and molecules

#### Measuring temperature

- Properties of materials change with temperature
  - Length
  - \* Volume
  - Electrical Resistance

### Hotter things become longer

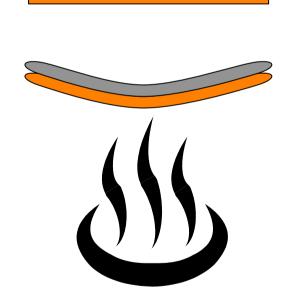
- Most solids get bigger when they get hot
  - A 1 metre long bar heated by 1 degree gets bigger by
    - Steel ≈0.01 mm
    - Glass ≈ 0.001 mm
    - Zerodur ≈ 0.0001mm



Rails expand and may buckle on a hot summer day

#### A bimetallic strip

 Join two metals with different coefficient of thermal expansion



e.g. fire alarm

# Hotter things take up more volume -1

- Most materials get bigger when they get hot (but not water 0°C -> 4°C gets smaller!)
  - Thermometer relies on a thermal expansion of a liquid (e.g.mercury)

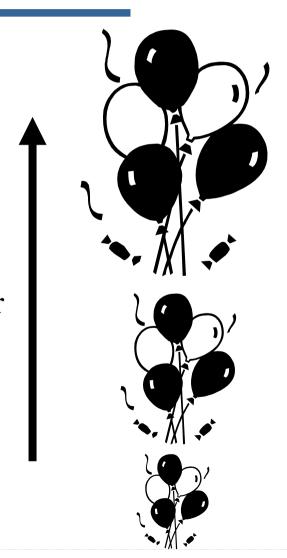
Thin tube
(Gives big
length change
for small
increase in
volume)

Large volume of reservoir

# Hotter things take up more volume -2

Gases (as we will see)can behave nearperfectly

Hotter



# Hotter things change their resistance

- All hotter metals have a higher electrical resistance
  - e.g. platinum resistance thermometer
- Hotter (undoped) semiconductors have a lower electrical resistance
  - \* key distinction between metals and insulators!
- Superconductors lose all electrical resistance at low T

# How long do you have to leave a thermometer in your mouth?

- Hot things stay hot if you insulate them, e.g.
  - coffee in a vacuum flask (keeps things cold too)
  - an explorer in a fur coat
- The mercury in the thermometer must reach the same temperature and you



#### Insulation

- Example of good (thermal) insulators
  - A vacuum, polystyrene, fibreglass, plastic, wood, brick
  - (low density/foam structure, poor electrical conductors)
- Examples of poor insulators, i.e. good conductors
  - Most metals (but stainless steel better than copper)
     e.g. gold contact used within IC chips to prevent
     heating
  - Gases, liquids
  - (high density, "mobile", good electrical conductors)

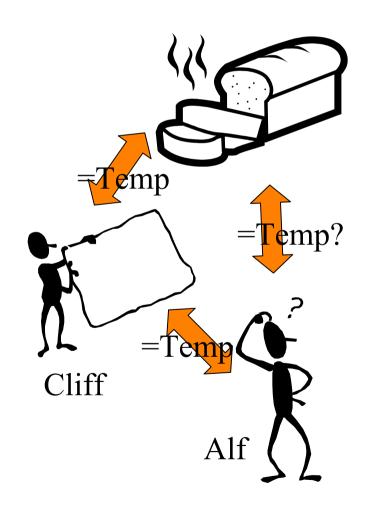
# Ask a friend if it's cool enough to eat

- Your friend eats the "hot" loaf and says it cool enough to eat (i.e it is "close" enough to their own temperature that it does not burn)
- Is it safe for you to eat too?
- If it is safe for them, it's safe for you!

### The 0<sup>th</sup> law of thermodynamics

If A and B are each in thermal equilibrium with C then A and B are in thermal equilibrium with each other

If Alfred and the Bread are the same temperature as Cliff then Alf is the same temperature as the Bread.



### So what is temperature?

- A State Variable
  - Independent of structure, path to current state etc.
- Describes thermal equilibrium
- Two objects in thermal contact will reach thermal equilibrium only when they have the same temperature
- Microscopically to do with motion or vibration of atoms and molecules

#### Temperature and scales

- We need a linear scale for temperature
- Could use any two fixed points
- Temperature scales in historical order (melting & boiling of water)
  - Degrees Fahrenheit (MP 32° F BP 212°F)
    - 0° F cold day, 100° F body temperature (with a cold!!)
  - Degrees Celsius (MP 0°C 100°C)
  - Degrees Kelvin (MP 273.15 K BP 373.15 K)

#### Converting between scales

#### Kelvin to Celsius

$$*K = C + 273.15$$

$$*C = K - 273.15$$

#### Fahrenheit to Celsius

$$F = C \times (9/5) + 32$$

$$*C = (F - 32) \times (5/9)$$

#### Example

- Convert the following temperatures into °F and K
- Boiling water, 100°C
- Freezing water, 0°C
- Absolute zero,-273.15°C

212°F, 373.15K

32°F, 273.15K

-460°F, 0K

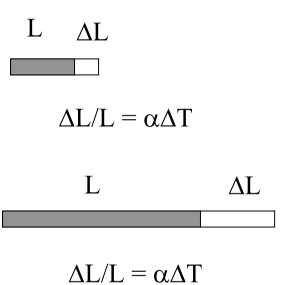
#### Types of thermometer

- Change in electrical resistance (convenient but not very linear)
- Change in length of a bar (bimetallic strip)
- Change in volume of a liquid
- Change in volume of gas (very accurate but slow and bulky)
- Thermocouple

#### Linear expansion

- Objects get longer when they get hotter
- Approximately linear over small range
- Their *fractional* change in length is proportional to the change in temperature
  - \*  $\Delta L/L = \alpha \Delta T$  or  $\Delta L = \alpha L \Delta T$

\* or 
$$\frac{dL}{dT} = \alpha L$$



## Thermal expansion ( $\alpha[K^{-1}]$ )

- Aluminium,  $\alpha = 2.4 \times 10^{-5} \text{ K}^{-1}$
- Steel,  $\alpha = 1.2 \times 10^{-5} \text{ K}^{-1}$
- Glass,  $\alpha \approx 5 \times 10^{-6} \text{ K}^{-1}$
- Invar,  $\alpha \approx 9 \times 10^{-7} \text{ K}^{-1}$
- Quartz,  $\alpha \approx 4 \times 10^{-7} \text{ K}^{-1}$

#### Example

- Metre rules are calibrated at 20°C
- What is the error in a measurement of 500mm if made at 45°C?
- $\alpha_{\text{steel}} = 1.2 \times 10^{-5} \text{ K}^{-1}$

$$\Delta L/L = \alpha \Delta T$$

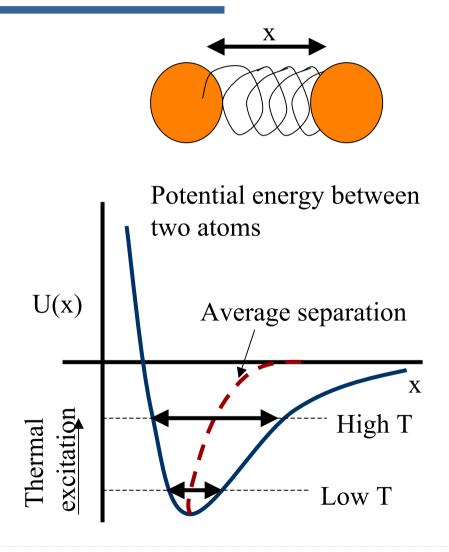
$$\Delta L = L \alpha \Delta T$$

$$\Delta L = 500 \times 10^{-3} \times 1.2 \times 10^{-5} \times 25$$

$$\Delta L = 1.5 \times 10^{-6} \text{m} = 1.5 \mu \text{m}$$

### Thermal expansion, why?

- Every microscopic object moves due to thermal excitation -Brownian motion
- Atoms too vibrate with respect to each other
- Hotter atoms vibrate more
  - Asymmetric potential means average separation increases



### Thermal Physics – EE1

Lecture 2
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#### **Volume Expansion**

- Every length goes from L to L+ $\Delta$ L = L + L $\alpha$   $\Delta$ T
- Old volume = L<sup>3</sup>
- New volume =  $(L + \Delta L)^3$
- Ignore terms like  $\Delta L^2$  and  $\Delta L^3$

• 
$$V_{\text{new}}$$
=  $(L + \Delta L)^3$  ≈  $L^3$  +  $3L^2$   $\Delta L$ 

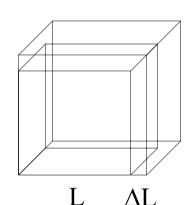


\* 
$$V_{new} = L^3 + 3L^2 \Delta L = L^3 + 3L^2 \alpha L \Delta T = L^3 + 3L^3 \alpha \Delta T$$

\* 
$$\Delta V = 3L^3 \alpha \Delta T$$

\* 
$$\Delta V / V = 3\alpha \Delta T$$
 or  $\Delta L = 3\alpha V \Delta T$ 

■  $3\alpha$  often called  $\beta$ 



#### Example

- If whisky bottles are made to be exactly 1 litre at 20°C
- but, whisky is bottled at 10°C
- How much whisky do you actually get if it is served at 20°C?

• 
$$\beta_{glass} = 2x10^{-5} \text{ K}^{-1}$$

• 
$$\beta_{\text{whisky}} = 75 \times 10^{-5} \text{ K}^{-1}$$

$$V_{\text{bottle@10}^{\circ}\text{C}} = V_{\text{bottle@20}^{\circ}\text{C}} (1 + \Delta T\beta)$$

$$V_{\text{bottle@}10^{\circ}\text{C}} = 1 (1 - 10 \times 2 \times 10^{-5})$$

$$V_{\text{bottle@10}^{\circ}\text{C}} = 0.9998 \text{ litres}$$

What does 0.9998 litres of whisky at 10°C occupy at 20°C?

$$V_{\text{whisky}@20^{\circ}\text{C}} = V_{\text{whisky}@10^{\circ}\text{C}} (1 + \Delta T\beta)$$

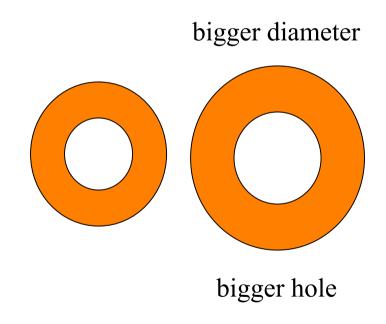
$$V_{\text{whisky}@20^{\circ}C} = 0.9998 (1+10 \times 2\times10^{-5})$$

$$V_{\text{whisky@20^{\circ}C}} = 0.9998 \ (1+10 \ x75x10^{-5})$$

$$V_{\text{whisky}@20^{\circ}\text{C}} = 1.0073 \text{ litres}$$

#### Shape change on expansion

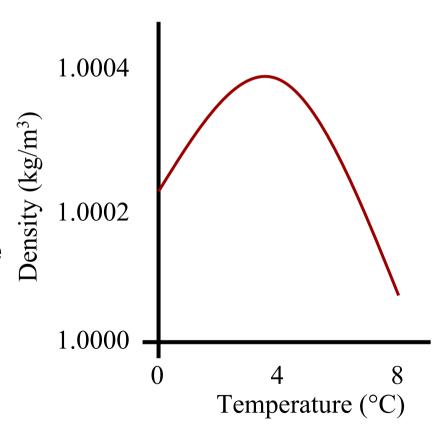
- This can be very complex for mismatched materials
- Single material (or matched α) much simpler



hotter

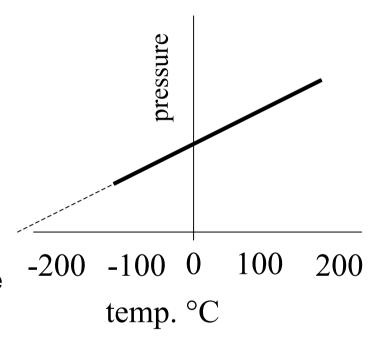
#### Thermal expansion of water

- Density of ice is less than water!!!
  - Icebergs float
- Density of water maximum at 4°C
  - Nearly frozen water floats to the top of the lake and hence freezes at surface



### Volume and pressure of a gas

- Gases (at constant pressure) expand with increasing temperature
  - ideal gases tend to zero volume at - 273.15°C!
- Gases (at constant volume) increase pressure with increasing temperature
  - ideal gases tend to zero pressure at - 273.15°C!
- In reality, gases liquefy when they get cold



#### Pressure

- Pressure is defined as force per unit area
  - Newtons per square metre N/m² = Pa
- The pressure exerted by a gas results from the atoms/ molecules "bumping" into the container walls
  - More atoms gives more bumps and higher pressure
  - Higher temperature gives faster bumps and higher pressure
- At sea level and 20°C, normal atmospheric pressure is
  - \*  $1atm \approx 1 \times 10^5 \text{ N/m}^2 = 0.1 \text{ MPa}$

#### Volume and Pressure of a Gas

- Ideal gas law:
  - ♣ P V = const.
- Can define PV = NkT
  - Where N is the number of molecules
  - ♣ And k is Boltzmann's constant, 1.38x10<sup>-23</sup> J K<sup>-1</sup>
- Alternatively PV = nRT
  - Where n is the number of moles of a gas
  - $*N = nN_A$
  - ♦ where N<sub>A</sub> is Avogadro's number, 6.02x10<sup>23</sup> mol<sup>-1</sup>
  - $R = k N_A = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

### Volume and pressure 2

Any two temperatures defined by the ratio

$$P_1 T_2 = P_2 T_1$$
 or  $V_1 T_2 = V_2 T_1$ 

 Can use this to calculate volume or pressure changes with temperature

### Example

- A bottle of hair spray is filled to a pressure of 1 atm at 20°C
- What is the canister pressure if it is placed into boiling water?

$$P_1 T_2 = P_2 T_1$$
  
 $1 \times 373 = P_2 \times 293$   
 $P_2 = 373/293$   
 $P_2 = 1.27 \text{ atm}$ 

### Absolute zero 1

- On the Kelvin scale, the lowest possible temperature is 0 K. (zero volume and zero pressure)
- The zero point is fixed -
  - Absolute Zero (≈ -273.15°C)
- Additional point defined at triple point of water (occurs at one temp and pressure where ice, steam and liquid all coexist (≈ 0.01°C and 0.006 atm)
- $T_{\text{triple}} = 273.16 \text{K}$
- $T = 273.16 \times (p/p_{triple})$

### Absolute zero 2

- Ideal gas has zero volume
- Resistance of metal drops to zero (superconductivity cuts in above 0K)
- Brownian motion ceases (kinetic energy due to thermal excitation ≈ 3/2 kT, see later)
- But lowest temperature yet attained in lab is ≈ 10<sup>-9</sup>K
- Very difficult to get really cold!!!

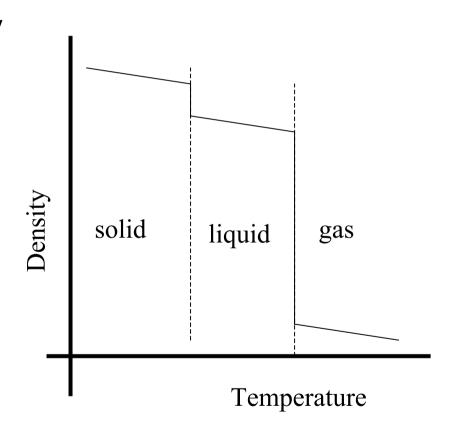
### Lord Kelvin

- William Thompson, born Belfast 1824
- Student in Natural Philosophy
- Professor at 22!
- Baron Kelvin of Largs in 1897
- Lived at 11 The Square
- A giant
  - Thermodynamics, Foams, Age of the Earth, Patents galore!



## Thermal expansion solid-liquidgas

Normally, density(ρ) changes as

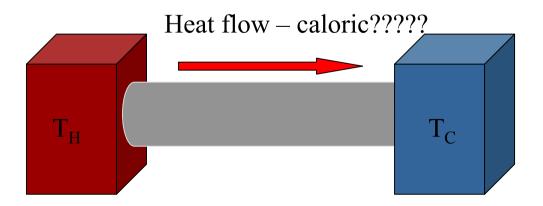


# How much energy required to heat object?

- Heat (energy) flows because of temperature difference
  - Bigger temperature difference bigger heat flow
  - Less insulation gives more heat flow for the same temperature difference
- Heat will not flow between two bodies of the same temperature

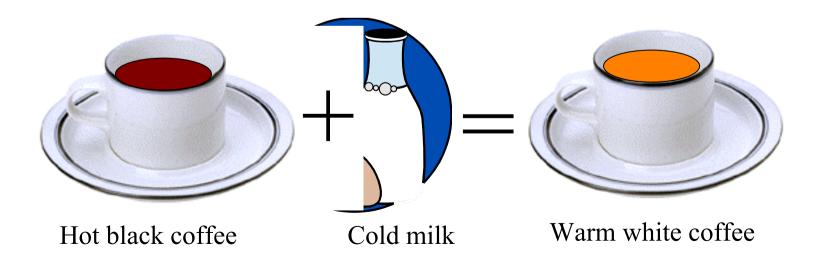
### What is heat?

- Heat flows from hot bodies to cold bodies until the temperature of the two is the same
- Early researchers thought it was a substance called caloric



### Equilibrium

Two objects of different temperature when placed in contact will reach the same temperature



### Sir James Joule

- James Joule 1818-1889
- Stirring water made it warm
  - Change in temperature proportional to work done
  - Showing equivalence of heat and energy
- Also that electrical current flow through a resistor gives heating



### Sir James Joule 2

- Heat and energy are the same thing
- Measured in Joules (J)
- Sometimes measured in calories
  - One cal raises one gram of water from 14.5°C to 15.5°C
  - \* 1 cal 4.186J



## 1st Law of Thermodynamics Heat transfer = energy transfer

- Doing work on something usually makes it hot
  - Splash in the bath and the water will get warmer!
- 1st law of thermodynamics heat and work are both forms of energy
- $\Delta U = Q W$
- Same as conservation of energy
  - Can neither be created nor destroyed, just changes form

## Thermal Physics – EE1

Lecture 3
Heat capacity and heat transfer
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## How quickly do things get hot?

- If you give heat to something
  - T increases
  - How fast?
  - Depends on the substance
  - \* Alcohol heats quicker than water
  - \* Also depends on how much you have of the substance

## Specific heat capacity

- $\blacksquare$  Q = mc  $\triangle$ T
  - Q is heat required
  - m is the mass of substance
- c is called the specific heat capacity
  - $\star$  c<sub>water</sub> = 4190 J kg<sup>-1</sup> K<sup>-1</sup> very difficult to heat
  - $c_{ice} = 2000 \text{ J kg}^{-1} \text{ K}^{-1}$
  - ★ c<sub>ethanol</sub> = 2428 J kg<sup>-1</sup> K<sup>-1</sup> easier to heat
  - \*  $c_{\text{mercury}} = 138 \text{ J kg}^{-1} \text{ K}^{-1}$  very easy to heat
- The higher c is, the more energy we need for heating

## Example – heat capacity

- "thrashing" around in the bath should heat up the water.
- How much will the water heat up after one minute of "thrashing"

Estimate volume of water  $\approx 0.5 \text{m}^3$ Estimate power of thrashing  $\approx 500 \text{W}$ 

 $\Delta T = Q/mc_{water}$   $\Delta T = 500 \times 60 /500 \times 4190$   $\Delta T = 0.015^{\circ}C$ 

### Reaching thermal equilibrium

- Total energy (heat) of a closed system is constant,  $\Delta Q_{coffee} = -\Delta Q_{milk}$  i.e  $\Sigma \Delta Q = 0$
- By convention heat flowing into a body ∆Q +ve



Hot black coffee at T<sub>H</sub>



Cold milk at T<sub>C</sub>



Warm white coffee at T<sub>w</sub>

$$(T_H - T_w)m_{coffee}c_{coffee} = -(T_c - T_w)m_{milk}c_{milk}$$

### Example 2 – heat capacity

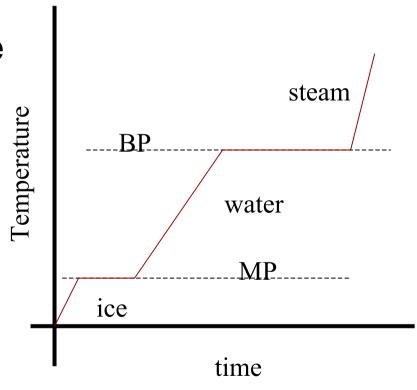
- A 2.5 kg steel bar is heated to 1000 °C
- It is then dropped into a 10 I tank of cold water at 10 °C (approx 10 kg water)
- What is the final temperature of the water?
- $c_{\text{steel}} = 420 \text{ J kg}^{-1} \text{ K}^{-1}$
- $c_{water} = 4190 \text{ J kg}^{-1} \text{ K}^{-1}$
- $\Delta T_{\text{steel}} / \Delta T_{\text{water}} = -39.9$
- $\Delta T_{water} \Delta T_{steel} = 990$
- $\Delta T_{water} + 39.9 \Delta T_{water} = 990$
- $\Delta T_{water} = 990/40.9 = 24.2 \, ^{\circ}C \implies T_{final} = 34.2 \, ^{\circ}C$
- $\Delta T_{\text{steel}} = -990/1.025 = 965.8 \, ^{\circ}\text{C} !!!!!!!$

### Molar heat capacity

- Quote Joules per mole rather than Joules per kilogram
- i.e.  $Q = nMc \Delta T$ 
  - n is the number of moles
  - Mc is the molar heat capacity (J mol<sup>-1</sup> K <sup>-1</sup>)
- Mc ≈ 25 J mol <sup>-1</sup> K <sup>-1</sup> for solids!
  - i.e. energy required to heat one atom of anything is about the same
  - Realised by Dulong and Petit

# Phase changes (e.g. solid to liquid)

- When heating ice into water and then into steam the temperature does not go up uniformly
  - Different gradients (c<sub>water</sub> > c<sub>ice</sub>)
  - Flat bits at phase changes
  - Need heat to convert
    - Solid to liquid
    - Liquid to vapour



# Energy required for phase change

- Heat of fusion (Q), solid -> liquid
  - $\bullet$  Q = mL<sub>f</sub> (L<sub>f</sub> is latent heat of fusion)
    - $L_{f \text{ (water)}} = 334 \text{ x} 10^3 \text{ J/kg}$
    - $L_{f \text{ (mercury)}} = 11.8 \times 10^3 \text{ J/kg}$
- Heat of vapourisation (Q), liquid -> gas
  - ♠ Q = mL<sub>v</sub> (L<sub>v</sub> is latent heat of vapourisation)
    - $L_{v \text{ (water)}} = 2256 \text{ x} 10^3 \text{ J/kg}$
    - $L_{v \text{ (mercury)}} = 272 \text{ x} 10^3 \text{ J/kg}$
- Heat of sublimation (Q), solid -> gas
  - $\bullet$  Q = mL<sub>s</sub> (L<sub>s</sub> is latent heat of sublimation)

### Other phase changes

- Magnetic transitions
  - Iron is paramagnetic at high temperature
    - Can be magnetised
    - Not permanently magnetised
  - Is ferromagnetic at lower temperatures
  - Change happens at the Curie Temperature
- Changes in crystal structure, ferroelectrics etc.
- May also have latent heats associated

## Using condensation to transfer energy

- Steam has two contributions to its stored thermal energy
  - The energy it took to heat it to 100°C
  - The energy it took turn it from water at 100°C to steam at 100°C
- Same idea with sweating
  - Sweat forms on the skin
  - It evaporates and this requires energy
  - Your skin gets cooler



Turning water into steam is a thermally efficient way of cooling things down

### Example

- If it takes 2 mins for your kettle to begin boiling how much longer does it take to boil dry?
  - Assume kettle is 3kW
  - Starting temp of water 20°C

```
Work done by kettle = power x time
= 2 \times 60 \times 3000 = 360000J
```

- = Work to boil water of mass M
  - $= \Delta T \times M \times C_{\text{water}}$
  - $= 80 \times M \times 4190 = 335200 M$

 $\rightarrow$  Mass of water = 1.07kg

Energy to boil water =  $M \times L_{v \text{ (water)}}$ = 1.07 x 2256 x10<sup>3</sup> = 2420 000J

Time required = Energy /power =  $2420\ 000/3000 = 808\ s \approx 13mins$ 

## Transferring heat energy

- 3 mechanisms
  - Conduction
    - Heat transfer through material
  - Convection
    - Heat transfer by movement of hot material
  - Radiation
    - Heat transfer by electromagnetic radiation (light, IR, etc.)

### Conduction of heat

- Conduction in solids
  - Heat energy causes atoms to vibrate, a vibrating atom passes this vibration to the next and so on
- Conduction in metal
  - Have free electron "gas"
  - Conduction electrons can move where they wish
  - Heat energy causes electrons to gain energy
  - This energy is rapidly spread out through entire free electron gas
    - Metals are good conductors of both heat and electricity

#### Rate of heat flow

- Heat flow (H) is energy transfer per unit time, depends on
  - Temperature difference
  - Thermal conductivity (κ)

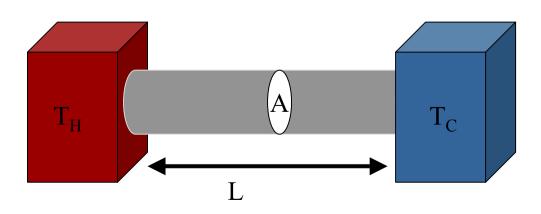
• 
$$k_{(copper)} = 385 \text{ W/(m K)}$$

• 
$$k_{(glass)} = 0.8 \text{ W/(m K)}$$

• 
$$k_{(air)} = 0.02 \text{ W/(m K)}$$

Thermal conductivity (
$$\kappa$$
)
•  $k_{\text{(copper)}} = 385 \text{ W/(m K)}$ 

$$H = \frac{dQ}{dt} = \kappa A \frac{T_H - T_C}{L}$$



### Example

- You poke a 1.2m long, 10mm dia. copper bar into molten lead
- How much heat energy flows through the bar to you?
  - Lead melts at 600K

Temperature difference along rod  $\Delta T = 600 - 311 = 289K$ 

$$H = k_{copper} A (\Delta T/L)$$

 $A=\pi \times r^2=3.142 \times 0.005^2=0.000078m^2$ 

$$H = k A (\Delta T/L) = 7.3$$
units?

Units =  $\{W/(mK)\}\ m^2 K / m = Watts$ 

## Thermal conduction vs thermal resistance

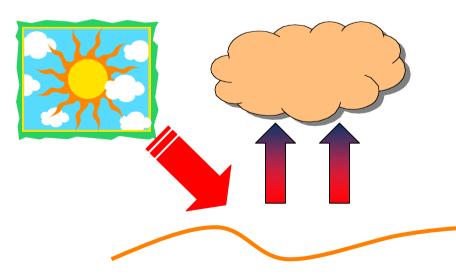
Can also use thermal resistance, R

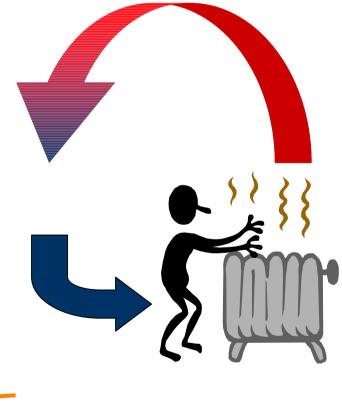
$$H = \frac{dQ}{dt} = \kappa A \frac{T_H - T_C}{L} = A \frac{T_H - T_C}{R}$$
 i.e.  $R = \frac{L}{\kappa}$ 

- R values often quoted for household insulation (in absurd imperial units!!)
- Can make equation of heat flow more general  $H = \frac{dQ}{dt} = \kappa A \frac{dT}{dx}$

### Convection of heat

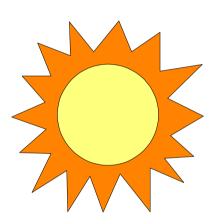
- "Hot air rises" (and takes its heat with it!)
  - Radiators
  - Cumulus clouds





### Radiation of heat

- Don't confuse with radioactivity
- Instead realise that EM radiation (light etc.) carries heat (e.g. the sun heats the earth)
- Anything above absolute zero radiates heat





## Not all things emit heat the same

Heat emission from an object of surface area A

- $H = Ae\sigma T^4$ 
  - $\sigma$  = Stafan's constant = 5.6x10<sup>-8</sup> W m<sup>-2</sup> K<sup>-4</sup>)
  - e = emissivity of a body, 0 -1
  - $e_{copper} = 0.3$
  - $e_{\text{charcoal}} \approx 1$

### Example

- Estimate the upper limit to the heat emission of the sun
  - Sun's surface temperature 6000k
  - ♣ Sun's radius 7x10<sup>8</sup>m

Emission,  $H = Ae\sigma T^4$ 

Area =  $4\pi r^2 = 6.2 \times 10^{18} \text{ m}^2$ 

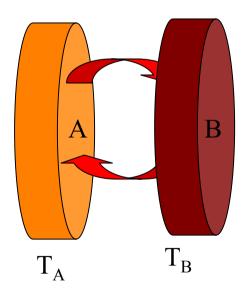
Emissivity  $\approx 1$ 

 $H = 6.2 \times 10^{18} \times 5.6 \times 10^{-8} \times 6000^{4}$ 

Sun's output =  $4.5 \times 10^{26} \text{ W}$ 

## Are heat emitters also good absorbers?

- Two bodies close
  - All heat emitted from A hits B
  - All heat emitted from B hits A
  - A is a perfect absorber & emitter
  - B emissivity e, absorptivity η
- B in thermal equilibrium with A, i.e. heat in = heat out
  - Area  $\eta_B \sigma T_A^4 = \text{Area } e_B \sigma T_B^4$
  - $T_A = T_B$  therefore  $e_B = \eta_B$



### The "colour" of heat

- Peak wavelength of EM radiation emitted depends on temperature
- Spectrum includes all wavelength longer than the peak but not many above
  - 20°C peak in infrared (need thermal imaging camera to see body heat)
  - \* 800°C peak in red (electric fire glows red)
  - 3000° peak in blue (but includes green and red light hence appears white)
  - 2.7K peak in microwave (background emission in the universe left over from the Big Bang)

## Thermal Physics – EE1

Lecture 4
Ideal gases
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### Equations of state

- State, identifies whether solid liquid or gas
- Key parameters or state variables
  - ♦ Volume, V (m³)
  - Pressure, P (N/m²)
  - Temperature, T (K)
  - Mass, M (kg) or number of moles, n
- Equation of state relates V, P, T, M or n

## Equation of state for a solid

- Increasing the temperature causes solid to expand
- Increasing the pressure causes solid to contract (0 subscript indicates initial value)
  - $V = V_0 [1 + \beta(T-T_0) k(p-p_0)]$ 
    - $\beta$  = thermal (volume) expansion coefficient
    - k = pressure induced volume expansion coefficient

# Amount of gas

- Better to describe gas in terms of number of moles (we shall see that all gases act the same!)
- Mass, m related to number of moles, n
  - \*m = nM
    - M = molecular mass (g/mole, 1mole = 6x10<sup>23</sup> atoms or molecules

# Equation of state for a gas

- All gases behave nearly the same
  - **\*** pV = *n*RT
    - R = 8.3 J mol<sup>-1</sup> K <sup>-1</sup>) for <u>all</u> gases (as long as they remain a gas)
    - T is in K!!!!!!
- Re-express
  - pV = (m/M) RT
- Density  $\rho$  = (m/V)
  - $\rho = pM/RT$

## Example

- What is the mass of a cubic metre of air?
  - Molecular weight of air ≈ 32 g

$$pV = nRT$$

Atmospheric pressure = 10<sup>5</sup> Pa Atmospheric temp. = 300 K

For a volume of 1 m<sup>3</sup>

$$n = pV/RT = 10^5 / (8.3 \times 300)$$
  
= 40 moles

$$M = 40 \times 0.032 = 1.3 \text{ kg}$$

## Constant mass of gas

- For a fixed amount of gas, its mass or number of moles remains the same
  - **♦** pV/T = nR = constant
- Comparing the same gas under different conditions
  - $p_1V_1/T_1 = p_2V_2/T_2$ 
    - Hence can use pressure of a constant volume of gas to define temperature (works even if gas is impure - since all gases the same)
    - Must use T in K!!!!!!

## Example

- A hot air balloon has a volume of 150m<sup>3</sup>
- If heated from 20°C to 60°C how much lighter does it get?
  - Molecular weight of air ≈32 g mol<sup>-1</sup>

$$pV/T = nR$$
  
 $n = pV/RT$ 

Balloon has constant volume and constant pressure

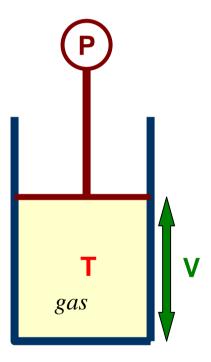
$$n_{cool} = 10^5 x 150 / (8.3 x 293) = 6168$$

$$n_{hot} = 10^5 x 150 / (8.3 x 333) = 5427.1$$
  
 $\Delta n = 7409 \text{ moles}$ 

$$\Delta M = 740.9 \times 0.032 = 23.7 \text{kg}$$

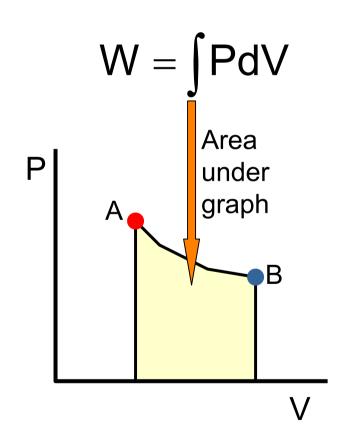
# Work on and by gases

- Compress
  - **\*** V → V-dV
  - Work done on the gas by the piston
  - W is -ve
- Expand
  - $*V \rightarrow V+dV$
  - Work done on the piston by the gas
  - \* W is +ve



# Work done on/by gas

- $\blacksquare$  dW = F dx
- P = F/A
- $\blacksquare$  dW = P Adx
- $\blacksquare$  Adx = dV
- dW = PdV
- P = nRT/V so P depends on V

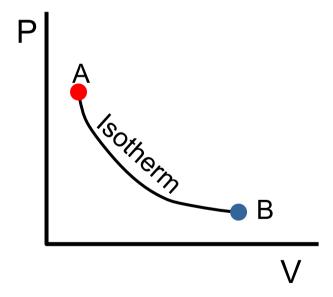


# Change of State processes

- How to get "there" from "here"
- Isothermal
  - Same temperature
- Isobaric
  - Same pressure
- Isovolumetric
  - Same volume
- Can relate to first law of thermodynamics
  - **★** ∆U=Q-W

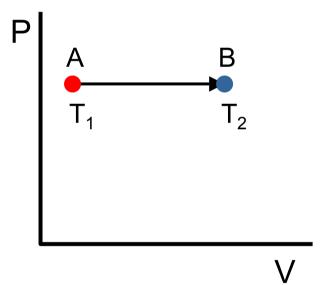
#### Isothermal

- Iso same
- Thermal temperature
- Pressure and volume change inversely
- PV = const
  - Boyle's law
- For ideal gas, if T is constant, U is constant
- $\Delta U=0=Q-W \Rightarrow Q=W$
- Heat input = Work done



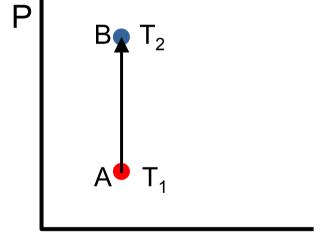
#### Isobaric

- Iso same
- Baric pressure
   V increases with T or vice versa



#### Isovolumetric

- Iso Same
- Volumetric volume
- As P increases, T increases
- $V_1 = V_2$   $W = \int PdV = 0$   $\Delta U = Q$
- All heat converted to internal energy

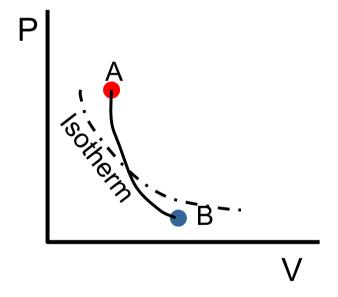


#### Adiabatic

- A not
- Dia through
- Batic passable
- i.e. No heat flow
- $\mathbf{Q} = \mathbf{Q} \Rightarrow \Delta \mathbf{U} = -\mathbf{W}$
- Process occurs fast

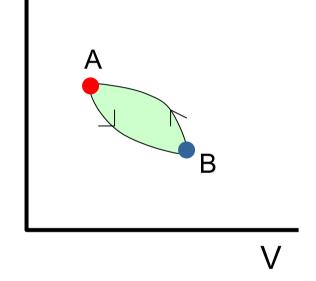
or

- Container is well insulated
- Adiabats obey PV<sup>γ</sup> = const.



# Cyclic processes

- Go from one state (point) to another and return by different route
- Net work: area of cycle (shaded)



- Quasistatic process
  - Slow change in state variables P, V, T

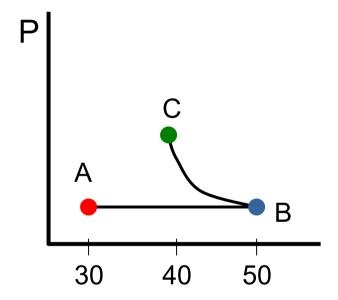
## Example

- Ideal gas expands isobarically at P = 120 kPa from A to B
- It is then compressed isothermally from B to C
- Find the work done

$$W = \int PdV$$

A to B: P const

$$W = P \int dV = P(V_2 - V_1)$$
  
= 120x10<sup>3</sup> x(50 - 30)x10<sup>-3</sup> = 2400J



## Example continued

B-C: T constant

$$W = \int_{V_1}^{V_2} PdV = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$
$$= nRT(\ln V_2 - \ln V_1)$$

C A B B 30 40 50

At B

$$\begin{aligned} PV &= 120x10^3x50x10^{-3} = 6000J = nRT \\ W &= 6000(ln\,V_2 - ln\,V_1) = 6000\,ln\frac{V_2}{V_1} = -1339J \\ W_{total} &= 2400 - 1339 = 1061J \quad \text{Done by the gas} \end{aligned}$$