The Design of PID Controllers using Ziegler Nichols Tuning

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1. Introduction

PID controllers are probably the most commonly used controller structures in industry. They do, however, present some challenges to control and instrumentation engineers in the aspect of **tuning** of the gains required for stability and good transient performance. There are several prescriptive rules used in PID tuning. An example is that proposed by Ziegler and Nichols in the 1940's and described in Section 3 of this note. These rules are by and large based on certain assumed models.

2. PID Controller Structure

The PID controller encapsulates three of the most important controller structures in a single package. The parallel form of a PID controller (see Figure 1) has transfer function:

$$\begin{aligned} C(s) &= K_p + \frac{K_i}{s} + K_d s \\ &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \end{aligned} \tag{1}$$

where: K_p := Proportional Gain

K_i := Integral Gain — T_i := Reset Time $= K_p/K_i$

K_d :=Derivative gain — T_d := Rate time or derivative time

Note these definitions!

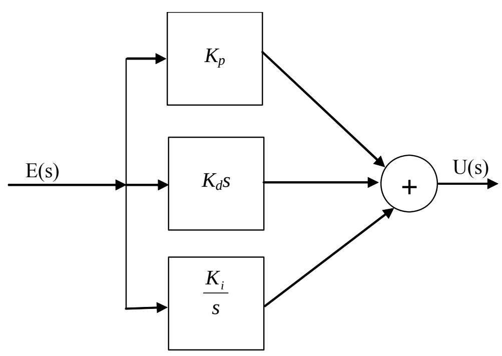


Figure 1: Parallel Form of the PID Compensator

The **proportional term** in the controller generally helps in establishing system stability and improving the transient response while the **derivative term** is often used when it is necessary to improve the closed loop response speed even further. Conceptually the effect of the derivative term is to feed information on the rate of change of the measured variable into the controller action.

The most important **term** in the controller is the **integrator term** that introduces a pole at $s = 0$ in the forward loop of the process. This makes the compensated[1] open loop system (i.e. original system plus PID controller) a type 1 system at least; our knowledge of steady state errors tells us that such systems are required for perfect steady state setpoint tracking. This is more formally stated in the following theorem:

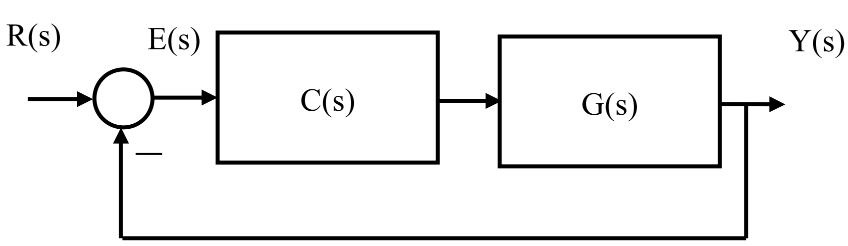


Figure 2

Theorem 1

For the unity feedback system of Figure 2, perfect setpoint control can only be achieved for the controller, C(s), if and only if

1. The open loop forward gain has a steady state gain of infinity i.e.

[1] The PID controller is often considered as one member of a family of **compensators** i.e., devices that can be added to an open loop system to change (compensate) for characteristics which make the achievement of the control objective difficult or impossible. As is the case for the PID controller, compensators are usually cascaded to the input of an existing plant before feedback is applied. This introduces a new set of poles and/or zeros to the picture. Control design proceeds by treating the cascaded pair as the new system to be controlled.

$$\lim_{s \to 0} C(s)G(s) = \infty$$

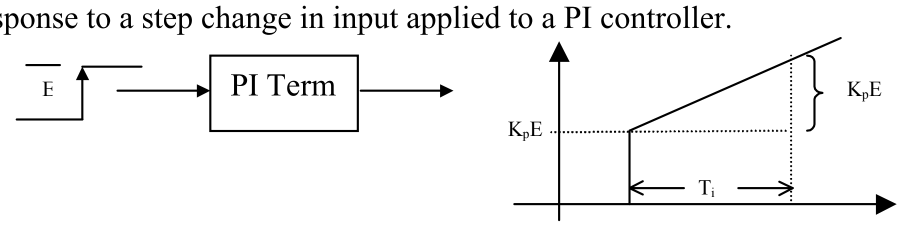
2. *The system is closed loop stable*
3. *Neither C(s) nor G(s) have zeros at the origin*

□

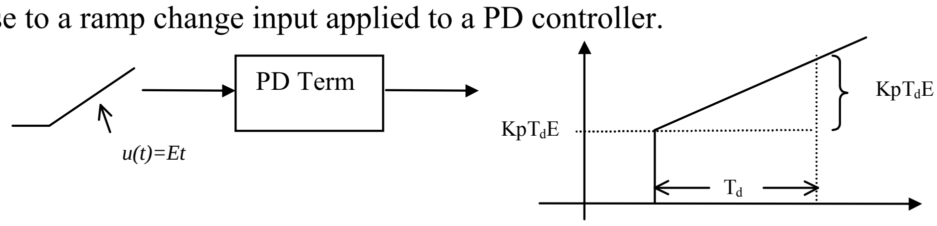
The final condition of the theorem eliminates the possibility of the loop transfer function cancelling the effect of the integrator pole.

Both forms of (1) are used with the second being the most common. The reset and rate times are of special significance in this regard:

- The **reset time** is the time taken for the integrator term output to equal the proportional term output in response to a step change in input applied to a PI controller.



- The **rate time** is the time taken for the proportional term output to equal the derivative term output in response to a ramp change input applied to a PD controller.



In addition, the Proportional gain, K_p, is often expressed as a proportional band (%):

$$PB = \frac{1}{K_p} \qquad (2)$$

PB is actually the fractional error change, relative to the error range, required to produce a 100% (full range) change in the proportional term output.

Mathematically, we have:

$$\left.\begin{aligned} K_p &= \frac{\Delta o}{\Delta e} = \frac{\text{Output Change}}{\text{Input Change}} \\ &= \frac{\Delta o / \Delta o_{\max}}{\Delta e / \Delta e_{\max}} \text{ x } \frac{\Delta o_{\max}}{\Delta e_{\max}} \\ &= \frac{1}{PB} \text{ x } \frac{\Delta o_{\max}}{\Delta e_{\max}} \leftarrow \text{This last term is usually} = 1. \\ &= \frac{1}{PB} \end{aligned}\right\} \qquad (3)$$

In practice PB is expressed as a percentage so

$$PB\% = \frac{100}{K_p} \qquad (4)$$

Thus a PB of 5% ⇔ Kp=20. We also note that:

1. The industry jargon is clearly more practical and useful in this case. The concept of gain, usually quite useful in analysis, is generally harder to grasp than the degree of signal variation (absolute or relative) required to obtain full output swing.

2. In addition, most controllers operate on relative (percentage or per unit) units. In this regard, quantities are scaled relative to their maximum range. This makes it easy to translate from one unit basis to another. For example, if our PV is a temperature in the range 0^o - 100^o mapped to a current range of *4-20*mA, then a PV *20%* translates to an absolute temperature of 20^o and an equivalent current signal reading of *4+ 16*.2=7.2*mA. NB: If analysis is performed on any system the units used will determine the gains in the various transfer boxes.

3. Popular variations of (1) are used to generate the following controllers:

Controller Type	K_p	K_i	K_d	$C(s)$
P (Proportional)	≠0	zero	zero	K_p
I (Integral)	zero	≠0	zero	$\frac{K_i}{s}$
PI (Proportional plus Integral)	≠0	≠0	zero	$\frac{K_p s + K_i}{s} = \frac{K_p\left(s + \frac{K_i}{K_p}\right)}{s}$
PD (Proportional plus Derivative)	≠0	zero	≠0	$K_d s + K_p = K_d(s + \frac{K_p}{K_d})$
PID (Proportional + Integral + Derivative)	≠0	≠0	≠0	$\frac{K_d s^2 + K_p s + K_i}{s} = \frac{K_d\left(s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}\right)}{s}$

Table 1: Common PID Controller Variations

The actual variation used depends on the specifications to be met. Purely derivative or integral plus derivative variations are almost never used. In all cases except proportional control, the PID compensator introduces one pole and at least one zero.

3. Ziegler-Nichols Tuning

In 1942 Ziegler and Nichols, both employees of Taylor Instruments, described simple mathematical procedures, the first and second methods respectively, for tuning PID controllers. These procedures are now accepted as standard in control systems practice. Both techniques make a priori assumptions on the system model, but do not require that these models be specifically known. Ziegler-Nichols formulae for specifying the controllers are based on plant step responses.

3.1 The First Method

The first method is applied to plants with step responses of the form displayed in Figure 4. This type of response is typical of a first order system with transportation delay, such as that induced by fluid flow from a tank along a pipe line. It is also typical of a plant made up of a series of first order systems. The response is characterised by two parameters, L the delay time and T the time constant. These are found by drawing a tangent to the step response at its point of inflection and noting its intersections with the time axis and the steady state value. The plant model is therefore

$$G(s) = \frac{Ke^{-sL}}{Ts+1} \tag{5}$$

Ziegler and Nichols derived the following control parameters based on this model:

PID Type	K_p	$T_i=K_p/K_i$	$T_d=K_d/K_p$
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Table 2: Ziegler-Nichols Recipe – First Method

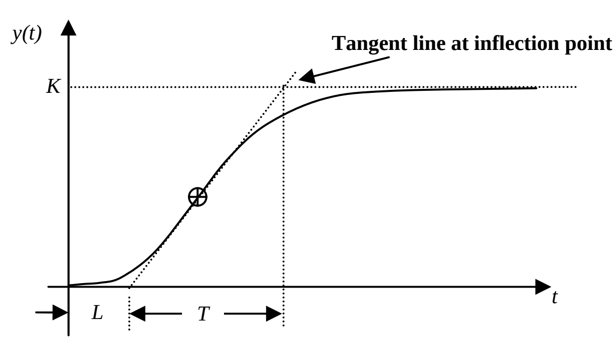


Figure 4: Response Curve for Ziegler-Nichols First Method

It should be noted that the response curve of Figure 4 is also typical of overdamped second order systems.

3.2 Second Method

The second method targets plants that can be rendered unstable under proportional control. The technique is designed to result in a closed loop system with 25% overshoot. This is rarely achieved as Ziegler and Nichols determined the adjustments based on a specific plant model.

Note: This is just like the Routh-Hurwitz stability range problems we studied

The steps for tuning a PID controller via the 2nd method is as follows:

Using only proportional feedback control:

1. Reduce the integrator and derivative gains to 0.
2. Increase K_p from 0 to some critical value $K_p=K_{cr}$ at which sustained oscillations occur. If it does not occur then another method has to be applied.
3. Note the value K_{cr} and the corresponding period of sustained oscillation, P_{cr}

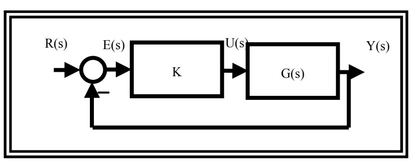
The controller gains are now specified as follows:

PID Type	K_p	T_i	T_d
P	$0.5\,K_{cr}$	∞	0
PI	$0.45\,K_{cr}$	$\frac{P_{cr}}{1.2}$	0
PID	$0.6\,K_{cr}$	$\frac{P_{cr}}{2}$	$\frac{P_{cr}}{8}$

Table 4: Ziegler Nichols Recipe – Second Method

3.3 An Example

The second method lends itself to both experimental and analytical study. Consider a process with transfer function $G(s)=\frac{1}{(s+1)(s+3)(s+5)}$ that is to be placed under PID control. We can determine the limiting gain for stability (before oscillations) by use of the Routh-Hurwitz condition. The characteristic equation, *p(s)*, with Proportional control is:



$$\left.\begin{aligned} &1+KG(s)=0 \Leftrightarrow \\ &(s+1)(s+3)(s+5)+K=0 \Leftrightarrow \\ &p(s)=s^3+9s^2+23s+15+K=0 \end{aligned}\right\} \quad (6)$$

The corresponding Routh array is

$$\begin{array}{llll} s^3 & 1 & 23 & 0 \\ s^2 & 9 & 15+K & 0 \\ s^1 & 192-K & 0 & \\ s^0 & 15+K & & \end{array}$$

From this we see that the range of K for stability is $15+K>0 \;=> K>-15$ and $192-K>0 \;=> K<192$. **So K_{cr}=192.** When K = 192, we have imaginary roots since the s^1 row is identically 0. The corresponding auxiliary equation is

$$9s^2+15+192=0 \quad (7)$$

with roots at s = ±j4.8. Since this is a quadratic factor of the characteristic polynomial => the sustained oscillation at the limiting value of K, K_{cr}, is at 4.8rad/s. Thus, P_{cr} = 1.31sec and K_{cr} = 192. This gives for full PID control from the table as $K_p = 0.6K_{cr} = 115.2$; $K_i = 2K_p/P_{cr} = 177.2$; $K_d = K_pT_d = K_p/8\,P_{cr} = 18.3$.

Analysis: The closed loop step response shows an overshoot performance of 50%, 100% over target. Given the dependence of the technique on a generic model, it is not surprising that the design objectives will almost always not be met. The technique, however, does provide an effective starting point for controller tuning.

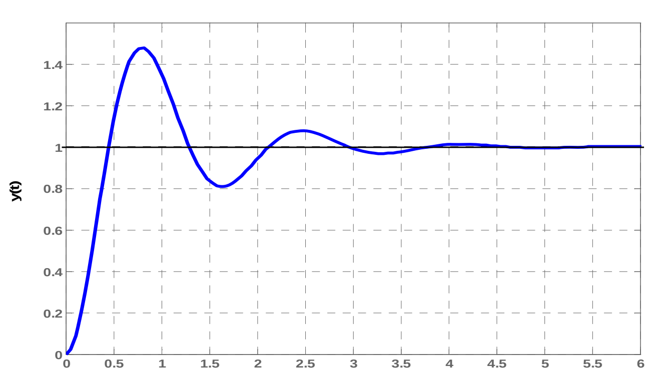


Figure 5: Step Response for System Tuned via the Second Method