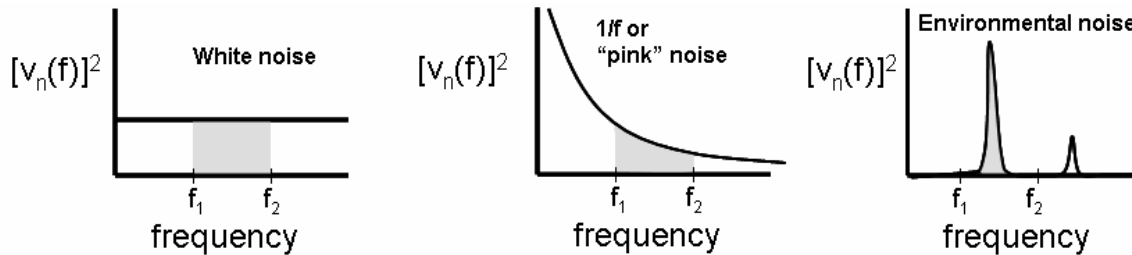


Noise in Analog Circuits

As we discussed earlier, the noise properties are often of paramount importance in analog circuit design. Noise is typically described in terms of its frequency spectrum, which describes how prevalent the noise is at different frequencies. Noise is typically described in terms of the noise power. However, in this respect the jargon makes somewhat sloppy use of terminology, and what is called “noise power” is often in units of V^2 . There are several types of noise.

- a) White noise: White noise has a constant frequency spectrum.
- b) $1/f$ (one-over-f) noise (sometimes called “pink” noise): The noise power varies like $1/f$.
- c) Environmental noise: This refers to any noise source that arises from well-defined sources in the environment; the most common would be pickup at 60 Hz, for example, from the 110 VAC power lines.



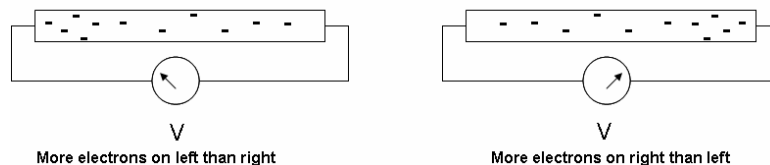
If a source that has a voltage noise spectrum given by $v_n(f)$ is measured over some range of frequencies extending from frequency f_1 to frequency f_2 , then the mean-square noise is given by:

$$V_n^2 = \int_{f_1}^{f_2} [v_n(f)]^2 df \quad \text{and the RMS noise voltage is:} \quad V_n = \sqrt{\int_{f_1}^{f_2} [v_n(f)]^2 df}$$

From this relationship, you can see that the noise voltage $v_n(f)$ has units of Volts/sqrt(Hz), commonly referred to as “volts per root Hertz”. The mean-square noise is then given by the area under the curves that are shaded in the figure above.

Fundamental origins of noise: Some sources of noise are fundamental and arise directly from thermodynamics and the quantized nature of electrical charge:

- 1) Johnson Noise: Johnson noise arises from the fact that in a resistive material, the spatial distribution of electrons is fluctuating in time. At any given point in time, there may be more electrons on the left than on the right. Since the electrostatic potential depends on the spatial distribution of charges, there is an accompanying fluctuation in the potential (voltage) measured across a resistor. So, if you put a perfect voltmeter across a resistor, you would see that the average potential would be zero, but it would be fluctuating above and below zero.



Johnson Noise arises straight out of thermodynamics. The fluctuations in potential have a frequency dependence, such that the squared voltage noise density is given by:

$$v_n^2 = 4kTR \text{ (units of volts/sqrt(Hz))}$$

$$\Delta V_{rms} = \sqrt{\int_{f1}^{f2} 4kTR df} = \sqrt{4kTR(\Delta f)} \text{ where } k \text{ is Boltzmann's constant, } T \text{ is absolute}$$

temperature, R is the resistance, and Δf is the bandwidth of the measurement.

Shot noise:

Similarly, "Shot Noise" arises from the fact that current is not a "fluid" flow, but rather the motion of discrete electrons. Current refers to the average rate of flow of electrons. Shot noise is the statistical fluctuation in the rate at which electrons move. Like Johnson Noise, shot noise is "white":

$$[i_n(f)]^2 = 2iq \text{ in units of amps}^2/\text{Hz} \text{ where } I \text{ is current in amps and } q \text{ is the charge on the electron}$$

(=1.602x10⁻¹⁹ Coulombs). Then,

$$\Delta I_{Shot}^{RMS} = \sqrt{2Iq(\Delta f)}$$

Noise in circuits:

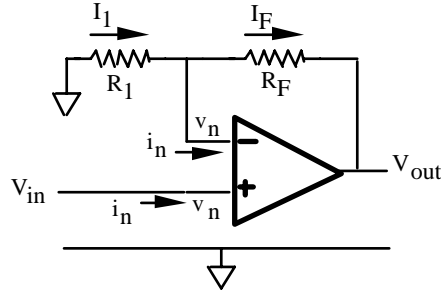
The noise characteristics of electronic components and circuits are virtually always expressed in terms of noise densities. Because electronic circuits always have some input voltage and some input current, we actually need two quantities to express the noise properties. These are the voltage noise density, in units of Volts/ $\sqrt{\text{Hz}}$, and the current noise density, in units of Amps/ $\sqrt{\text{Hz}}$. You should further note that we can express the noise in terms of its effect on the output of a circuit (or op-amp), or we could express the noise as if it were superimposed on the input signal. In the former case, we speak of noise "referred to output", or "RTO", while in the latter case we speak of noise "referred to input", or "RTI". In understanding the overall behavior of an electronic circuit we will usually be more interested in noise referred to the output (or simply, the total output noise). However, individual circuit elements usually have their noise properties specified referred to the *input*.

In any real circuit, there will almost always be several sources of noise. In order to determine the total noise, we must know whether the individual sources of noise are correlated, or whether they are uncorrelated. That's equivalent to saying that we must know whether the circuit elements generating the noise behave independently. If they behave independently, then the noise generated by the components will be uncorrelated. For uncorrelated sources of noise, the total noise is determined as:

$$V_{noise,total} = \sqrt{V_{N1}^2 + V_{N2}^2 + V_{N3}^2 + \dots}$$

Let's look at how to predict the output noise for an electronic circuit. We'll start with the non-inverting amplifier. The overall noise performance can be determined by considering the noise sources one by one, assuming that they are uncorrelated, and finally performing a sum-of-squares as above.

Input voltage Noise:



Let's consider first the effect of input voltage noise at the inverting input:

$\frac{0 - V_-}{R_1} = \frac{V_- - V_{out}}{R_F}$. Because of the input voltage noise, the voltage at the inverting input will

be fluctuating by the amount v_n . If $V_+ = 0$, and the op-amp is functioning properly, then the op-amp will adjust V_{out} in order to keep $V_- = V_+ = 0$. This requires that the output of the op-amp fluctuate. We have:

$$\frac{0 - v_n}{R_1} = \frac{v_n - V_{out}}{R_F}$$

This can be easily solved for V_{out} to give:

$$V_{out} = v_n \left(1 + \frac{R_F}{R_1} \right).$$

More correctly, we should remember that v_n is a fluctuating voltage and

write this as: $\Delta V_{out}(\text{voltage}, V_-) = v_n \left(1 + \frac{R_F}{R_1} \right)$. This number represents the fluctuations in

the output voltage arising from the noise voltage at the inverting input. What about noise at the

non-inverting input? Again, we have: $\frac{0 - v_n}{R_1} = \frac{v_n - V_{out}}{R_F}$, which again

gives: $V_{out} = v_n \left(1 + \frac{R_F}{R_1} \right)$ or, $\Delta V_{out}(\text{voltage}, V_+) = v_n \left(1 + \frac{R_F}{R_1} \right)$. This number represents

the fluctuations in the output voltage arising from the noise voltage at the non-inverting input.

Input Current Noise:

Input current noise can be interpreted as a fluctuating current going into the inverting and non-inverting inputs. At the inverting input, we have:

$$\frac{0 - V_-}{R_1} = i_n + \frac{V_- - V_{out}}{R_F},$$

where i_n represents the fluctuating current into the inverting input.

But $V_- = V_+ = 0$, so: $\Delta V_{out} = i_n R_F$.

What is the effect of current noise at the non-inverting input? It depends on the output impedance of whatever is driving the op-amp. If the non-inverting input is held at ground, then noise current at V_+ cannot create any voltage at V_+ , and the output fluctuation will be zero. However, if we write the output impedance of the signal source as Z , then through a similar equation we would get: $\Delta V_{out} = i_n Z$.

Now, let's look at the total noise arising from both voltage and current noise at both the inverting and non-inverting inputs: we have:

$$\Delta V_{out}^{total} = \sqrt{\left[v_n \left(1 + \frac{R_F}{R_1} \right) \right]^2 + \left[v_n \left(1 + \frac{R_F}{R_1} \right) \right]^2 + [i_n R_F]^2 + [i_n Z]^2}$$

$$= \sqrt{2v_n^2 \left(1 + \frac{R_F}{R_1} \right)^2 + i_n^2 (R_F^2 + Z^2)}$$

There are several important points here: First of all, look at what happens if the source impedance is high. In the limit of large Z, the output noise becomes: $\Delta V_{out} \cong i_n Z$. From this, we can see a general rule:

At high source impedances, the noise in electronic circuits is almost always dominated by the input *current* noise.

At low source impedances, noise is usually dominated by input *voltage* noise.

Let's put in some typical values: We'll consider the Burr-brown OP627 Operational amplifier. It has an input current noise density of 1.7 fA/Hz and an input voltage noise density of about 5 nV/Hz. Let's assume that we made an amplifier with a gain of 10000 using a 100 ohm and a 1Megohm resistor. The gain-product bandwidth is 100 MHz, so a gain of 100 amplifier will operate out to frequencies of approximately 10 kHz. Thus, the "bandwidth" Δf will be 10^4 Hz. The input voltage noise will then be 5×10^{-6} volts, and the input current noise will be 1.7×10^{-13} amp. Let's assume first that we simply ground the input, so that the non-inverting input is connected directly to ground. The output noise will then be:

$$V_{out}^{total} = \sqrt{2 \left(5 \times 10^{-7} \right)^2 \left(1 + \frac{10^6}{10^2} \right)^2 + \left(1.7 \times 10^{-13} \right)^2 \left(10^6 \right)^2} = 7.1 \text{ millivolts.}$$

Looking at the magnitudes of the numbers in the equation above, you will see that the total noise is almost completely dominated by voltage noise because the source has zero impedance. In this case, you will see that the voltage noise continues to dominate the performance as long as Z_{source} is less than about 10^{10} ohms.

We have thus far forgotten about the Johnson noise of the resistors. How does this contribute?

Let's consider the Johnson noise of R_1 . We have:

$$\frac{0 - V_-}{R_1} = \frac{V_- - V_{out}}{R_F}, \text{ but now the voltage across the resistor (0-V-) has some uncertainty in it.}$$

Let's for the moment assume that $V_{in} = 0$. Then, $0 - V_- = V_{Johnson}(R_1)$.

$$\frac{v_{Johnson}(R_1)}{R_1} = \frac{-V_{out}}{R_F}, \text{ or } \Delta V_{out}^{Johnson}(R_1) = -\frac{R_F}{R_1} v_{Johnson}(R_1)$$

Now, the effect of Johnson noise of the feedback resistor R_F : This appears as fluctuating voltage between V_{out} and V_- , and so appears as:

$$\frac{0 - V_-}{R_1} = \frac{V_- - V_{out} + v_{Johnson(RF)}}{R_F}. \text{ Using } V_- = 0 \text{ gives: } \Delta V_{Johnson(R_2)}^{out} = -v_{Johnson(R_2)}$$

Note that the Johnson noise of R_1 gets "amplified" by the ratio of R_F/R_1 , while the Johnson noise of R_2 is not. Through a similar analysis we'd find that the Johnson noise of the source resistance Z would appear at the output as: $\Delta V_{Johnson(Z)}^{out} = -v_{Johnson(Z)}$

For a 100 ohm resistor at 300 Kelvin, $\Delta V_{Johnson} = 1.2 \times 10^{-7}$ Volt. For R_F , $\Delta V_{Johnson} = 1.27 \times 10^{-5}$ Volt. For ΔV_{Source} , we have: $1.7 \times 10^{-8} * \text{Sqrt}(Z)$.

Since again these noise sources are uncorrelated, we can take them and using the sum-of-square approach, calculate the total noise:

$$\Delta V_{total}^{out} = \sqrt{\Delta V_{op-amp}^2 + \Delta V_{Johnson}^2} = \sqrt{(7.1 \times 10^{-3})^2 + (1.27 \times 10^{-5})^2 + (1.2 \times 10^{-7})^2 + (1.2 \times 10^{-8})^2 Z_{source}}$$

Provided that $Z_{source} < 10^{10}$ ohms, this again reduces to about 7.1 millivolts.

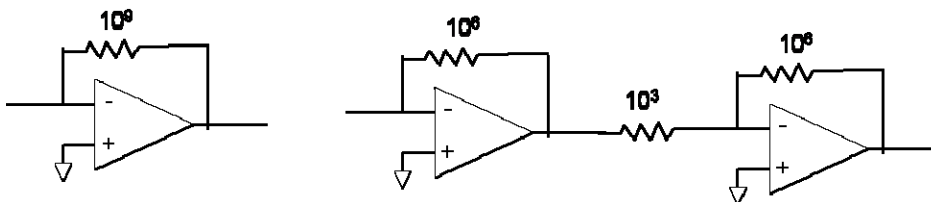
Noise in the current-to-voltage converter:

In the current-to-voltage converter, it is very common to use quite large resistor values in the feedback loop. In that case, the Johnson noise of the feedback network becomes limiting. Since Kirchoff's law says that the current input and the current through the feedback resistor are the same, this Johnson noise generates a current that looks just like a fluctuating input current:

$$\Delta I_{in} = \frac{V_- - V_{out} + v_{Johnson(RF)}}{R_F}$$

$$\Delta V_{in} = v_{Johnson(RF)} = \sqrt{4kTR\Delta f}$$

One important point is that the noise in a current-to-voltage converter scales like \sqrt{R} . However, the signal gets amplified by a factor of R . So, the signal-to-noise ratio of a current-to-voltage converter scales like \sqrt{R} . This has an important consequence. It is very common in current-to-voltage converter to need a very high gain, on the order of 10^9 volts/amp. You could do this using a single-stage amplifier with a 10^9 ohm resistor, or using a two-stage amplifier using an I-V converter with a 10^6 ohm resistor followed by a regular op-amp voltage amplifier with a 1000x gain.



Both these systems would have the same overall amplification. However, they would give different signal-to-noise ratios. The single stage amplifier gives a noise level of:

$$\Delta V_{in} = v_{Johnson(RF)} = \sqrt{4kT(10^9)\Delta f}$$

while the two-stage amplifier has a noise level closer to:

$$\Delta V_{in} = v_{Johnson(RF)} = 1000 * \sqrt{4kT(10^6)\Delta f}$$

. You can see that the noise level in the single-amplifier configuration will be 30 times smaller than that of the two-stage design.

This leads to the general rule:

In any circuit, the noise will be almost completely controlled by the transducer and the first stage of amplification.

You will almost always achieve the best signal-to-noise in any measurement by amplifying as much as possible in the very first stage. In a two-stage current-to-voltage converter, for example, the signal-to-noise is controlled almost entirely by the first op-amp, because whatever noise is generated by this op-amp is amplified by the second stage.

Chem 628 Grounding and Shielding

In general, extrinsic "noise" in instruments comes from three sources:

- 1) Poor grounding and/or ground loops (usually at high frequency)
- 2) Magnetic pickup from transformers and power lines.
- 3) Electrostatic pickup from various points in a circuit

Grounding in electronic circuits and between instruments:

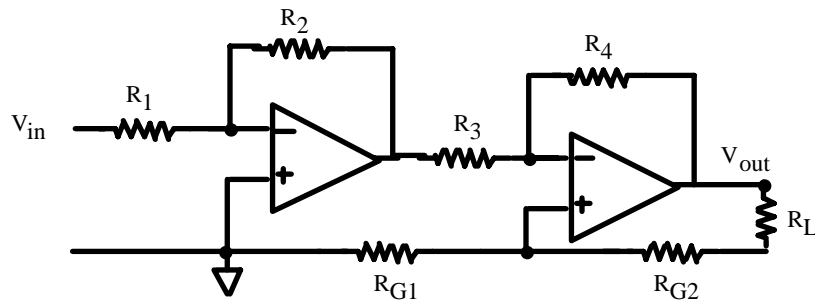
Grounding is a very important element in making electronic circuits work correctly. Since all that we can ever measure are differences in voltages between two points, our "reference points" (i.e., "grounds") are just as important as our "signal" wires. Although we have a tendency in schematics to only include the signals and generally leave the ground as an "understood" reference point, the details of how ground wires are wired and connected can drastically alter the way a circuit performs. Particularly in high-gain circuits, poor grounding techniques can lead to offset, non-linearity, and oscillation problems. With a few simple rules, most grounding problems can be avoided. However, in some cases you can't optimize the grounding the way you'd like. For example, even though you can do the grounding within any given circuit box correctly, you might find grounding problems that arise when you start connecting several instruments together. Some times, the very act of trying to observe a circuit using an oscilloscope can introduce new problems because of the grounds involved. With an understanding of how grounding problems arise, you will be able to solve most grounding problems scientifically.

The "golden rule" for proper grounding is:

Circuits should be constructed such that the flow of current in the ground lines does NOT produce IR drops at undesired places within a circuit. This can be restated as: "All grounds are created equal, but some are more equal than others".

To illustrate this, we'll consider two circuits in detail. We'll consider the circuit with BAD grounding technique first.

Bad grounding technique: The following circuit is a two-stage amplifier which delivers an output voltage to a load, represented by the resistor R_L . The output voltage which will be measured is the voltage drop across this resistor. The wires connecting the grounds have resistances R_{G1} and R_{G2} . The input to the first op-amp is connected directly to ground, (we'll assume that this wire has no resistance). The "bad technique" part of this circuit is that the "ground" on the output connector is connected to the input of op-amp 2, which is then connected to the ground of op-amp 1. This kind of "linear" chaining of grounds (from the output connector to the op-amp 2 to op-amp 1 to ground) is called "daisy-chaining". We'll see that it can easily give problems.



For op-amp 1, there is no problem at all: we have $V_{out1} = -(R_2/R_1) \cdot V_{in}$. As always, we write the equations for op-amp 2 as:
 $(V_{out1} - V_{+2})/R_3 = (V_{+2} - V_{out2})/R_4$

Note, however, that V_{+2} is connected to ground through a wire with resistance, and the current flow through this wire gives rise to an IR drop at the non-inverting input. The voltage at V_{+2} is determined by the "voltage-divider equation", since R_L , R_{G2} , and R_{G1} form a voltage divider (we'll assume that the op-amps have zero input bias current). The voltage at V_{+2} is then:

$$V_{+2} = V_{out2} \frac{R_{G1}}{R_{G1} + R_{G2} + R_L}$$

Substituting in the equation above then gives:

$$\frac{-\frac{R_2}{R_1} V_{in} - V_{out2} \frac{R_{G1}}{R_{G1} + R_{G2} + R_L}}{R_3} = \frac{V_{out2} \frac{R_{G1}}{R_{G1} + R_{G2} + R_L} - V_{out2}}{R_4}$$

Finally, we get:

$$V_{out2} = V_{in} \frac{R_2 R_4}{R_1 R_3} \left\{ \frac{1}{1 - \frac{R_{G1}}{R_{G1} + R_{G2} + R_L} \left(1 + \frac{R_4}{R_3} \right)} \right\}$$

The output voltage measured by whatever we connect is given by the voltage drop across the "load" resistor R_L . Thus, $V_m = I_L R_L = V_{out2} \frac{R_L}{R_{G1} + R_{G2} + R_L}$

This can be written as:

$$V_m = V_{in} \frac{R_2 R_4}{R_1 R_3} \frac{R_L}{R_L + R_{G2} - R_{G1} \left(\frac{R_4}{R_3} \right)}$$

The first two factors are what we would have if there was no load; indeed, if $R_L = \infty$, the last factor goes to one, and we have that $V_{out} = V_{in} * (R_2 R_4 / (R_1 R_3))$, as we hoped for in the first place.

To understand how the ground current affects the circuit, we need to look at the last factor. If $R_L = \infty$, then this factor becomes 1 and there's no problem. To understand what happens when there is a finite load, note that the denominator contains a term $R_{G1} * (R_4 / R_3)$. This term reflects the effect of the IR drop across R_{G1} , which is amplified by the op-amp circuit. If R_{G1} is small compared to R_L , then as R_{G1} is increased, the current flow through the ground line remains approximately constant, but the voltage at V_+ increases. We can also look at this as a circuit having two feedback loops: the feedback from V_{out2} to V_- is "negative" feedback, which stabilizes the circuit. The ground connection from V_{out2} to V_+ represents positive feedback, which tends to destabilize the circuit.

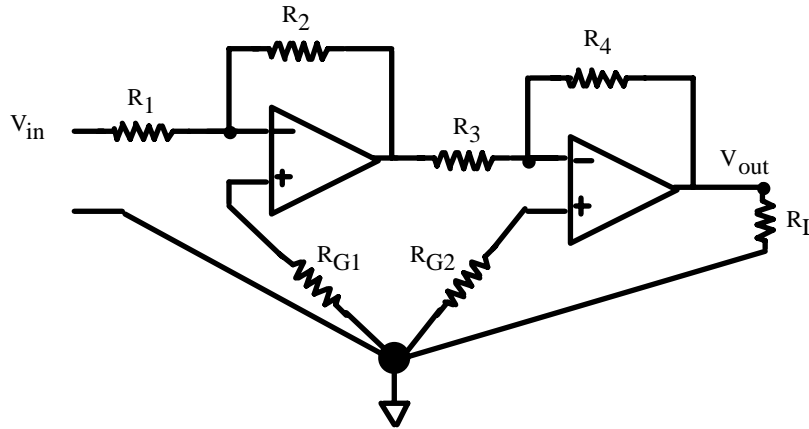
To see how serious the back grounding is, let's assume that the first stage has a gain of 10 and the second stage has a gain of 100, so $R_4 / R_3 = 100$. Let's also assume that the load resistor R_L is 1000 ohms, and that the "ground" resistances R_{G1} and R_{G2} are just 1 ohm each (it's easy to get 1 ohm of resistance in a solder connection). According to our result above,

$V_{measured} / V_{in} = (1000) * (1000 / (1000 - 100)) = 1111$. Instead of the expected gain of 1000, we got a gain of 1111, or an "error" of more than 11%. You can see that the output voltage is increased by our bad grounding, because a fraction of the input voltage is returned to the NON-inverting input, where it tends to increase the output voltage.

To see a more drastic effect, consider what happens if the load resistor is only 100 ohms. Now,

$V_m / V_{in} = (1000) * (100 / 1) = 10,000$!!! The gain is 100 times larger than it should be; if R_L is slightly smaller than 100 ohms, the output gain goes to INFINITY!. Again, it's easy to understand what's happening: as the output current flows, it increases the voltage at V_+ , further increasing the output voltage. We've got POSITIVE feedback, and the op-amp will easily saturate at one of the power supply voltages or else oscillate between +15 and -15 Volts. Either way, you won't get what you want.

Now, consider the "correct" way of grounding, as shown below. The main difference here is that instead of "chaining" the grounds together, we can think of the grounds as coming together into a "star" configuration. This type of grounding, in which all the individual grounds come together at a single point, is called single-point grounding and is generally the best solution to grounding problems. Let's analyze the circuit:



$(V_{out1} - V_{-2})/R_3 = (V_{-2} - V_{out2})/R_4$. As before, $V_{out1} = -V_{in} * R_2/R_1$. Now, however, $V_{+2} = 0$, since it's connected directly to the "single-point" ground and because the op-amp inputs draw no current (at least, for an ideal op amp). We get $V_{out2} = R_2 R_4 / (R_1 R_3) * V_{in}$. The voltage measured at the output is still going to be the voltage drop across R_L , which is:

$$V_m = V_{out2} * (R_L / (R_L + R_{G2})) = (R_2 R_4 / (R_1 R_3)) * (R_L / (R_L + R_G)) * V_{in}$$

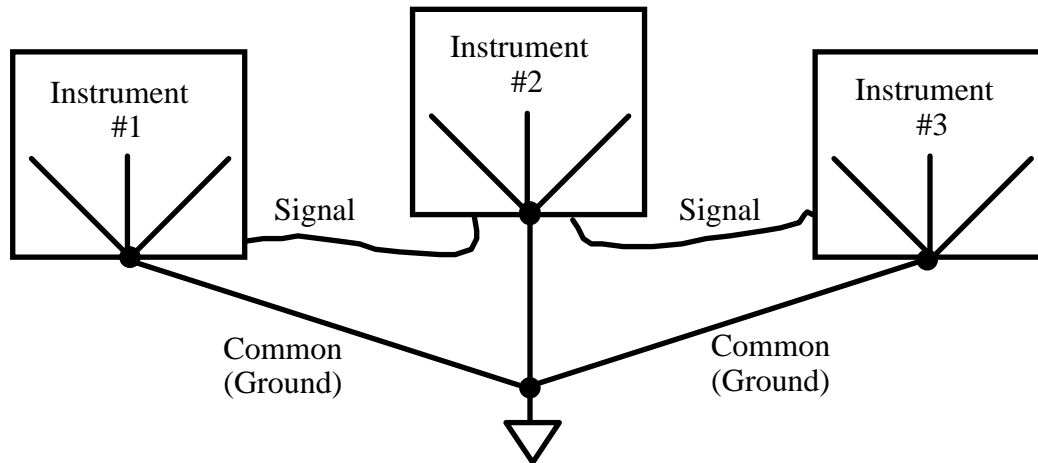
For the gain of 1000 amplifier we considered earlier with a load resistance of 1000ohms and a "ground" resistance of 1 ohm, we find that $(V_m/V_{in}) = (1000) * (1000/1001) = 999$ compared with the "ideal" value of 1000, or an error of only 0.1% (compared with an error of more than 11% for the daisy-chained arrangement). Even from a load resistance as small as 100 ohms, the error is only 1%, whereas in the daisy-chained arrangement the gain was off by a factor of 1000. Additionally, you will note that this circuit is stable for all values of R_L , whereas the daisy-chained arrangement would oscillate for low values of R_L .

From the above discussion, you can see that the best grounding arrangement is one which minimizes the possibility of having current flow in one part of a circuit create IR drops which wind up at the inputs to other parts of the circuit (particular in high-gain amplifiers).

What happens when you connect two instruments together? Unfortunately, this is where things get a bit more difficult, and grounding sometimes takes on more an aspect of "art" than "science". The techniques which one might use to get close to "single-point" grounding are not always those that lead to the best rejection of interference from noise sources, for example. Exactly how the "best" grounding is done between instruments requires an understanding of where the noise originates, the frequencies of interest (i.e., the bandwidth) of the measurement, and other considerations.

Ideal configuration:

The ideal configuration of grounding between instruments, as far as grounding is concerned, would involve having each instrument grounded internally to a single point, and then having each instrument joined together at a second single point. Signals would then be coupled from one instrument to another through single-wire cables, since all instruments would use one universal ground as their reference.

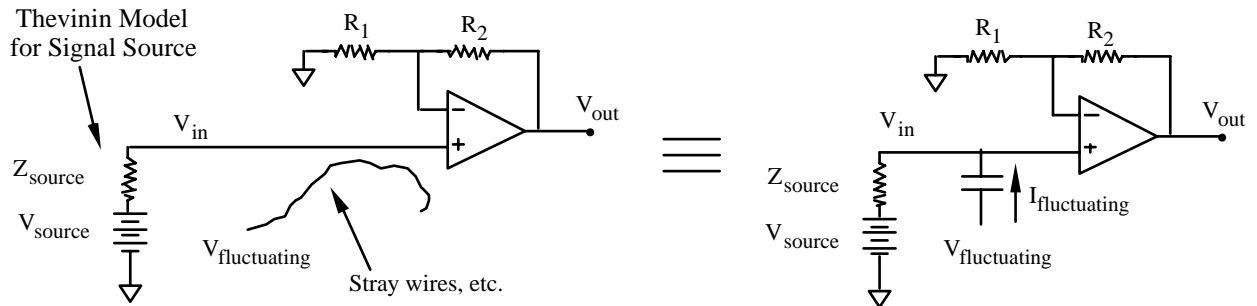


Unfortunately, this configuration has some problems with magnetic and electrostatic pickup. Most of the interferences that arise, whether from poor grounding, or electrostatic and/or magnetic pickup, get worse as the distances between the instruments increases. Therefore, the best solution to solving grounding and /or interference problems is usually to place "sensitive" instruments close to the signal source, where the lowest level signals usually occur. In many cases, one can improve the grounding situation by making a very low resistance ground connection from instrument to instrument, such that resistances on ground lines become negligible. Note here that you generally want a very low resistance, and so you want to use thick wire and good, solid mechanical contacts. (In my research group, we routinely connect all our instruments together with 3/4"-wide "grounding strap" to eliminate small ground loops which cannot be eliminated any other way.)

Electrostatic Pickup

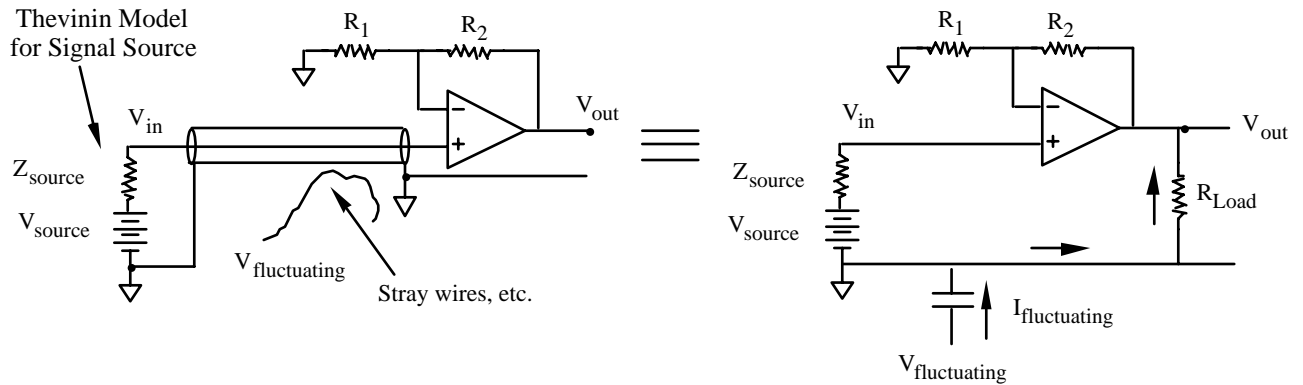
Electrostatic pickup arises because signal (and ground) wires have capacitance to other wires in the circuit. Voltages changing in one wire will therefore induce a current flow in adjacent wires according to the normal capacitance relationship: $I=C*dV/dt$. Because the currents are proportional to the rate of change of voltage, electrostatic pickup is generally most serious in high-frequency circuits, but low-frequency circuits can also be affected by electrostatic pickup. If you imagine inserting a capacitor between a "signal" source (with finite output impedance) and an amplifier (assumed to have infinite input impedance) as shown below, it's easy to understand electrostatic pickup. Changing the voltage on one plate of the capacitor (which is actually an adjacent wire in the circuit)

induces a current $I_{fluctuating} = C \frac{dV_{fluctuating}}{dt}$. This current in turn induces a voltage drop equal to $\Delta V_{in} = Z_{source} I_{fluctuating} = Z_{source} C \frac{dV_{fluctuating}}{dt}$ where Z_{source} is the output impedance of the signal source. Here, we see one immediately important consequence of electrostatic pickup: **its importance in voltage-amplification circuits scales with the impedance of the signal source.**



Thus, high impedance voltage sources (such as pH electrodes and some kinds of optical detectors) are particularly susceptible to electrostatic pickup. In contrast, electrostatic pickup between op-amp stages, for example, is small because the output impedance of op-amps is small. In current amplification circuits (such as an I-V converter), the capacitively-coupled current looks identical to a "real" signal current, and will be amplified; again, this is easy to understand if you think of a current source as an ideal battery in series with a very large resistance (infinite output impedance).

Electrostatic pickup can be minimized in several ways. First, one can put a "shield" around signal wires, with the shield connected to a **good** ground. In that case, the capacitive currents can flow through the shield without inducing an IR drops; the signal wire sees that it is surrounded by a ground potential and does not see any fluctuating electrostatic potentials. This is basic idea behind "coax" cable and "shieded" twisted pair. In both cases, by surrounding the signal wires by a good conductor at ground potential, the effects of electrostatic pickup can be reduced. It's worth noting that the typical "shield" of coax cable doesn't actually cover the entire wire - usually it covers 90-95% of the wire, but you will still get some electrostatic coupling if you place another wire with a fluctuating potential very close to your signal wires.



Note that external fluctuating voltages now induce currents in the ground line, rather than the signal line. Since the external "load" usually connected to an op-amp is on the order of kilohms or tens of kilohms (rather than the Megohms often associated with devices such as pH electrodes, photomultiplier tubes, optical detectors, etc.), the effects are reduced by many orders of magnitude. If $R_{Load} \ll Z_{source}$, then it is easy to see that all this induced current will flow through R_{Load} . The op-amp will keep V_{out} constant, so that current induced on the ground line will NOT produce ANY change in the output voltage measured with respect to ground (assuming, of course, that the ground line has zero resistance!).

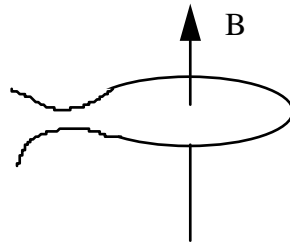
One way of further reducing electrostatic pickup is to use "triax" and/or "shielded twisted pair" wire, with a differential input on the voltage amplifier. The idea here is that there are essentially two "grounds": one serves as a reference point for the voltage measurement (and should have only small amounts of current flowing through it), and the other "shield" acts as a capacitor plate to establish a constant potential and carry off the capacitively-induced currents to a good ground. Any remaining electrostatic coupling which propagates into the two central conductors will usually affect both wires similarly, so that a differential amplifier (which responds to the difference in voltage between the two inputs) can often get rid of the remaining "common-mode" pickup.

Magnetic Pickup

Magnetic pickup is more insidious than electrostatic pickup and isn't talked about too much in polite company. For low-noise circuits operating at audio frequencies (1 Hz - 100 kHz or so), it's usually more problematic than the more commonly-discussed electrostatic pickup. Magnetic pickup usually occurs at a few well-defined frequencies: 60 Hz and multiples thereof (120 Hz, 180 Hz, etc.) from 110-Volt power lines, 30 kHz - 50 kHz or so from computer monitors, and possibly higher frequencies (hundreds of MHz) from instruments like NMR's.

From electrostatics, you might remember that a loop of wire in a time-varying magnetic field will have an induced voltage. (This is, of course, how transformers work and how electricity is generated at electric power plants). If we assume that the magnetic field B is constant in space but varies in time (appropriate for a small loop of wire, for example), the voltage induced around the loop will be:

$$V = -A \cdot (dB/dt),$$



where V is the voltage induced in the loop, B is the magnetic field strength (in Tesla, where $1 \text{ Tesla} = 10^4 \text{ Gauss}$) and A is the area of the loop, in square meters. (The earth's magnetic field is 0.6 Gauss , or $6 \times 10^{-5} \text{ Tesla}$)

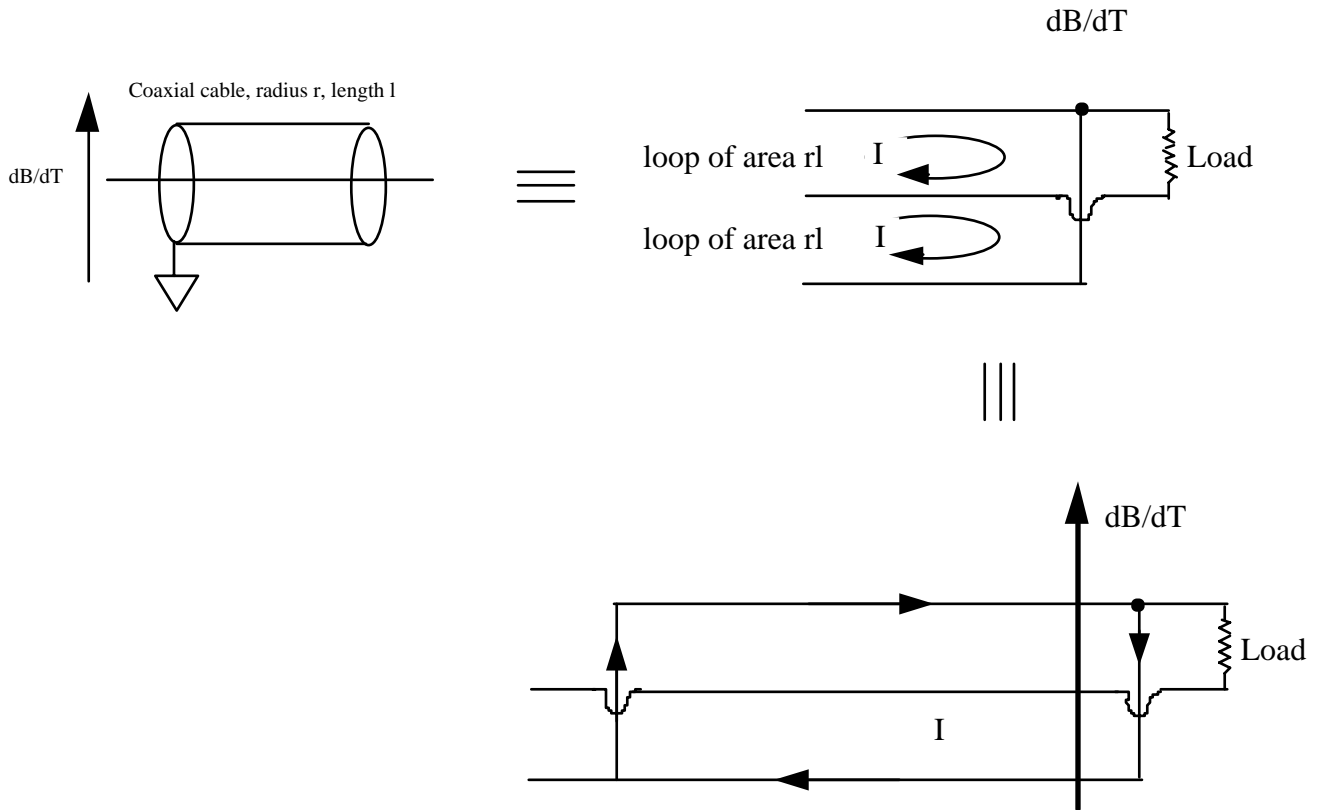
First, let's consider how big this effect is. The magnetic field in the vicinity of a transformer can easily be 100 times the Earth's magnetic field, or approximately $100 \text{ Gauss} = 10^{-2} \text{ Tesla}$. If we assume that the magnetic field is varying at $f = 60 \text{ Hz}$, then $B(t) = 0.01 \sin(2\pi ft)$ and $dB/dt = 0.01 \cdot 2\pi f \cos(2\pi ft)$. For a loop wire $10 \text{ cm} \times 10 \text{ cm} = 0.01 \text{ m}^2$, the induced voltage is 6 millivolts. Magnetic pickup is also quite serious from some other sources; for example, computer monitors use electrostatic coils to deflect an electron beam which strikes the phosphor screen; the "raster-scan" of the electron beam is typically at $30 - 50 \text{ kHz}$ for a super-VGA monitor, which means that there's a time-varying magnetic field at $30 - 50 \text{ kHz}$. Although B might be small, at this higher frequency dB/dt can be quite large, again producing voltages in your circuitry at 30 kHz (If you don't believe this can be significant, just ask any of my students about 30 kHz monitor noise!!).

Magnetic pickup of this sort is quite problematic in circuits where one needs to measure time-varying signals in the range between 60 Hz and about 240 Hz . While a lot of attention is typically given to grounding and shielding for high-frequency circuits, the magnetically-induced voltages can be a far more serious (and difficult to eliminate) problem. The stray magnetic fields from transformers and other coils basically looks like a magnetic dipole, and decays rather slowly with distance: approximately r_0/r , where r_0 is the size of the coil generating the magnetic field. Because $1/r$ isn't a particularly strong function of distance, it means that it's generally impractical to eliminate magnetically-induced voltage by simply moving the power supplies farther away, although sometimes it solves the problem.

In general, the primary solution to magnetically-induced voltages is to make the circuit immune to pickup by making all circuit paths small, effectively decreasing the area A . Again, when considering magnetic noise pickup in your circuit you must think about how your signal propagates through the circuit *and returns through the ground line !!* You must think in terms of current loops, not just voltages on "signal" wires.

One way of minimizing the area A is to use "coax" cable, in which the signal is applied to the central element and the return current flows through the "shield". The insulator between the shield and the wire is about 1 mm thick ($= 0.001 \text{ meter}$), making the effective "area" of the loop about $0.001 \cdot \text{cable length}$ (in meters). Because the shield completely

surrounds the central conductor, you can think of this (at least in two dimensions) as being like two loops, one constituting the central conductor and the top part of the shield, and the second loop comprised of the central conductor and the bottom part of the shield. The current induced in each of these loops will be either clockwise or counter-clockwise (depending on whether the magnetic field is increasing or decreasing), which means the the currents induced in the top loop and the bottom loop will *cancel* one another. Of course, if the magnetic field is inhomogeneous (such as a coax wire running right next to a transformer), then the fields will not cancel and you will still have to contend with the magnetic pickup.



Another way of reducing the loop area A is to use "twisted-pair" conductors, in which the signal and ground wires are twisted around one another. You can think of each "twist" as being two half-twists of 180 degrees each.



Any current induced in the area of the first half-twist will be cancelled by the current induced in the next half-twist because the "signal" and "ground" wires are exchanged (in space). As long as the magnetic field B doesn't change significantly on the distance scale of one twist, the magnetic induction will be zero. Twisted-pair usually does a better job than coax cable because the insulators are usually thinner (making loop areas smaller).

Ways of reducing magnetically-coupled "pickup" in electronic circuits:

- 1) Keep loop areas small by using coax or shielded twisted-pair wire whenever possible and keeping wires short.
- 2) Keep 110 V power lines away from your circuit!! Use twisted-pair wire (shielded twisted-pair is even better) for all 110 volt lines.. Assuming that you bring the 110-V into an instrument through the back panel, putting the 110-volt "on-off" switch on the *rear* panel instead of the front panel will save you from having to snake 110-V lines near your circuit and will *almost always* give you reduced 60-Hz pickup.
- 3) If you need really low noise at 60 Hz (and multiples thereof), consider getting rid of all AC voltages by removing the (usually 110 VAC - 15VDC) power supplies from the box containing sensitive, high-gain electronics and putting them in a separate box. (This may create other problems due to pickup along the wire connecting the supplies and/or IR drops along the wire connecting the power supply to your circuit). Alternatively, consider powering your circuit with batteries. Small sealed, rechargeable lead-acid batteries are available in a range of voltages and can easily power a few op-amps for 8-10 hours before being recharged. It's worth noting that commercial low-noise voltage and current amplifiers available from EG&G-Princeton Applied Research, Ithaco, Krohn-Hite, and others are all available with operation from 12-15VDC sealed lead-acid batteries. Contrary to what you might have expected, they use batteries not because the DC voltage produced is any quieter, but simply because eliminating all 110-VAC from the instrument reduces the 60 Hz *magnetic* pickup.
- 4) If you want to keep the power supplies in the same box, consider use power supplies with "toroidal" transformers instead of normal "square-frame" transformers. The more symmetric design of the toroidal transformers reduces the fringing fields
- 5) Use specially-designed "low-noise" power supplies (available from AAK and others); these are usually "potted" supplies which internally use toroidal transformers; the designers have usually paid attention to magnetic shielding. These cost about 2-3 times as much as regular supplies, but have about 10x lower noise and reduced magnetic fringing fields might be worth it if you need really low noise.
- 6) Magnetic shielding can help pieces of soft iron or "mu-metal" strategically placed can reduce the fringe fields from power supplies. This doesn't always work as well as you might think it should; dipolar magnetic fields can easily "squirt" through holes in boxes, and putting a magnetic shield in one place will often increase the field somewhere else; this generally requires some experimentation, and can do more damage than good if you just do things blindly. With some attention, this can reduce magnetic fields by a factor of 3 or so without too much trouble.
- 7) Pay attention to what else is in the vicinity of low-noise, high-gain electronics. A computer monitor 3 feet away can be spewing out magnetic fields at 30 kHz; oscilloscopes and other power supplies can also couple magnetically into your circuit over distances of a couple feet. Because magnetic fields are usually dipolar fields, turning a box on its side can often change the pickup dramatically; this isn't always a recommended solution, but it sometimes helps in tracking things down.

