ME 417
Design of Alternative Energy Systems

Wind Energy Problems Solutions

1. A downtown city terrain is under consideration for a wind farm. At 15 meters the wind velocity is found to be 4.3 m/s. Develop a graph of wind power density versus height for this locale.

Solution:

By definition we have

$$\dot{w}'' = \frac{\dot{W}}{A} = \frac{1}{2}\rho\vec{v}^3$$

Hence to graph the wind power density against height, we need to determine the velocity at different heights. Recall that we have the relationship

$$\vec{v}_2 = \vec{v}_1\left(\frac{h_2}{h_1}\right)^n$$

where for a downtown city terrain we have n = 0.4. Then we could write

$$\vec{v} = (4.3)\left(\frac{h}{15}\right)^{0.4}$$

For the air density we will use the ideal gas law

$$\rho = \frac{P}{RT}$$

We will assume that we are at standard conditions, so that

$$\rho = \frac{101}{(8.314/29)(298)} = 1.18\ \text{kg/m}^3$$

So we can now go to Excel and make the following table

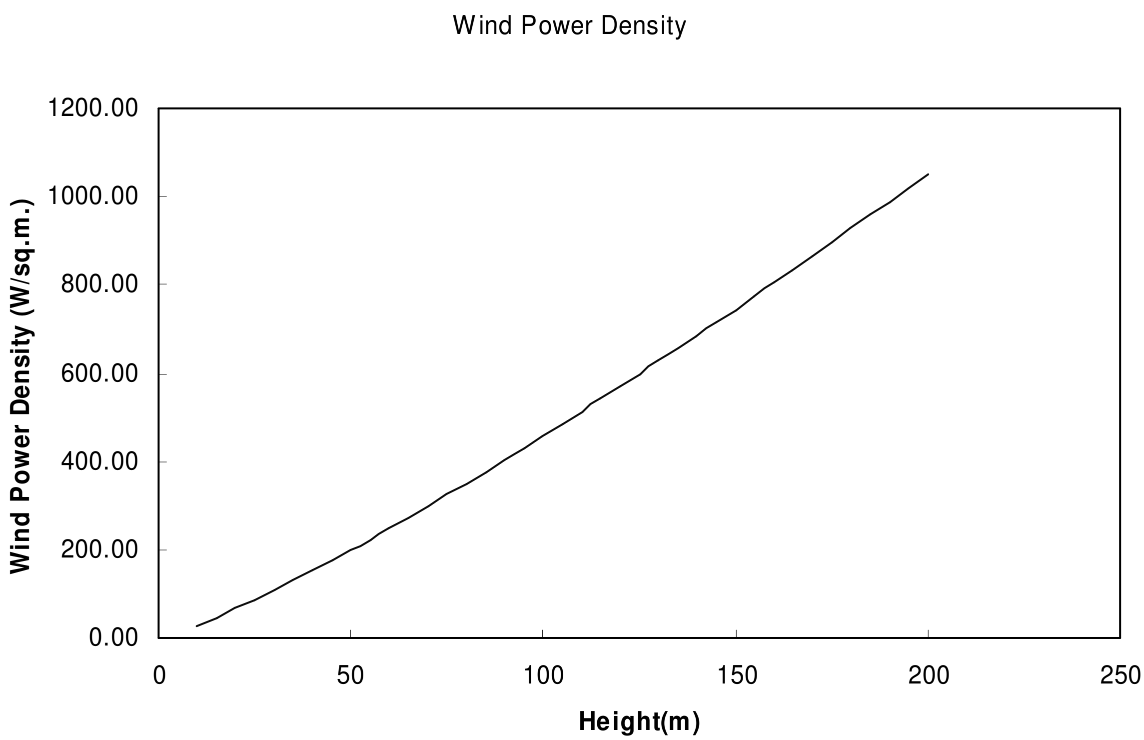
h (m)	$\vec{v}$ (m/s)	$\dot{w}''$ (W/m^2)
10	3.66	28.89
15	4.30	47.00
20	4.82	66.37
25	5.27	86.75
30	5.67	107.97
35	6.03	129.91
40	6.37	152.49
45	6.67	175.64
50	6.96	199.31

h (m)	$\vec{v}$ (m/s)	$\dot{w}''$ (W/m^2)
55	7.23	223.46
60	7.49	248.05
65	7.73	273.06
70	7.96	298.45
75	8.19	324.21
80	8.40	350.32
85	8.61	376.76
90	8.80	403.51
95	9.00	430.55

h (m)	$\vec{v}$ (m/s)	$\dot{w}''$ (W/m^2)
100	9.18	457.89
105	9.36	485.50
110	9.54	513.37
115	9.71	541.50
120	9.88	569.87
125	10.04	598.48
130	10.20	627.32
135	10.36	656.39
140	10.51	685.67
145	10.66	715.16
150	10.80	744.85

h (m)	$\vec{v}$ (m/s)	$\dot{w}''$ (W/m^2)
155	10.94	774.74
160	11.08	804.83
165	11.22	835.10
170	11.36	865.56
175	11.49	896.20
180	11.62	927.01
185	11.75	958.00
190	11.87	989.15
195	12.00	1020.47
200	12.12	1051.95

which gives the following graph



2. Determine the power generated from a Darrieus wind turbine of rotor diameter 4 meters, blade length of 7 meters, and speed of 180 rpm in a wind velocity of 6 m/s.

Solution:

The power will be given by

$$\dot{W}_{wt} = \eta_{wt} \frac{1}{2} \rho A \vec{v}^3$$

where

$$\rho = 1.18 \text{ kg/m}^3$$
$$A = D_{rotor}L_{blade} = (4)(7) = 28 \text{ m}^2$$
$$\vec{v} = 6 \text{ m/s}$$

The efficiency will come from our tip speed ratio (TSR) equation. Then

$$\text{TSR} = \frac{\omega R_{rotor}}{\vec{v}} = \frac{(180/60)(2\pi)(4/2)}{6} = 6.28$$

Our appropriate efficiency equation is

$$\eta_{wt} = -0.078369(\text{TSR})^2 + 0.92146(\text{TSR}) - 2.3532$$

which gives an efficiency of 0.34. Then substituting

$$\dot{W}_{wt} = (0.34)\frac{1}{2}(1.18)(28)(6)^3 = 1.092 \text{ kW}$$

3. A modern three blade wind turbine with fiberglass (ρ= 1200 kg/m^3) blades of length 50 m, width 0.3 m, and thickness of 0.05 m is powered by 20°C air with a velocity of 7.5 m/s. Determine

   (a) power generated
   (b) turbine speed in rpm

Solution:

The power generated will be given by

$$\dot{W}_{wt} = \eta_{wt}\frac{1}{2}\rho A\vec{v}^3$$

To determine the efficiency we must know the tip speed ratio (TSR)

$$\text{TSR} = \frac{\omega R_{rotor}}{\vec{v}}$$

This requires us to know the rotational speed of the turbine, which means we must solve the problem iteratively using the following three equations:

$$\eta_{wt}\frac{1}{2}\rho A\vec{v}^3 = \frac{1}{2}I_{shaft}\omega^3$$

$$\text{TSR} = \frac{\omega R_{rotor}}{\vec{v}}$$

$$\eta_{wt} = -0.020554(\text{TSR})^2 + 0.18327(\text{TSR}) + 0.023286$$

with the following iterative loop

1. Guess η_{wt}

2. Calculate ω from $\omega = \left\{ \dfrac{\eta_{wt} \rho A \vec{v}^3}{I_{shaft}} \right\}^{1/3}$

3. Calculate TSR from $TSR = \dfrac{\omega R_{rotor}}{\vec{v}}$

4. Calculate η_{wt} from $\eta_{wt} = -0.020554(TSR)^2 + 0.18327(TSR) + 0.023286$

5. Check η_{wt} for convergence (say less than 1% change)

6. Repeat steps 2-5 until convergence is achieved, using η_{wt} calculated in 5 as the guess for each iterative loop.

Before we begin the iteration; let's calculate some of the parameters that do not change during the iteration.

$$\rho = \frac{P}{RT} = \frac{101}{(8.314/29)(293)} = 1.20\,\text{kg/m}^3$$

$$A = \pi L_{blade}^2 = \pi(50)^2 = 7854\,\text{m}^2$$

$$I_{shaft} = \frac{N_B \rho_B (L_B W_B t_B) L_B^2}{3} = \frac{(3)(1200)(50)(0.3)(0.05)(50)^2}{3}$$

$$= 2.25 \text{ x } 10^6 \text{ kg}\cdot\text{m}^2$$

Now carrying out our iteration

1. Guess $\eta_{wt} = 0.30$

2. Calculate $\omega = \left\{ \dfrac{\eta_{wt} \rho A \vec{v}^3}{I_{shaft}} \right\}^{1/3} = \left\{ \dfrac{\eta_{wt}(1.2)(7854)(7.5)^3}{2.25 \times 10^6} \right\}^{1/3} = (1.7672\eta_{wt})^{1/3}$

$= 0.809\,\text{rad/sec}$

3. Calculate TSR from $TSR = \dfrac{\omega R_{rotor}}{\vec{v}} = \dfrac{(0.809)(50)}{7.5} = 5.39$

4. Calculate $\eta_{wt} = -0.020554(5.39)^2 + 0.18327(5.39) + 0.023286 = 0.414$

5. $\Delta = |0.414 - 0.30| = 0.114$

Must continue!

Loop 2

1. Guess $\eta_{wt} = 0.414$
2. Calculate $\omega == (1.7672\eta_{wt})^{1/3} = 0.901 \text{rad/sec}$
3. Calculate TSR from $\text{TSR} = \dfrac{(0.901)(50)}{7.5} = 6.01$
4. Calculate $\eta_{wt} = -0.020554(6.01)^2 + 0.18327(6.01) + 0.023286 = 0.382$
5. $\Delta = |0.382\text{-}0.414| = 0.032$

Must continue!

Loop 3

1. Guess $\eta_{wt} = 0.382$
2. Calculate $\omega == (1.7672\eta_{wt})^{1/3} = 0.877 \text{rad/sec}$
3. Calculate TSR from $\text{TSR} = \dfrac{(0.877)(50)}{7.5} = 5.85$
4. Calculate $\eta_{wt} = -0.020554(5.85)^2 + 0.18327(5.85) + 0.023286 = 0.392$
5. $\Delta = |0.392\text{-}0.382| = 0.010$

Close enough!

Then

$$\dot{W}_{wt} = \eta_{wt} \frac{1}{2} \rho A \vec{v}^3 = (0.392)(0.5)(1.2)(7854)(7.5)^3 = 779 \text{ kW}$$