Operational Amplifier Stability
Part 1 of 15: Loop Stability Basics

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Wherever possible the overall technique used for this series will be "definition by example" with generic formulae included for use in other applications. To make stability analysis easy we will use more than one tool from our toolbox with *data sheet information*, *tricks*, *rules-of-thumb*, *SPICE Simulation*, and *real-world testing* all accelerating our design of stable operational amplifier (op amp) circuits. These tools are specifically targeted at voltage feedback op amps with unity-gain bandwidths <20 MHz, although many of the techniques are applicable to any voltage feedback op amp. 20 MHz is chosen because as we increase to higher bandwidth circuits there are other major factors in closing the loop: such as parasitic capacitances on PCBs, parasitic inductances in capacitors, parasitic inductances and capacitances in resistors, etc. Most of the rules-of-thumb and techniques were developed not just from theory but from the actual building of real-world circuits with op amps <20 MHz.

This first part reviews some fundamentals essential to ease of stability analysis and defines some nomenclature which will be used consistently throughout the entire series.

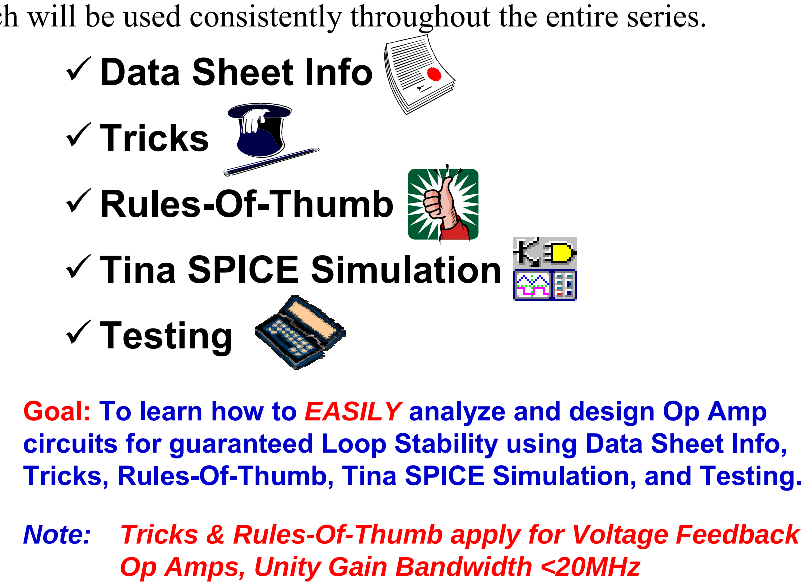


Fig. 1.0: Stability Analysis Toolbox

Bode Plot Basics

The frequency response for the magnitude plot is the change in voltage gain as frequency changes, specified on a Bode plot as voltage gain in dB vs frequency in Hz. Bode and plotted semi-log with frequency (Hz) on the x-axis, log scale, and voltage gain (dB) on the y-axis, linear scale. Preferred y-axis scaling is a convenient 20 dB per major division. The other half of the Bode plot is the phase plot (phase shift vs frequency) and is plotted as degrees phase shift vs frequency. Bode phase plots are semi-log with frequency (Hz) on the x-axis, log scale, and phase shift (degrees) on the y-axis, linear scale. Preferred y-axis scaling is a convenient 45° degrees per major division.

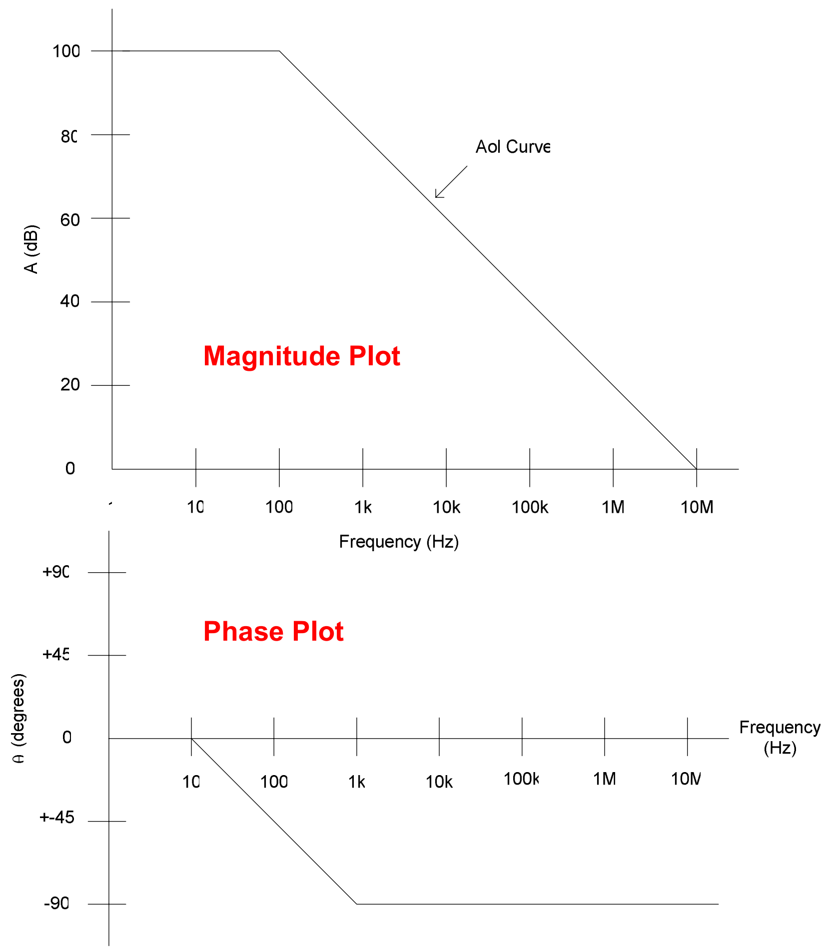


Fig. 1.1: Magnitude And Phase Bode Plots

Magnitude Bode Plots require voltage gain to be converted to dB, defined as $20Log_{10}10A$, where A is the voltage gain in volts/volts (V/V).

dB → A(dB) = $20Log_{10}A$ where A = Voltage Gain in V/V

A (V/V)	A (dB)
0.001	-60
0.01	-40
0.1	-20
1	0
10	20
100	40
1,000	60
10,000	80
100,000	100
1,000,000	120
10,000,000	140

Fig. 1.2: dB Definition For Magnitude Bode Plots

Fig. 1.3 defines some commonly-used Bode plot terms.

- Roll-Off Rate → Decrease in gain with frequency
- Decade → x10 increase or x1/10 decrease in frequency. From 10Hz to 100Hz is one decade.
- Octave → X2 increase or x1/2 decrease in frequency. From 10Hz to 20Hz is one octave.

Fig. 1.3: More Bode Plot Definitions

The slope of voltage gain with frequency is defined in +20 dB/decade or -20 dB/decade increments on a magnitude Bode plot. They can also be described as +6 dB/octave or -6 dB/octave (see Fig. 1.4) which can be proved by:

$\Delta A\ (dB) = A\ (dB)\ at\ fb - A\ (dB)\ at\ fa$

$\Delta A\ (dB) = [Aol\ (dB) - 20\log_{10}(fb/f1)] - [Aol\ (dB) - 20\log_{10}(fa/f1)]$

$\Delta A\ (dB) = Aol\ (dB) - 20\log_{10}(fb/f1) - Aol\ (dB) + 20\log_{10}(fa/f1)$

$\Delta A\ (dB) = 20\log_{10}(fa/f1) - 20\mathrm{Log}_{10}(fb/f1) = 20\log_{10}(fa/fb) = 20\log_{10}(1\ k/10\ k)$

so, $\underline{\Delta A\ (dB) = -20\ dB/decade}$

Also, $\Delta A\ (dB) = 20\log_{10}(fb/fc) = 20\log_{10}(10\ k/20\ k)$

so, $\underline{\Delta A\ (dB) = -6\ dB/octave}$

That is, -20 dB/decade = -6 dB/octave

And also: +20 dB/decade = +6 dB/octave; -20dB/decade = -6dB/octave
And: +40 dB/decade = +12 dB/octave; -40 dB/decade = -12 dB/octave
And: +60 dB/decade = +18 dB/octave; -60 dB/decade = -18 dB/octave

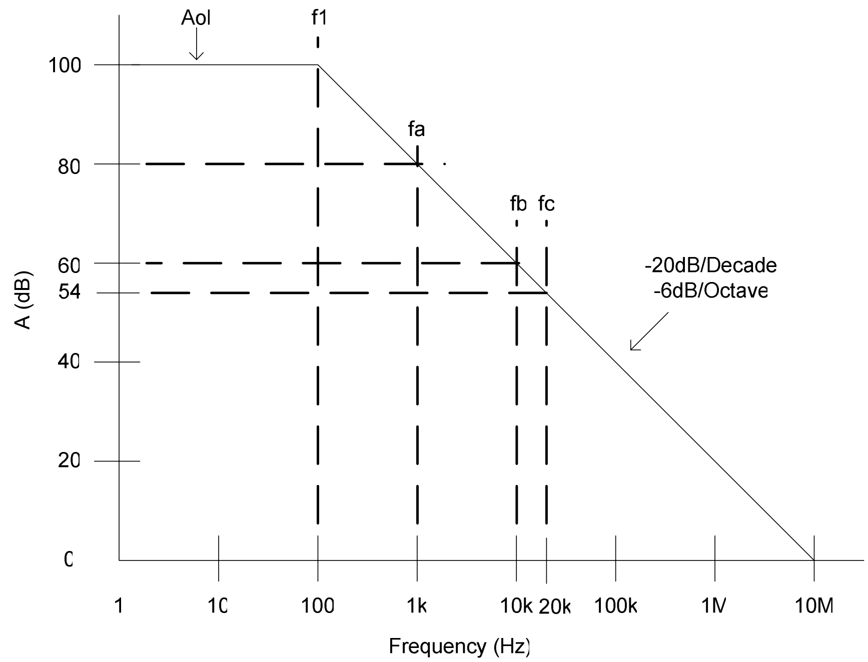


Fig. 1.4: Magnitude Bode Plot: 20 dB/Decade = 6 dB/Octave

Pole

A single pole response has a -20 dB/decade, -6 dB/octave rolloff in the Bode magnitude plot. At its location (fP) the gain is reduced by 3dB from the dc value. In the phase plot the pole has a -45° phase shift at fP. The phase extends on either side of fP to 0° and -90° at a -45°/decade slope. A single pole may be represented by a simple RC low pass network as shown in Fig. 1.5. Note how the phase of a pole affects frequencies up to one decade above and one decade below the pole frequency.

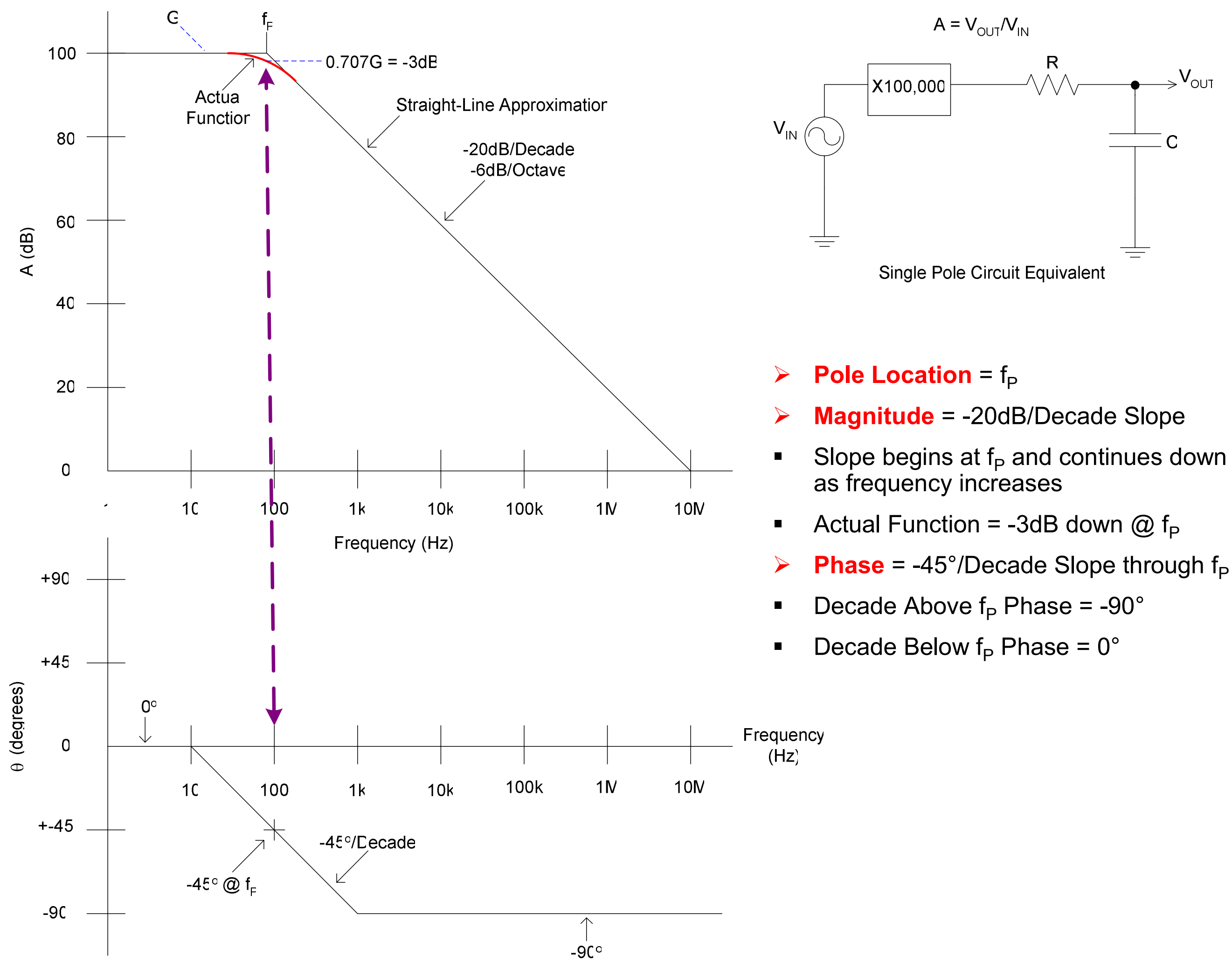
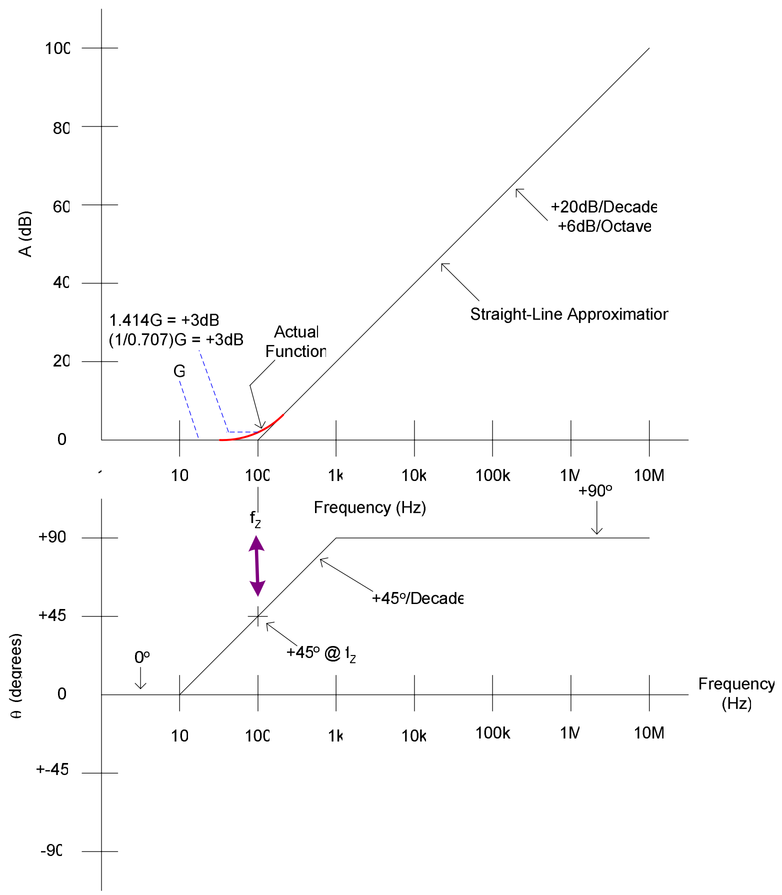
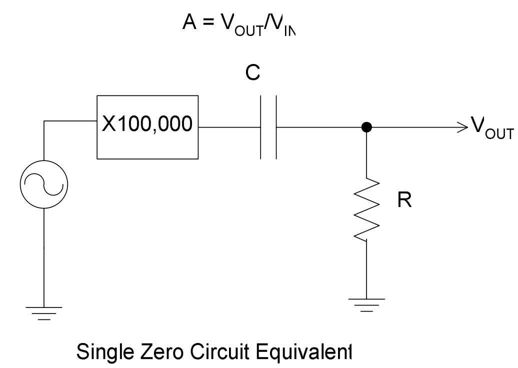


Fig. 1.5: Poles: Bode Plot Magnitude and Phase

Zero

A single zero response has a +20 dB/decade, +6 dB/octave "roll-up" in the Bode magnitude plot. At the zero location (fZ) the gain is increased by 3 dB from the dc value. In the phase plot the zero has a +45° phase shift at fZ. The phase extends on either side of fZ to 0° and +90° at a +45°/decade slope. A single zero may be represented by a simple RC high pass network (Fig. 1.6). Note how the phase of a zero affects frequencies up to one decade above and one decade below the zero frequency.

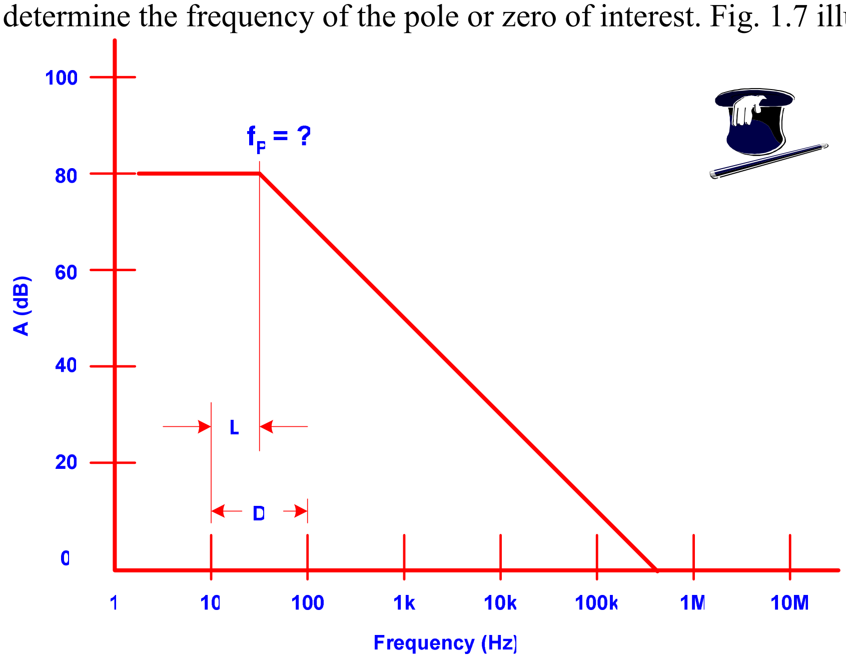




- **Zero Location** = f_Z
- **Magnitude** = +20dB/Decade Slope
  - Slope begins at f_Z and continues up as frequency increases
  - Actual Function = +3dB up @ f_Z
- **Phase** = +45°/Decade Slope through f_Z
  - Decade Above f_Z Phase = +90°
  - Decade Below f_Z Phase = 0°

Fig. 1.6: Zeros: Bode Plot Magnitude and Phase

On a Bode magnitude plot it is easy to find the frequency location of a given pole or zero. Since the x-axis is a log scale of frequency this technique allows a ratio of distances to accurately and quickly determine the frequency of the pole or zero of interest. Fig. 1.7 illustrates this "Log Scale Trick."



Log Scale Trick (f_P = ?):

1) Given: L = 1cm; D = 2cm
2) $L/D = \text{Log}_{10}(f_P)$
3) $\mathbf{f_P = Log_{10}^{-1}(L/D) = 10^{(L/D)}}$
   $\mathbf{f_P = 10^{(L/D)} = 10^{(1cm/2cm)} = 3.16}$
4) Adjust for the decade range working within – 10Hz-100Hz decade → f_P = 31.6Hz
5) $\mathbf{L = Log_{10}(fp') \times D}$
   $\mathbf{L = Log_{10}(3.16) \times 2cm = 1cm}$
   where fp' = fp normalized to the 1-10 decade range – f_P = 31.6 → f_P' = 3.16

Fig. 1.7: Log Scale Trick

Intuitive Component Models

Most op amp applications use combinations of four key components-- op amp, resistor, capacitor, and inductor -- and to facilitate stability analysis it is convenient to have "intuitive models" for them.

Our intuitive op amp model for ac stability analysis is defined in Fig. 1.8. The differential voltage between the IN+ and IN- terminals will be amplified by x1 and converted to a single-ended ac voltage source, V_{DIFF}, which is then amplified by K(f) (representing the data sheet Aol Curve: open-loop gain vs frequency). The resultant voltage, V_O, is then followed by the open-loop, ac small-signal, output resistance, R_O, with the output voltage appearing as V_{OUT}.

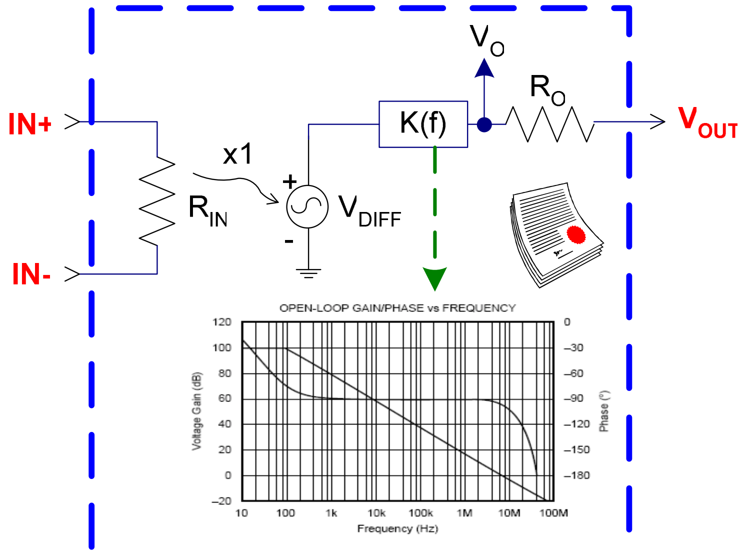


Fig. 1.8: Intuitive Op Amp Model

Our intuitive resistor model for ac stability analysis is defined in Fig. 1.9. The resistor has a constant resistance value regardless of the operating frequency.

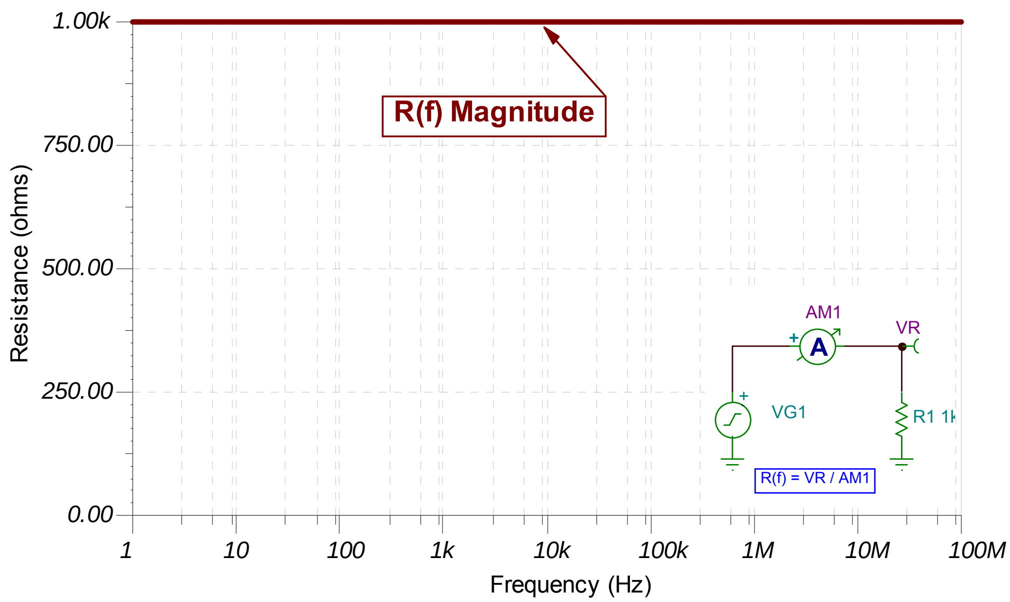


Fig. 1.9: Intuitive Resistor Model

Our intuitive capacitor model for ac stability analysis is defined in Fig. 1.10 and contains three distinct operating areas. At dc the capacitor is open-circuit. At "high" frequencies it is short-circuit. In between the capacitor is a frequency-controlled resistor with a $1/X_C$ decrease in impedance as frequency increases. The SPICE simulation in Fig. 1.11 depicts our intuitive capacitor model over frequency.

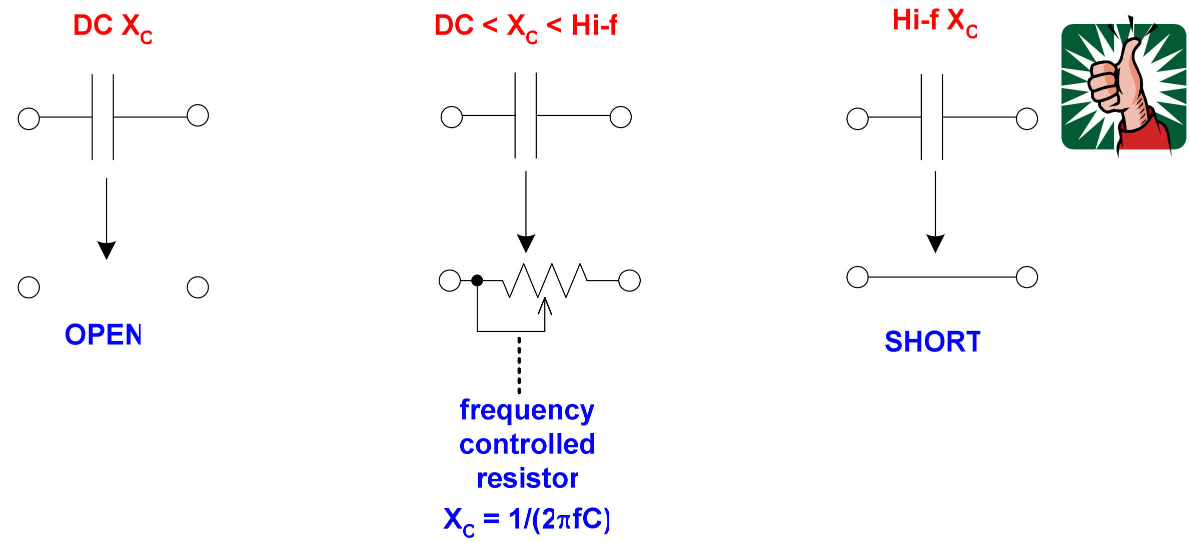


Fig. 1.10: Intuitive Capacitor Model

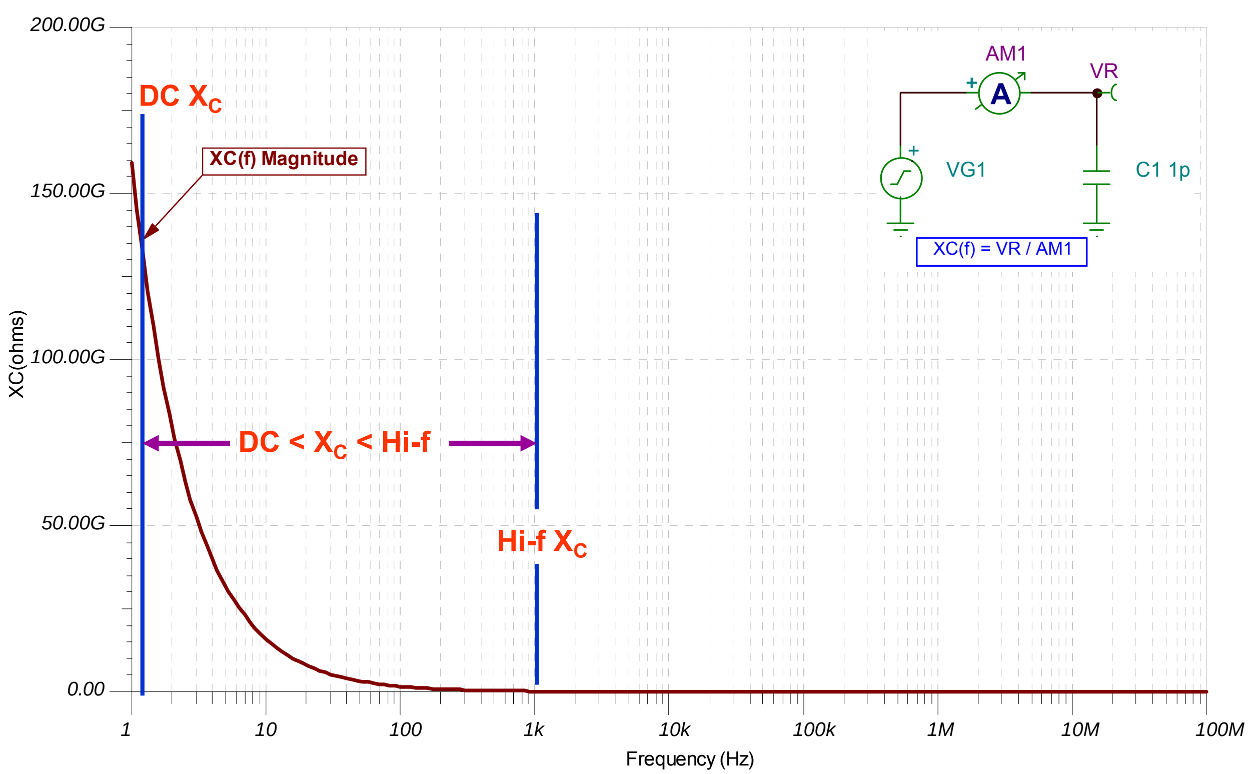


Fig. 1.11: Intuitive Capacitor Model SPICE Simulation

Our intuitive inductor model for ac stability analysis is defined in Fig. 1.12 with three distinct operating areas. At dc the inductor is short-circuit. At "high" frequencies it is open-circuit. In between the inductor is a frequency-controlled resistor with an X_L increase in impedance as frequency increases. The SPICE simulation in Fig. 1.13 depicts our intuitive inductive model over frequency.

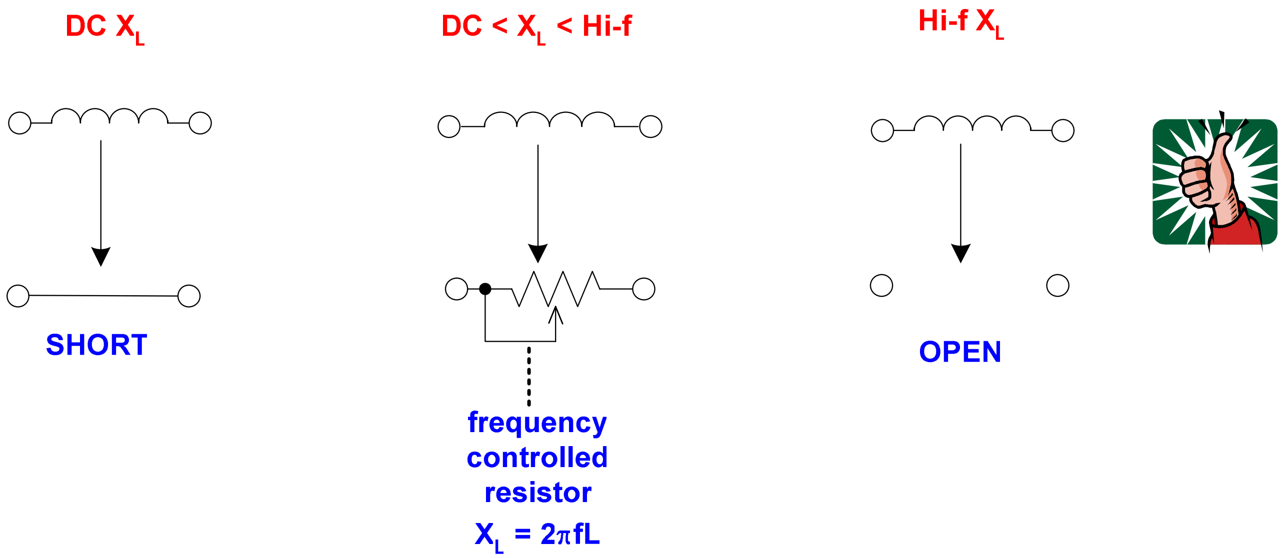


Fig. 1.12: Intuitive Inductor Model

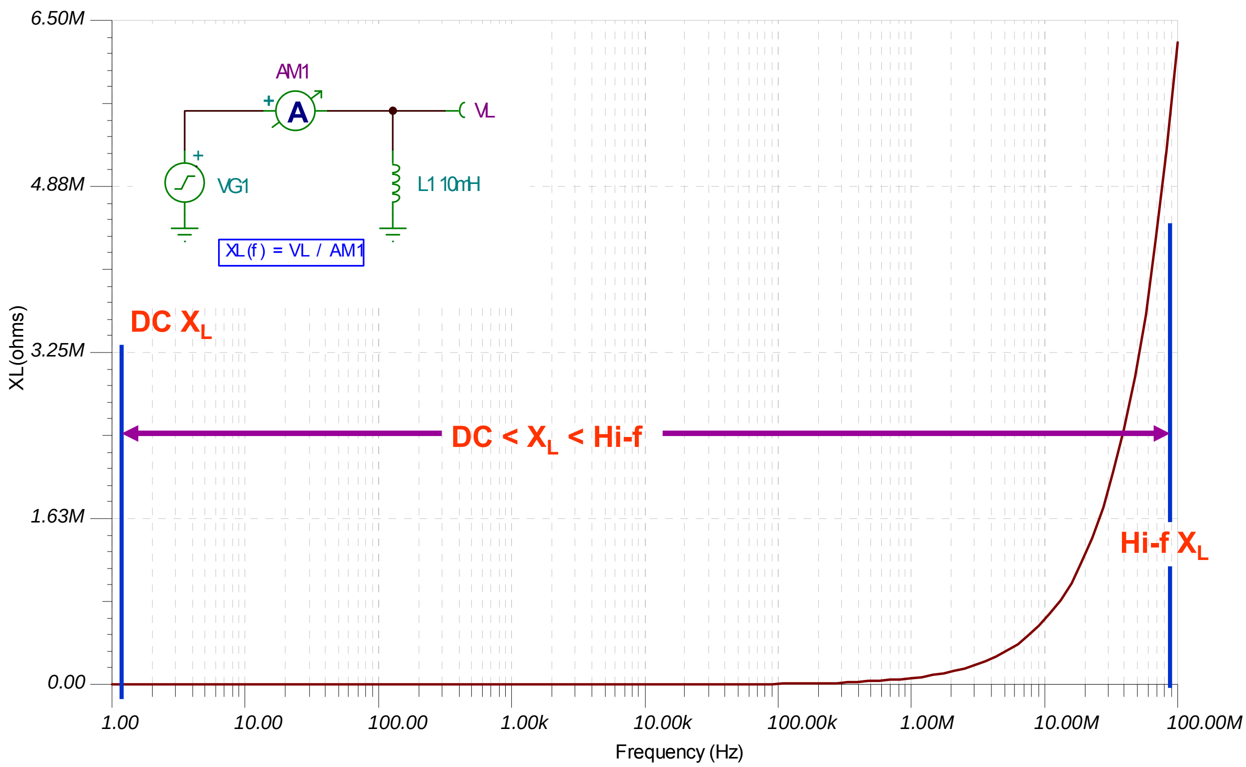
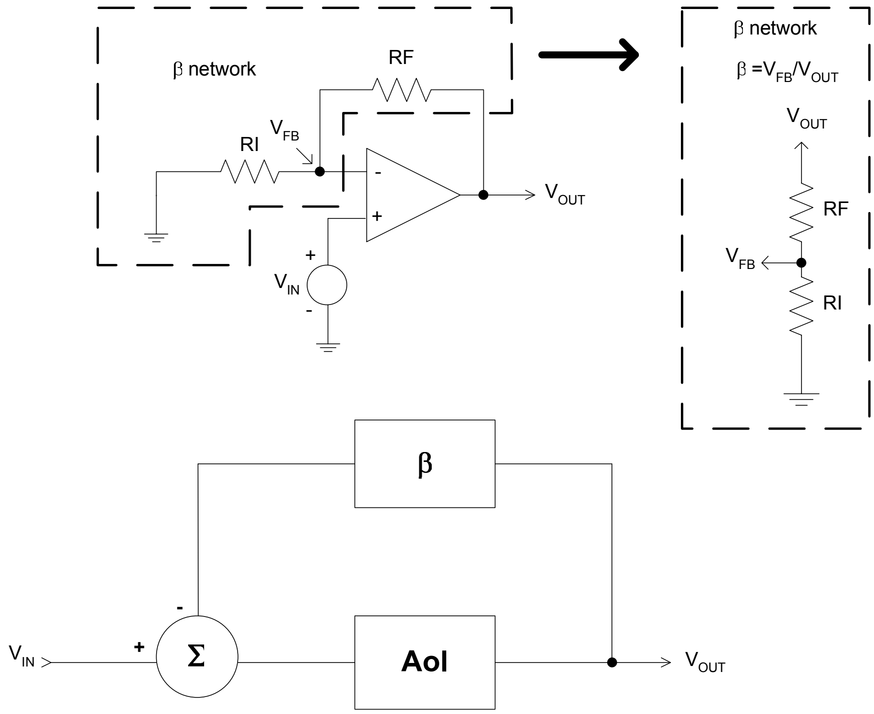


Fig. 1.13: Intuitive Inductor Model SPICE Simulation

Stability Criteria

The lower part of Fig. 1.14 illustrates the traditional control-loop model which represents a gain circuit with feedback. The top part of Fig. 1.14 depicts the sections of a typical op amp circuit with feedback which correspond to the control-loop model. This model can be called the op amp loop-gain model. Note that the Aol is the op amp data sheet parameter Aol, and is the open-loop gain. β is the amount of output voltage, V_{OUT}, that gets fed back, created in this example by a resistor network. Deriving V_{OUT}/V_{IN} we see that the closed-loop gain function is directly defined by Aol and β.



V_{OUT}/V_{IN} = Acl = Aol/(1+Aolβ)

If Aol >> 1 then Acl ≈ 1/β

Aol: Open Loop Gain

β: Feedback Factor

Acl: Closed Loop Gain

Fig. 1.14: Op amp Loop-Gain Model

From this model we can derive the criteria for stability in a closed-loop op amp circuit:

V_{OUT}/V_{IN} = Aol / (1+ Aolβ)
If: Aolβ = -1
Then: V_{OUT}/V_{IN} = Aol / 0 → ∞

If V_{OUT}/V_{IN} = ∞ → Unbounded Gain

Any small changes in V_{IN} will result in large changes in V_{OUT} which will feed back to V_{IN} and result in even larger changes in V_{OUT} → *OSCILLATIONS* → *INSTABILITY !!*

Aolβ: Loop Gain
Aolβ = -1 → Phase shift of ±180°, Magnitude of 1 (0dB)
fcl: frequency where Aolβ = 1 (0dB)

Stability Criteria:
At fcl, where Aolβ = 1 (0dB), Phase Shift < ±180°
Desired Phase Margin (distance from ±180° Phase Shift) ≥ 45°

Fig. 1.15: Derivation Of Stability Criteria

Loop Stability Tests

Since loop stability is defined by the magnitude and phase plot of loop gain (Aolβ) we analyze its magnitude and phase by breaking into the closed-loop op amp circuit, injecting a small-signal ac source into the loop, and then measuring amplitude and phase to plot the complete loop-gain picture. Fig. 1.16 shows the equivalent control-loop block diagrams for the op amp loop-gain model and the technique we will use for the loop-gain test.

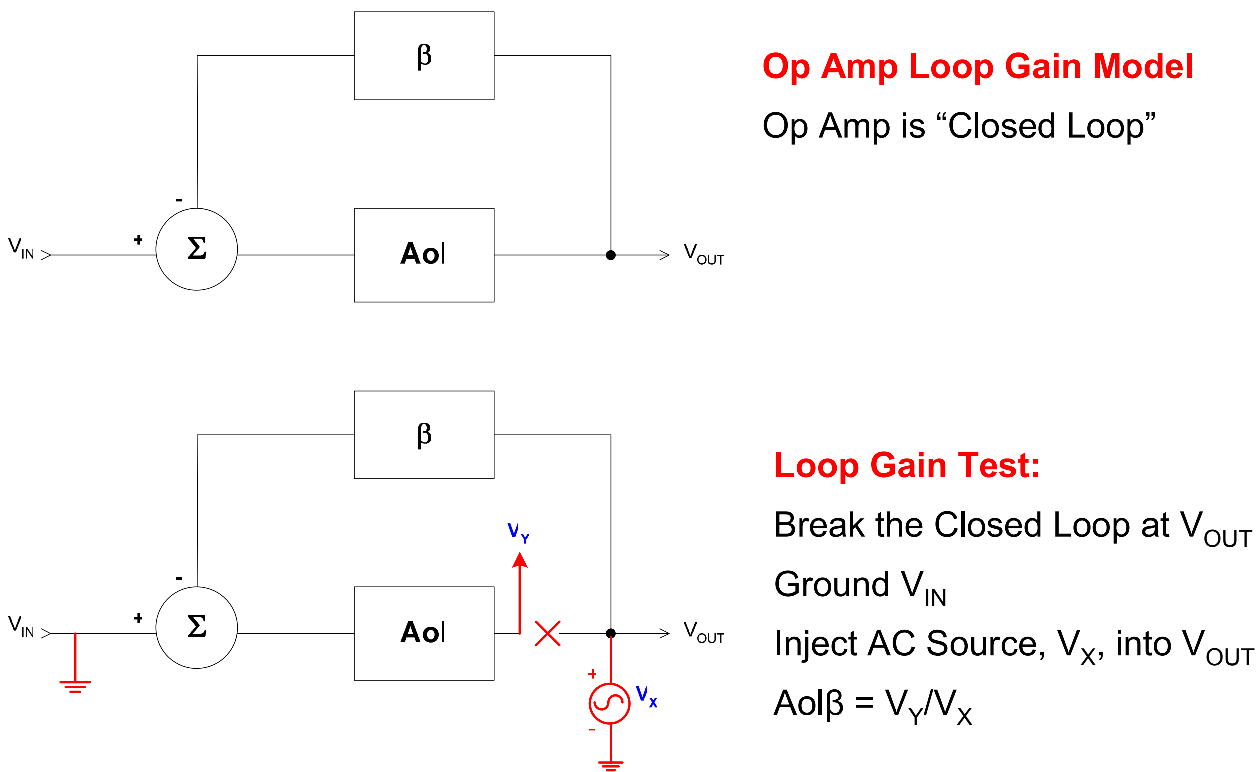


Fig. 1.16: Traditional Loop Gain Test

When analyzing a circuit built in SPICE for simulation, the traditional loop-gain test breaks the closed-loop op amp circuit using an inductor and capacitor. A very large value of inductance ensures the loop is closed at dc (a requirement for SPICE simulation is to be able to calculate a dc operating point before performing an ac Analysis) but open at the ac frequencies of interest. A very large value of capacitance ensures that our ac small signal source is not connected at dc but is directly connected at the frequencies of interest.

Fig. 1.17 illustrates the SPICE setup schematic for the traditional loop-gain test.

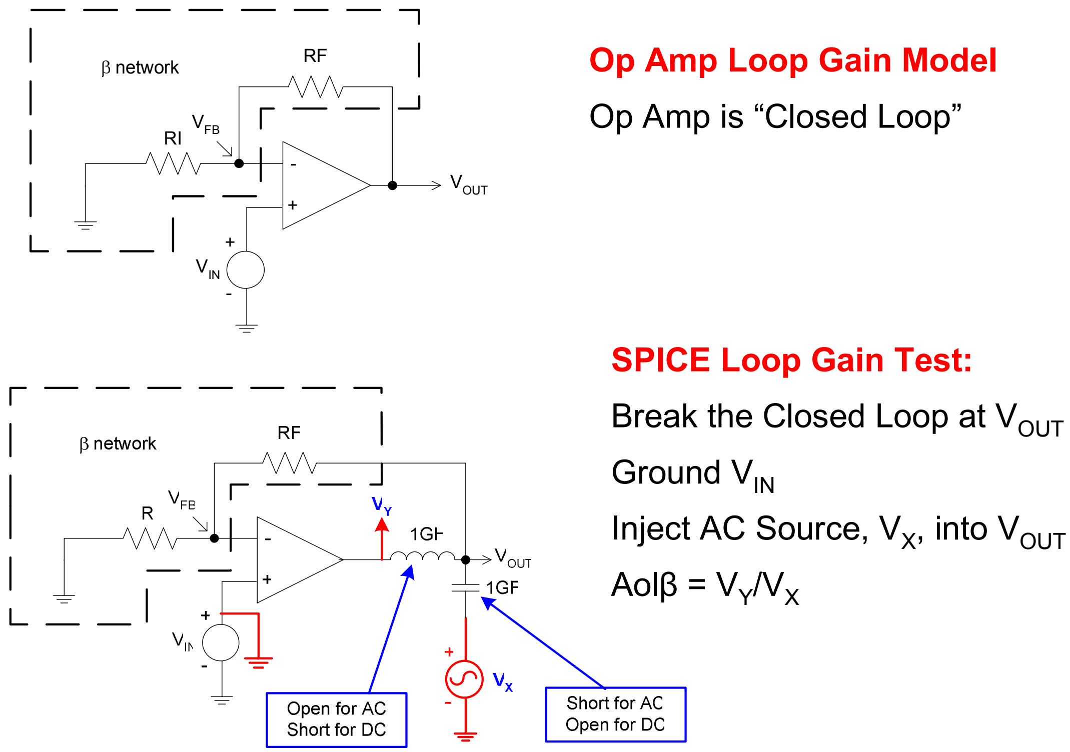


Fig. 1.17: Traditional Loop-Gain Test: SPICE Setup

Before simulating a circuit in SPICE we want to know the approximate outcome. Remember GIGO (garbage-in-garbage-out)! β and 1/β, along with the data sheet Aol curve, provide a powerful method for first-order approximation of loop-gain analysis. In future sections tricks and rules-of-thumb will be presented for computing β and 1/β. Fig. 1.18 defines the β network for op amp circuits.

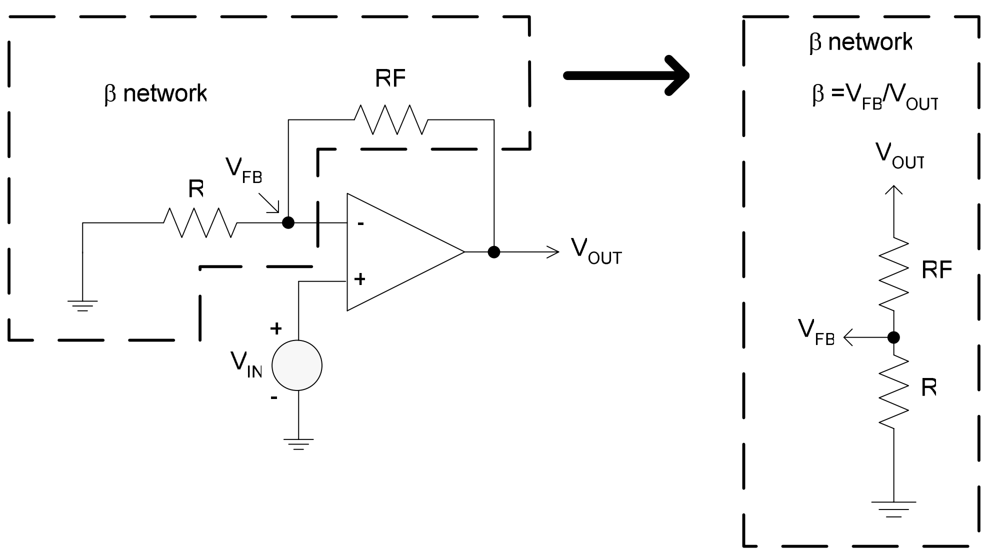


Fig. 1.18: Op Amp β Network

The 1/β plot imposed on the Aol curve will provide a clear picture of exactly what the loop-gain (Aolβ) plot is. From the derivation in Fig. 1.19 we clearly see that the Aolβ magnitude plot is simply the difference between Aol and 1/β when we plot 1/β in dB. Note that as frequency increases Aolβ decreases. Aolβ is the gain left to correct for errors in the V_{OUT}/V_{IN} or closed-loop response, so as Aolβ decreases the V_{OUT}/V_{IN} response will become less accurate until Aolβ goes to 0 dB when the V_{OUT}/V_{IN} response simply follows the Aol curve.

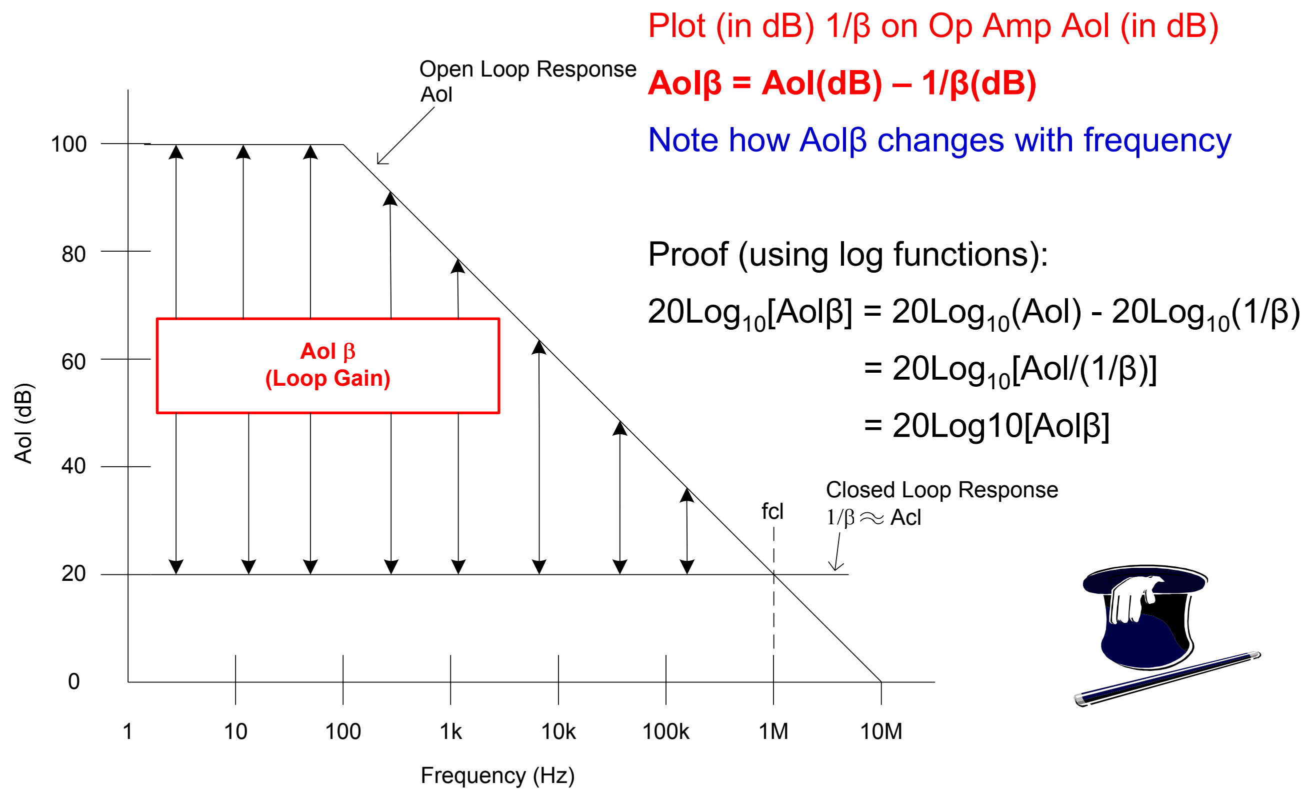


Fig. 1.19: Loop Gain Information From Aol Plot And 1/β Plot

Plotting the 1/β on the Aol curve there is an easy first-order check for stability called rate-of-closure, defined as the "rate-of-closure" of the 1/β curve with the Aol curve at fcl, where the loop gain goes to 0 dB. A 40 dB/decade rate-of-closure implies an UNSTABLE circuit, because it implies two poles in the Aolβ plot before fcl which can mean a 180° phase shift; a 20 dB/decade rate-of-closure implies a STABLE circuit. Four examples are shown in Fig. 1.20 with their respective rate-of-closure computed below.

fcl1: Aol - 1/β1 = -20 dB/decade - +20 dB/decade = -40 dB/decade rate-of-closure: **Unstable**
fcl2: Aol - 1/β2 = -20 dB/decade - 0 dB/decade = -20 dB/decade rate-of-closure: **Stable**
fcl3: Aol - 1/β3 = -40 dB/decade - 0 dB/decade = -40 dB/decade rate-of-closure: **Unstable**
fcl4: Aol - 1/β4 = -40 dB/decade - -20 dB/decade = -20dB/decade rate-of-closure: **Stable**

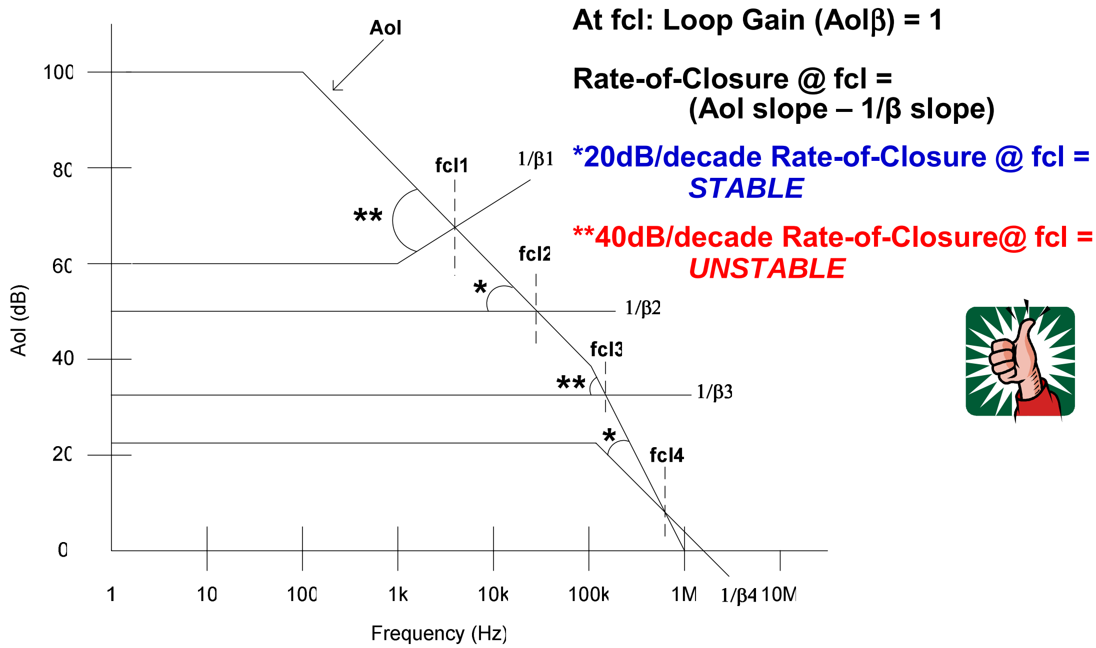


Fig. 1.20: Rate-Of-Closure Test for Loop Gain Stability

Loop Gain Stability Example

A loop gain analysis example (see Fig. 1.21) serves to relate how we can analyze the stability of an op amp circuit from the 1/β plot plotted on the Aol curve. As frequency increases the capacitor, CF, goes towards zero in impedance lowering the magnitude of the β plot with frequency (less voltage feedback) and raising the 1/β curve. From our rate-of-closure criteria we predict an **Unstable** circuit.

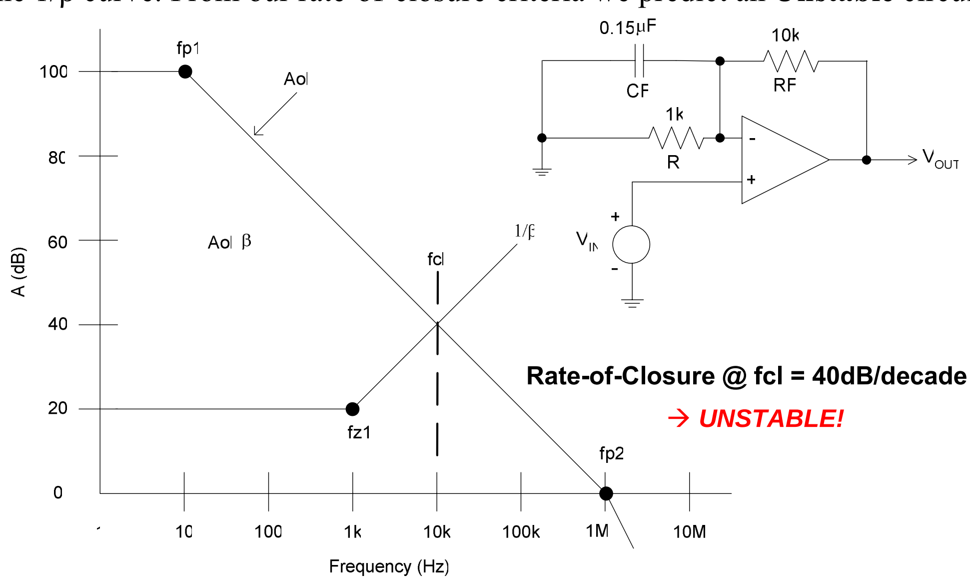


Fig. 1.21: Loop Gain Stability Example

From our 1/β plot on the Aol curve we can plot the Aolβ (loop-gain) magnitude plot (see Fig. 1.22) and we can then plot the loop gain phase plot. The rules to create an Aolβ plot from the 1/β plot on the Aol curve are simple: Poles and zeros from the Aol curve are poles and zeros in the Aolβ plot. Poles and zeros from the 1/β plot are opposite in the Aolβ plot. One easy way to remember this is β is used in the Aolβ plot and 1/β is the reciprocal of β and so we would expect the Aolβ curve to use the reciprocal of poles and zeros from the 1/β plot. Reciprocal of a pole is a zero and reciprocal of a zero is a pole.

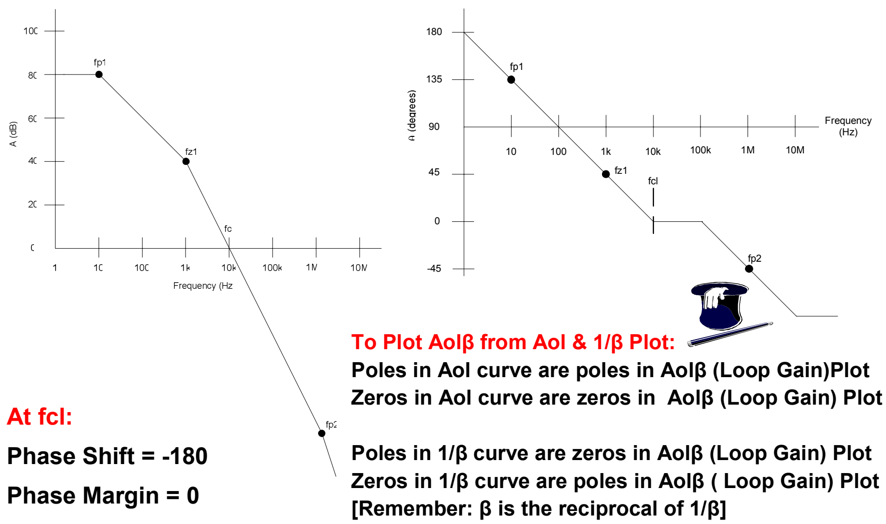


Fig. 1.22: Loop Gain Plot From Aol Curve & 1/β Plot

1/β and Closed-Loop Response

The V_{OUT}/V_{IN} closed-loop response is not always the same as 1/β. In the example in Fig. 1.23 we see that the ac small-signal feedback is modified by the Rn-Cn network in parallel with RI.

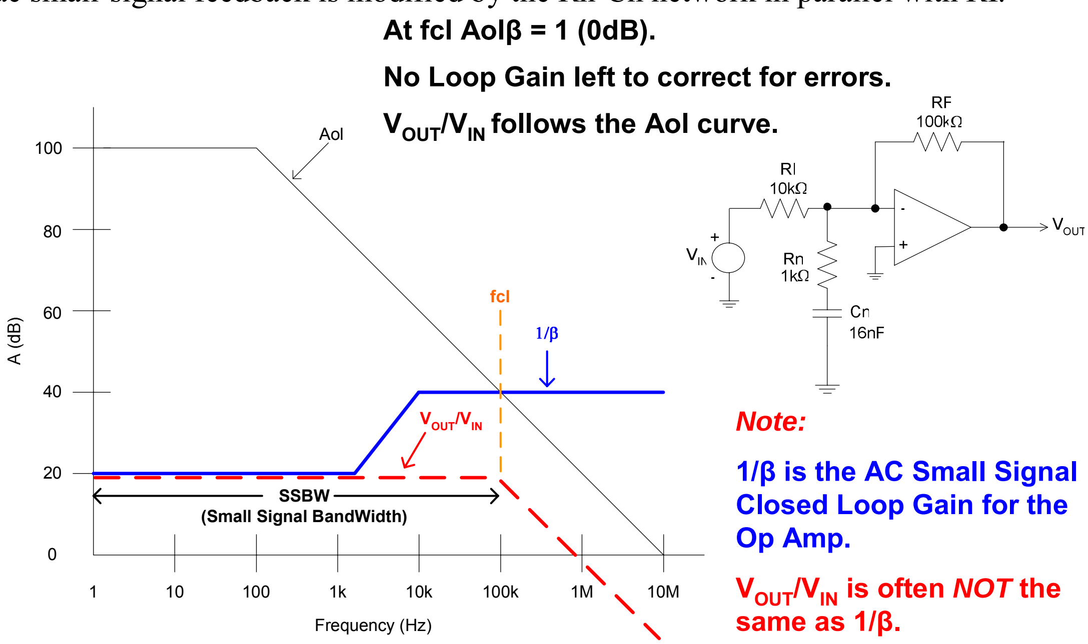


Fig. 1.23: V_{OUT}/V_{IN} Vs 1/β

As frequency increases we see the results of this network reflected in the 1/β plot on the Aol curve. Think of this example as an inverting summing op amp circuit. We are summing in V_{IN} through RI and ground through the Rn-Cn network. V_{OUT}/V_{IN} will not be affected by this Rn-Cn network at low frequencies and the desired gain is seen as 20 dB. As loop gain (Aolβ) is forced to 1 (0 dB) by the Rn-Cn network there is no loop gain left to correct for errors and V_{OUT}/V_{IN} will follow the Aol curve at frequencies above fcl.

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About The Author

After earning a BSEE from the University of Arizona, Tim Green has worked as an analog and mixed-signal board/system level design engineer for over 23 years, including brushless motor control, aircraft jet engine control, missile systems, power op amps, data acquisition systems, and CCD cameras. Tim's recent experience includes analog & mixed-signal semiconductor strategic marketing. He is currently a Strategic Development Engineer at Burr-Brown, a division of Texas Instruments, in Tucson, AZ and focuses on instrumentation amplifiers and digitally-programmable analog conditioning ICs.