Capacitive Sensor Interfaces

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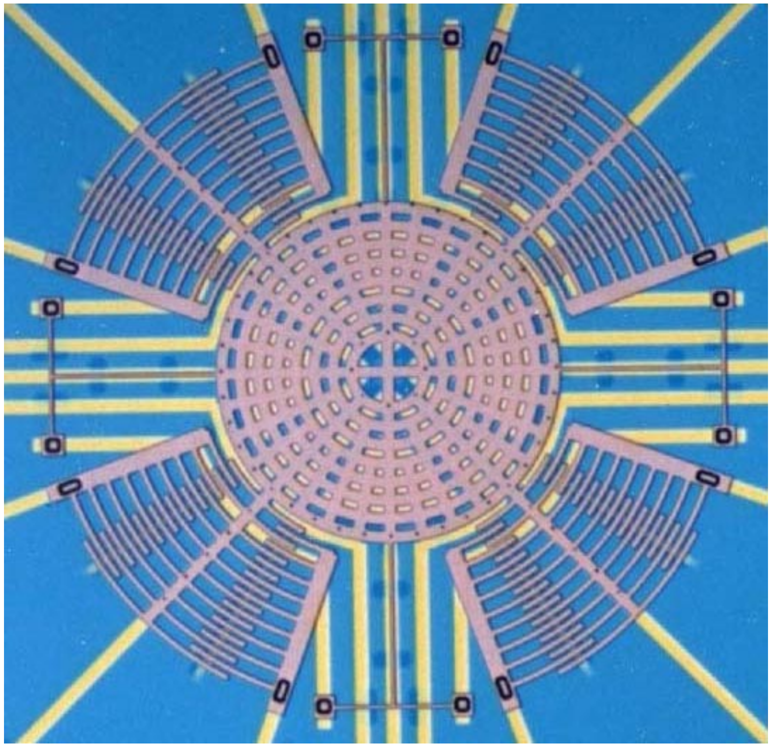
Example: Vibratory Gyroscope



Ref: T. Juneau et al., "Micromachined Dual Input Axis Angular Rate Sensor", Solid-State Sensor and Actuator Workshop, Hilton Head, SC, June 1996.

Electrostatic Interfaces for:

- vibration (about z-axis)
- x/y-axis tilt
- x/y-axis force feedback
- x/y-axis frequency tuning
- quadrature error cancellation
- . . .

Outline

- Capacitor Basics
- MEMS Capacitor Configurations
  - parallel plate
  - transverse comb
  - lateral comb
- Simulation
- Summary

Capacitor Basics

- Definition: $$C = \frac{Q}{V}$$
- Energy: $$E = \frac{1}{2} C V^2$$
- Force: $$F = \frac{\partial E}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} V^2$$
- Spring Constant: $$k = \frac{\partial F}{\partial x}$$
- Figure of Merit (for sensing): $$FM = \frac{{}^{\partial C}\!/_{\partial x}}{C}$$

Capacitors in MEMS

- always present …
  - no special fabrication steps required
  - must deal with in any case …
- versatile
  - sensor & actuator
  - negligible temperature coefficient
  - high accuracy: position measurements < 0.01Å demonstrated
- challenges
  - small signals, parasitics
  - undesired electrostatic actuation

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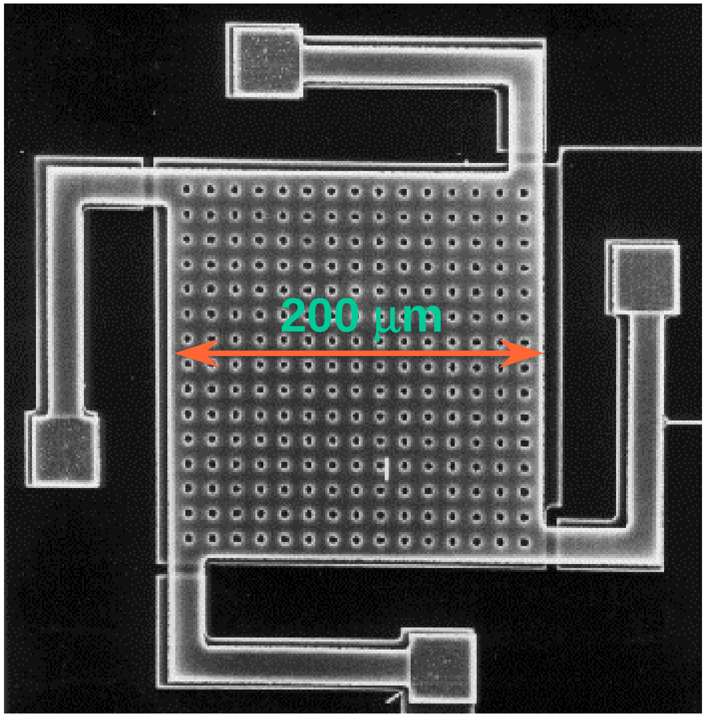
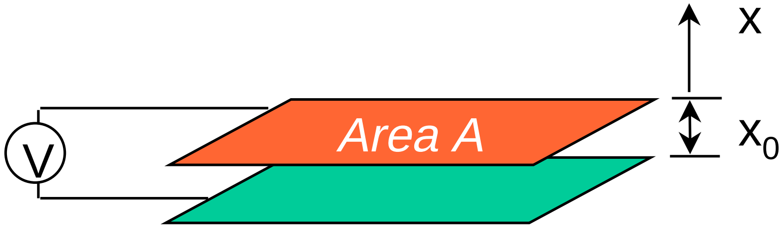
Parallel Plate Capacitor



Ref: D. Young et al., “A Micromachined Variable Capacitor for Monolithic Low-Noise VCOs”, Solid-State Sensor and Actuator Workshop, Hilton Head, SC, June 1996.



- MEMS Applications
  - accelerometers
  - gyroscopes
  - actuators
  - varactor replacement

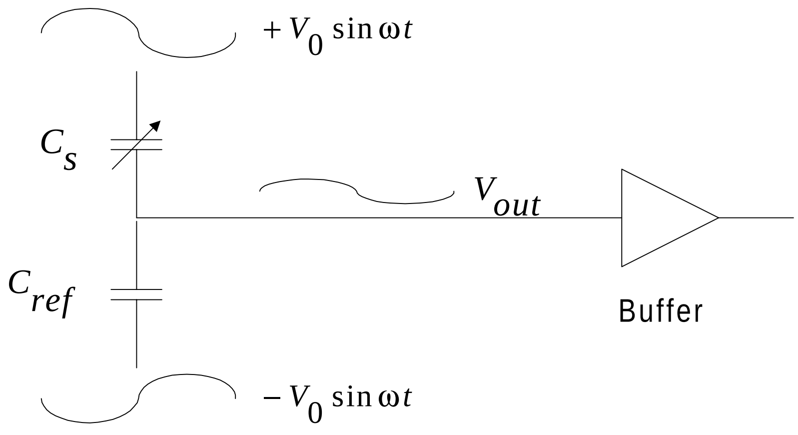
Parallel Plate Capacitor (cont.)

		Example
• Capacitance	$C = \frac{\varepsilon_0 A}{x_0 + x}$	1400 fF
• Sensitivity	$\frac{\partial C}{\partial x} = -\frac{C}{x_0 + x}$	-1400 fF/μm
• Force	$F = -\frac{1}{2}\frac{CV^2}{x_0 + x}$	-18 μN
• Spring Constant	$k = \frac{CV^2}{(x_0 + x)^2}$	35 N/m
• Figure of Merit	$FM = \frac{-1}{x_0 + x}$	-1μm^{-1}

(for $x_0 = 1\ \mu m$, $A = (400\ \mu m)^2$, $V = 5\ V$ (constant), $\varepsilon_0 = 8.85\ aF/\mu m$)

- *large capacitance & force (large area)*
- *nonlinear electrostatic spring*

Application: Position Sensing



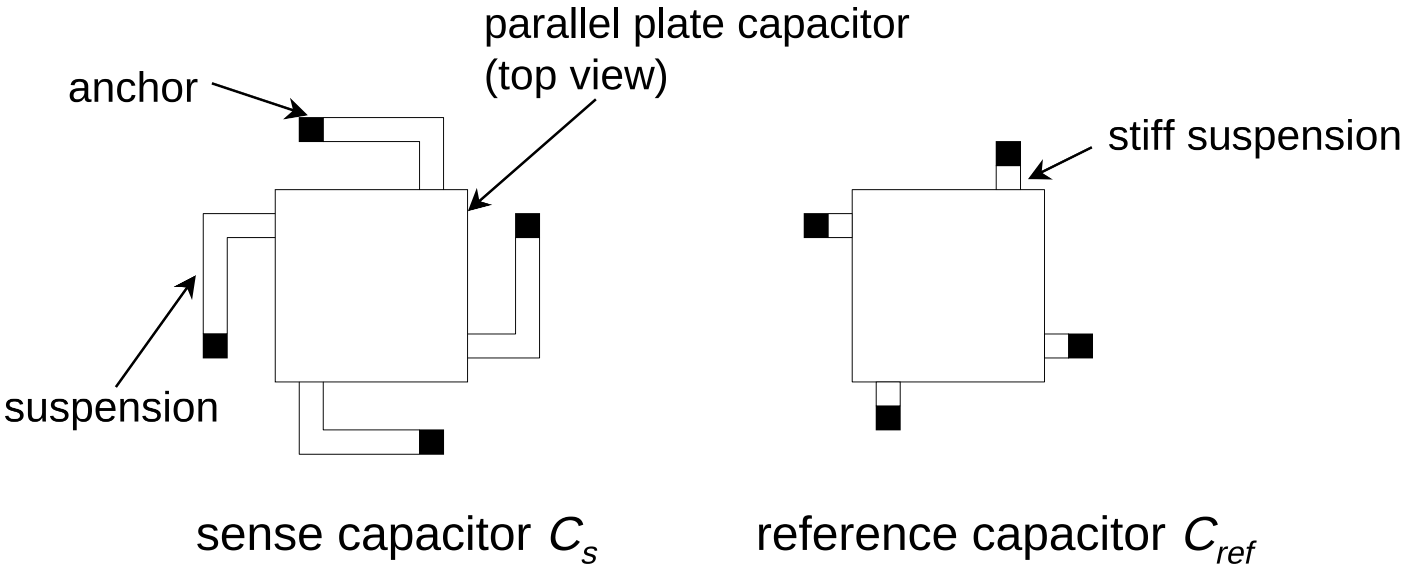
$$C_s = \frac{\varepsilon_0 A}{x_0 + x} \approx \underbrace{C_0}_{C|_{x=0}} - \frac{\partial C}{\partial x} x \propto x \qquad (x << x_0)$$

$$C_{ref} = C_0$$

$$V_{out} \approx \frac{V_0}{2}\left(\underbrace{FM \times x}_{signal} + \underbrace{\frac{C_0 - C_{ref}}{C_0}}_{offset}\right)$$

- minimize offset: match C_{ref} to C_0
- output proportional to x ($x << x_0$)
- sense voltage V_0: parasitic force

Matching C_{ref} to C_0

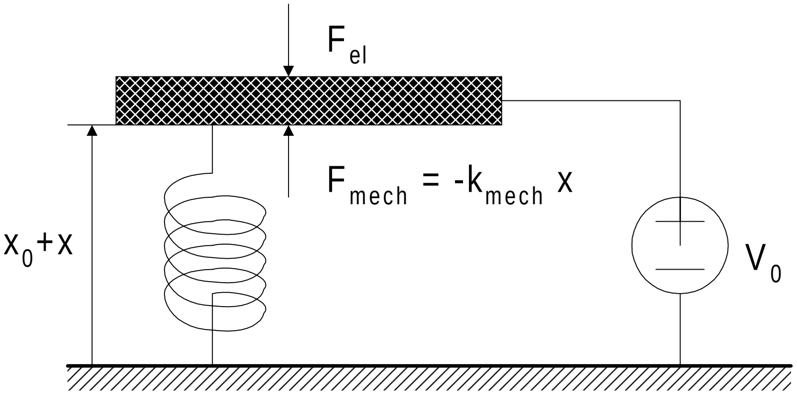


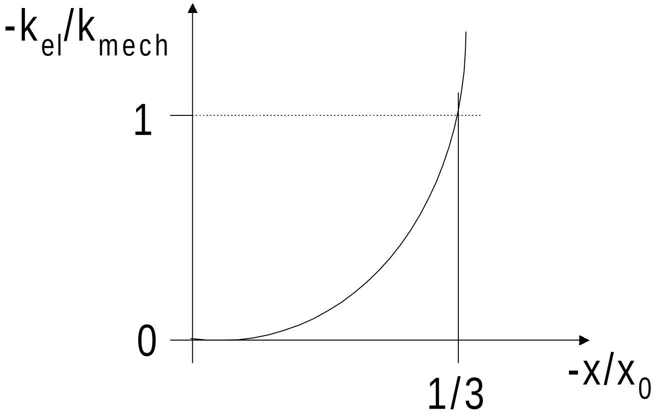
sense capacitor C_s

reference capacitor C_{ref}

Ref.: W. Yun et al., “Surface micromachined, digitally force-balanced accelerometer with integrated CMOS detection circuitry”, Solid-State Sensor and Actuator Workshop, Hilton Head, 1992, pp. 21-25.

Pull-In Voltage





$$F_{el} = F_{mech} \quad \rightarrow \quad -V_0^2 = \frac{2k_{mech}x(x_0 + x)}{C}$$

$$k_{el} = -k_{mech}\frac{2x}{x_0 + x}$$

Pull-In Voltage (cont.)

- electrostatic force always positive, $F_{el} \geq 0$
- pull-in occurs when $|k_{el}| > |k_{mech}|$

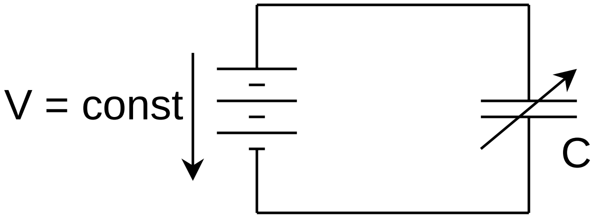
$$|x| > \frac{x_0}{3}$$

$$V_0 > \frac{2x_0}{3}\sqrt{\frac{k_m}{1.5C_0}}$$

e.g. $x_0 = 1\mu m$, $C_0 = 1pF$, $k_m = 1N/m$ → $|V_0| < 0.54$ V to avoid pull-in

Voltage versus Charge

constant voltage



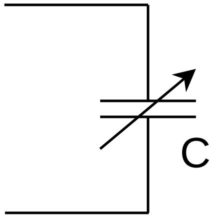
$$E = \frac{1}{2} C V^2$$

$$F = -\frac{1}{2} \frac{C V^2}{x_0 + x}$$

$$k = \frac{C V^2}{(x_0 + x)^2}$$

force dependent on *x*
(quadratic in *V*)

constant charge



$$E = \frac{1}{2} \frac{Q^2}{C}$$

$$F = \frac{1}{2} \frac{Q^2}{\varepsilon_0 A}$$

$$k = 0$$

force independent of *x, no pull-in*
(quadratic in *Q*)

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  - parallel plate
  - ⇨transverse comb
  - lateral comb
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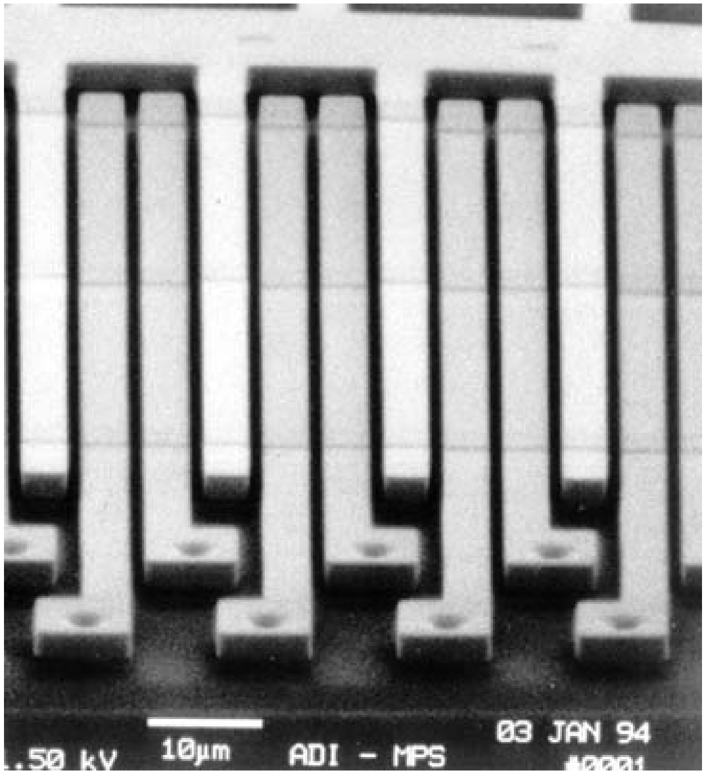
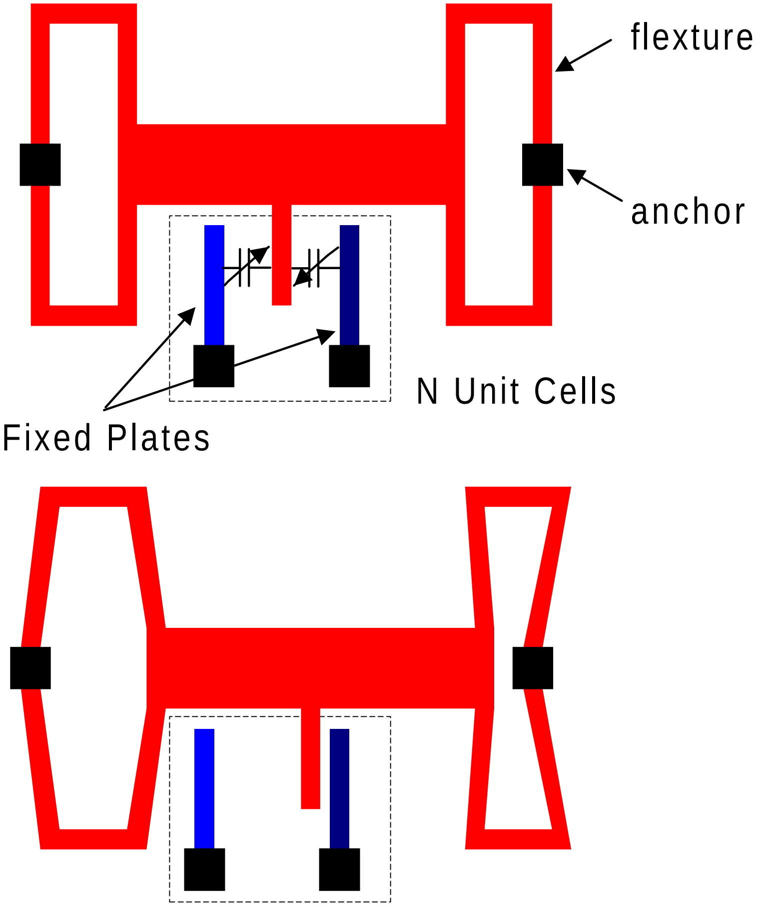
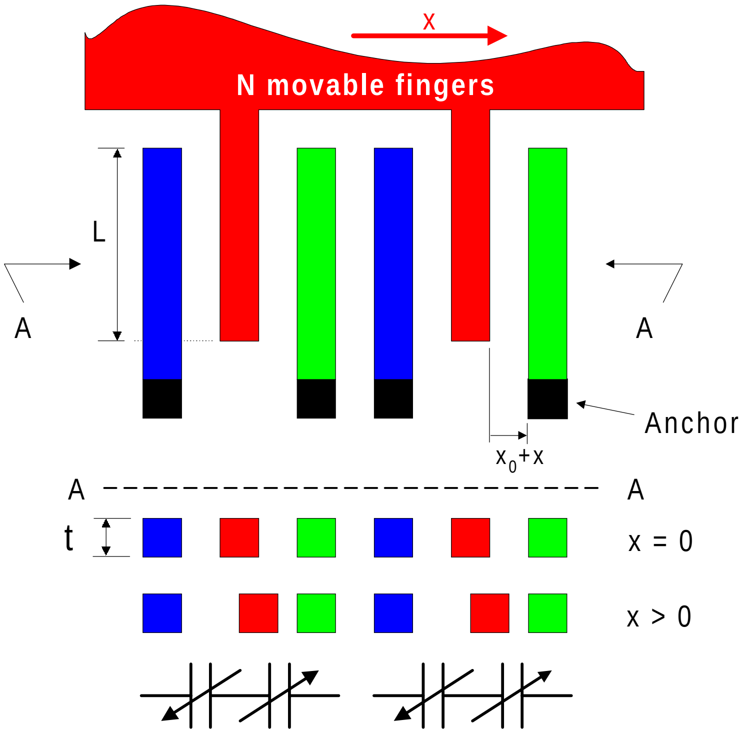
Transverse Comb

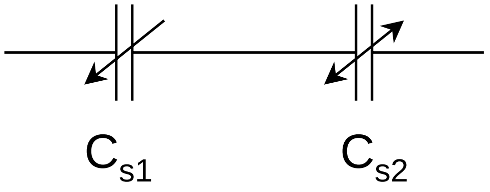


Ref: Analog Devices ADXL-50



Transverse Comb (cont.)



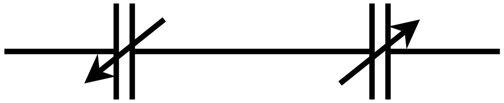


$$C_{s1} = N\left[\frac{\varepsilon_0 L t}{x_0 + x} + C_{fringe}\right]$$

$$C_{s2} = N\left[\frac{\varepsilon_0 L t}{x_0 - x} + C_{fringe}\right]$$

Transverse Comb (cont.)

$C_{s1} \approx C\big|_{x=0} - \frac{\partial C}{\partial x} x$

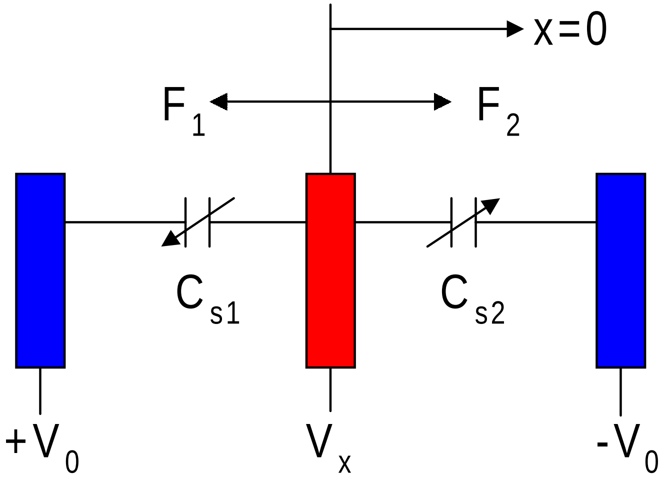


$C_{s2} = C\big|_{x=0} + \frac{\partial C}{\partial x} x$

		Example
• Capacitance	$C\big|_{x=0} = \underbrace{\frac{\varepsilon_0 NLt}{x_0}}_{C_0} + C_{fringe}$	125 fF
• Sensitivity	$\frac{\partial C}{\partial x} \approx \frac{C_0}{x_0}$	105 fF/μm
• Figure of Merit	$FM \approx \frac{1}{x_0}\left(1 - \frac{C_{fringe}}{C_0}\right)$	$0.8\mu m^{-1}$

for $x_0 = 1\ \mu m$, L = 150 μm, t = 2 μm, N = 40, V = 5 V (constant), $C_{fringe}/C_0 = 0.2$

Differential Force (x=0)



$$\Delta F = F_1 - F_2$$

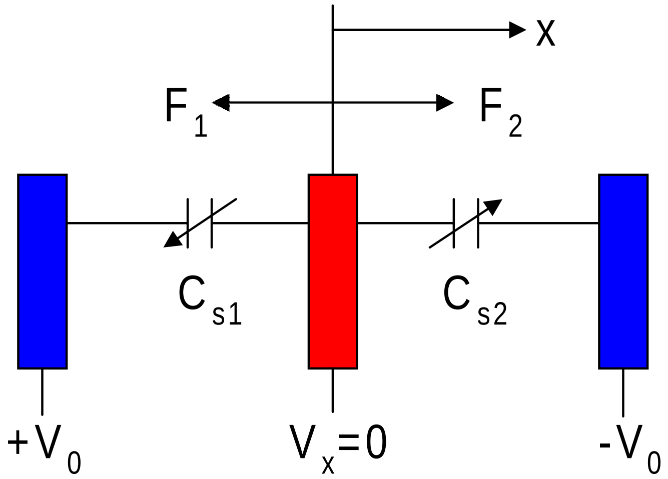
$$\approx -\frac{1}{2}\frac{C_0}{x_0}\left[\left(V_0 - V_x\right)^2 - \left(V_0 + V_x\right)^2\right]$$

$$\approx \frac{2C_0 V_0 V_x}{x_0}$$

e.g. $\Delta F/V_x = 1.25\mu N/V$ for $V_0 = 5V$, $C_0 = 125fF$, $x_0 = 1\mu m$

- linear voltage-force relationship

Electrostatic Spring ($V_x = 0$)



$$k_{el} = \frac{d}{dx}\left(F_1 - F_2\right)$$

$$= \frac{d}{dx}\frac{\varepsilon_0 A V_0^2}{2}\left[\frac{1}{\left(x_0 + x\right)^2} - \frac{1}{\left(x_0 - x\right)^2}\right]$$

$$\approx -\frac{2C_0 V_0^2}{x_0^2} \qquad x << x_0$$

e.g. $k_{el} = -0.1\ N/m$ for V_0 = 1V, C_0 = 100fF, x_0 = 1µm

Resonant Frequency Shift

- linear second order mechanical system:

$$\omega_r = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{k_{mech} + k_{el}}{m}}$$

$$= \omega_0 \sqrt{1 + \frac{k_{el}}{k_{mech}}}$$

e.g.

ω_0 = 50 krad/sec

m = 0.1 μg

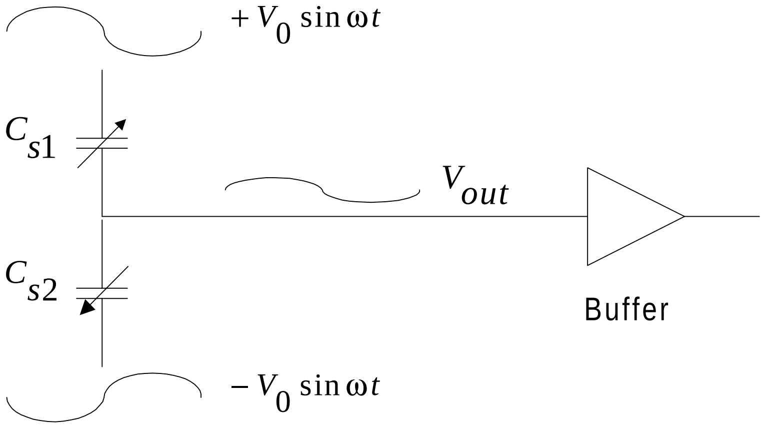
$k_{mech} = m\,\omega_0^2$ = 0.25 N/m

k_{el} = -0.1 N/m

ω_r/ω_0 = <u>0.78</u>

- substantially reduced resonance
- negative ω_r possible

Differential Position Sensing



$$V_{out} \approx V_0 \frac{\partial C / \partial x}{C_0} x = V_0 \times FM \times x$$

$V_{out}/x = V_o \times FM = 4V/\mu m = 400\mu V/Å$

- output proportional to *x* for $x \ll x_0$
- sense force almost canceled: $F_1 \approx F_2$

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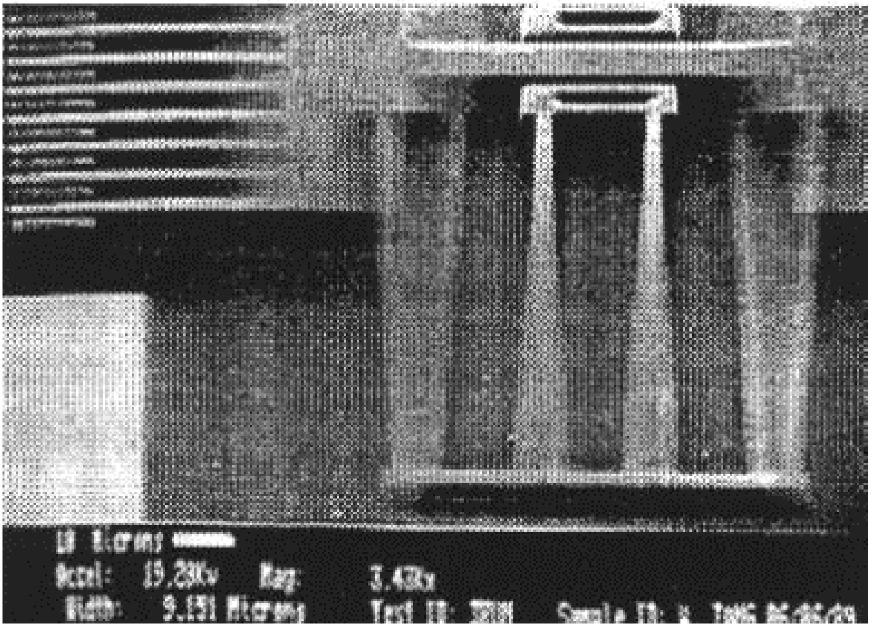
Lateral Comb

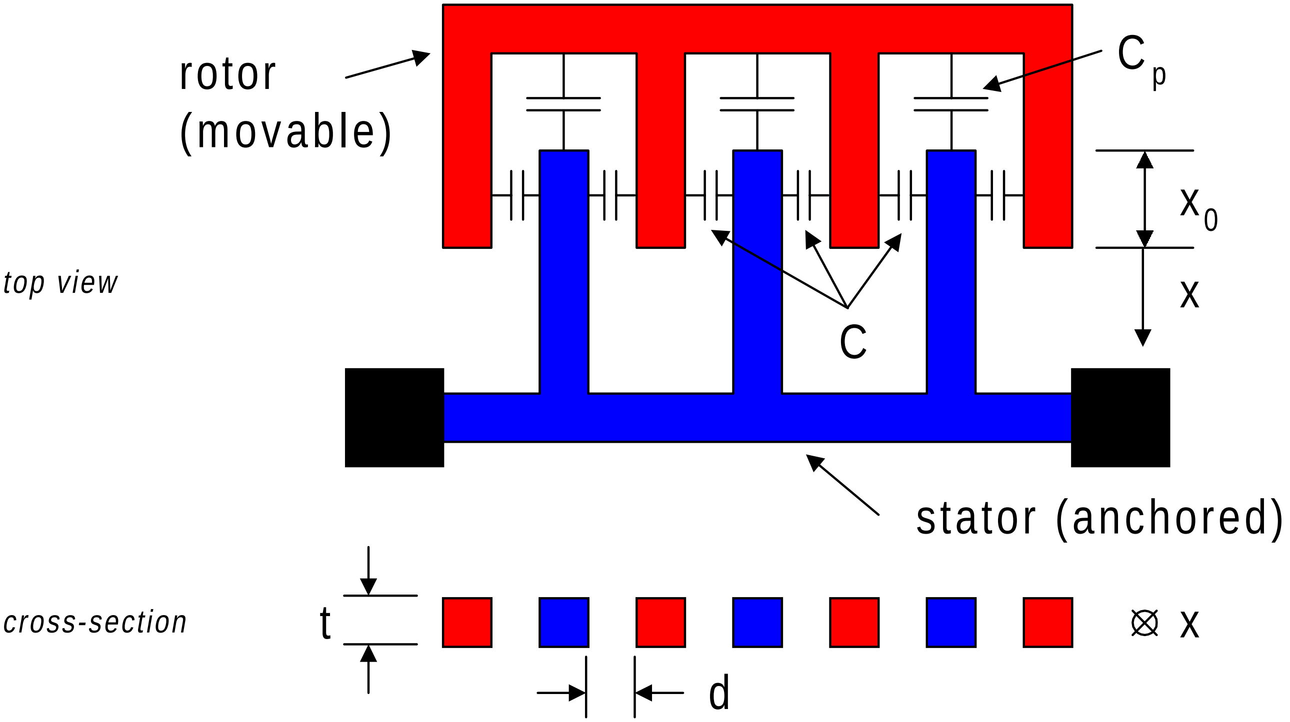


lateral comb drive

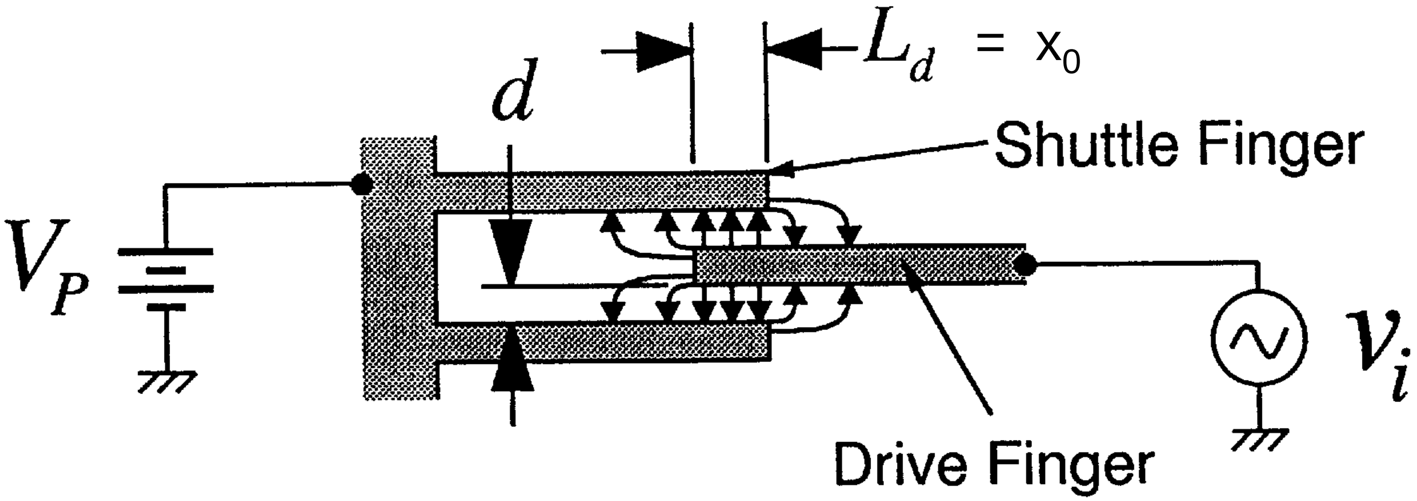
resonator

Ref.: W. Tang, “Electrostatic comb drive for resonant sensor and actuator applications”, Ph.D. Thesis, UC Berkeley, EECS, 1990.

Lateral Comb Geometry



Capacitance per Finger



Mutual Capacitance: $$C = N\frac{\varepsilon_0 t(x_0 + x)}{d} + \underbrace{NC_p}_{parasitic}$$

Lateral Comb

		Example
• Capacitance	$C = N\frac{\varepsilon_0 t(x_0 + x)}{d}\left[+C_p\right]$	9 fF
• Sensitivity	$\frac{\partial C}{\partial x} \approx N\frac{\varepsilon_0 t}{d}$	1.8 fF/µm
• Force	$F = -\frac{1}{2}\frac{N\varepsilon_0 t}{d}V^2$	22 fN
• Spring Constant	$k = 0$	0
• Figure of Merit	$FM \approx \frac{1}{x_0 + x}$	$0.2\mu m^{-1}$

(for $x_0 = 5\ \mu m$, $t = 2\ \mu m$, $d = 1\ \mu m$, $N = 100$, $V = 5V$, $C_p = 0$)

Lateral Comb Characteristics

- linear: C proportional to x
- no electrostatic spring
- main application:

  linear forcer, e.g. in resonator
  (use differential setup to cancel nonlinearity in voltage)
- challenges:

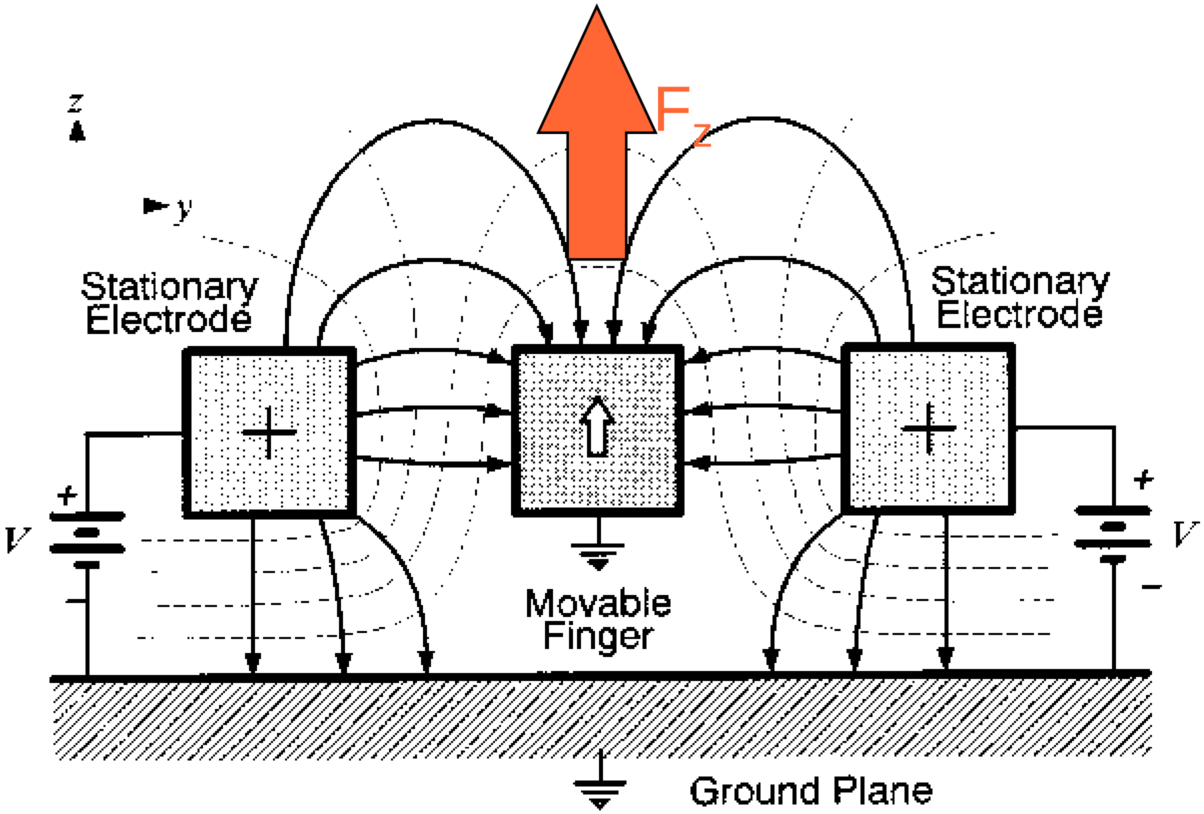
  parasitics introduce nonlinearity
  poor sensitivity dC/dx
  small forces for standard supply voltages (5V)

Exact Analysis of Lateral Force

- W.A. Johnson et al., “Electrophysics of Micromechanical Comb Actuators”, IEEE J. Electromech. Systems, pp. 49-59, March 1995. (includes fringing field effects)

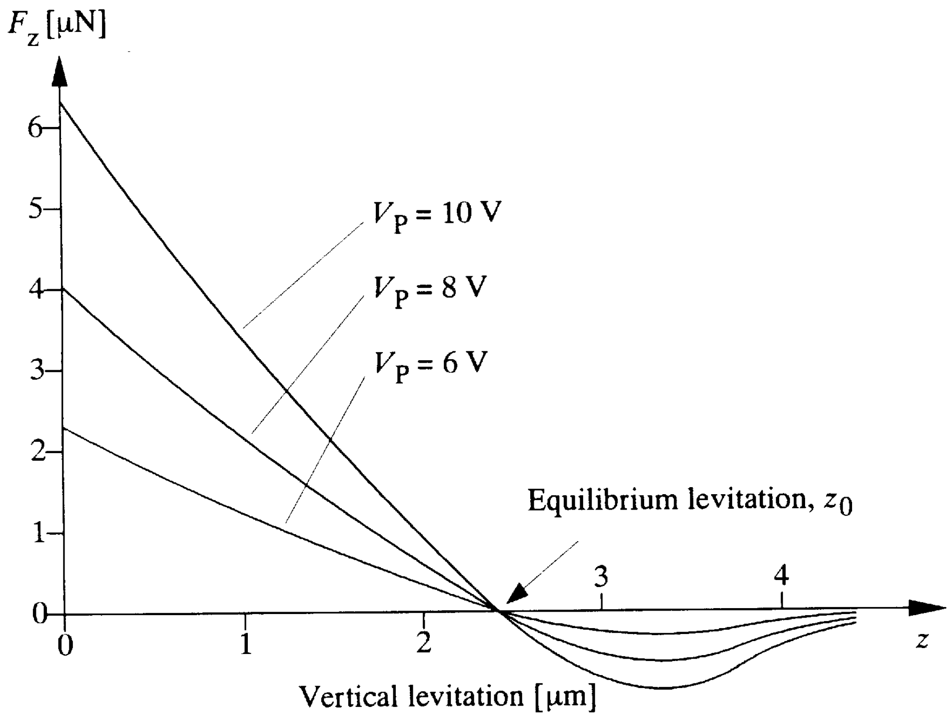
- G. Fedder, “Simulation of Microelectromechanical Systems”, Ph.D. thesis, UC Berkeley, EECS, 1994.

Levitation Effects

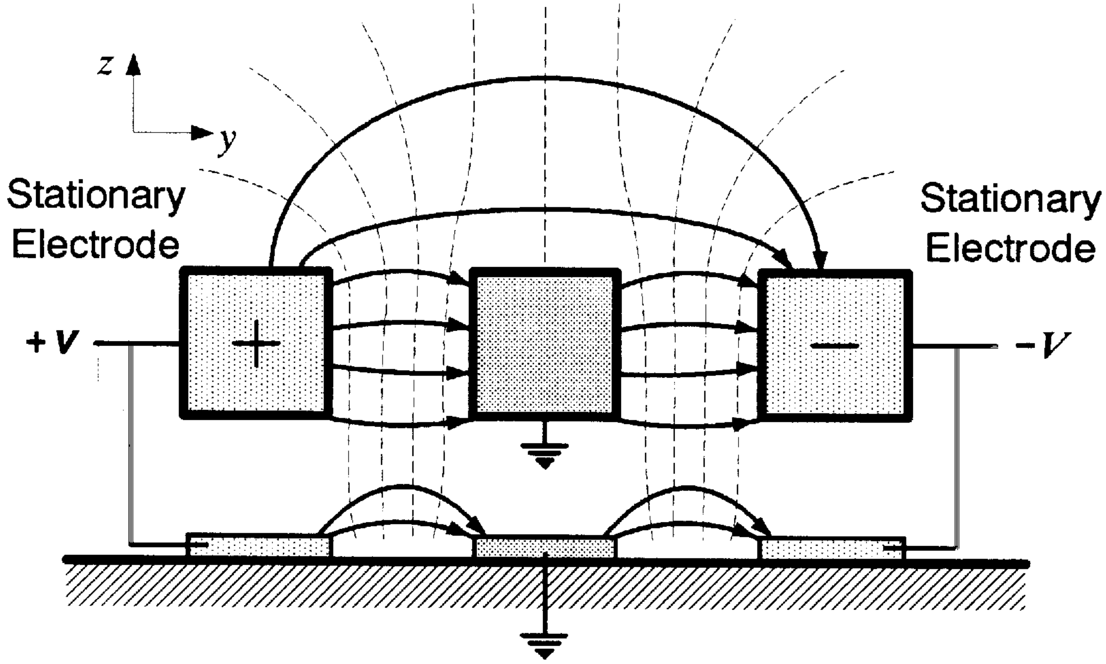


Ref: W. Tang, “Electrostatic Comb Drive for Resonant Sensor and Actuator Applications”, Ph.D. thesis, UC Berkeley, EECS, 1990.

Levitation Force

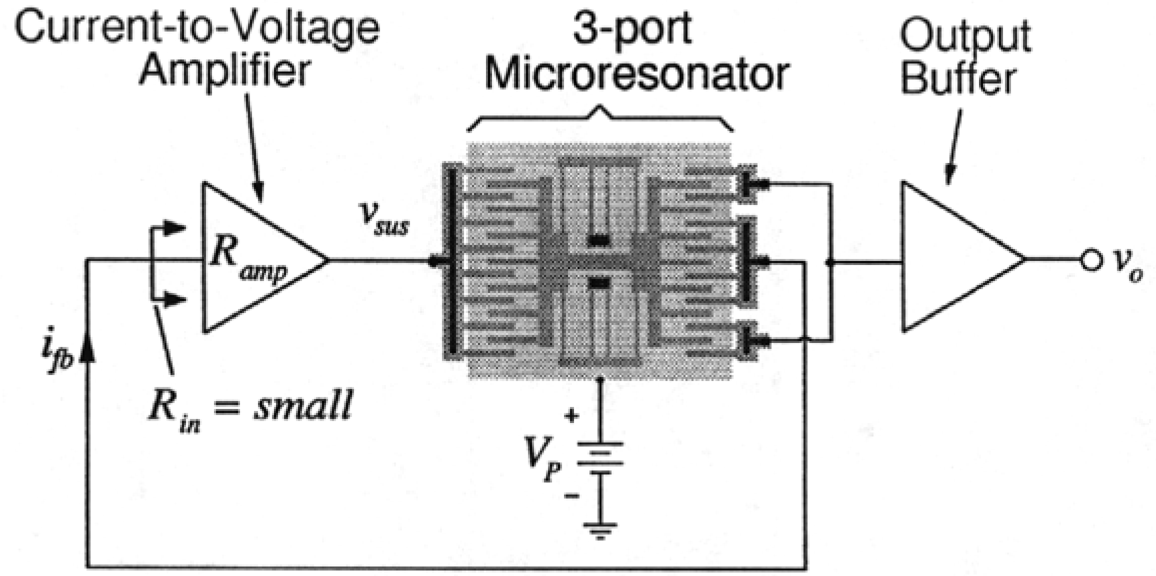


Levitation Suppression



- sliced ground-plane reduces levitation force by order-of-magnitude

Lateral Comb Resonator



Ref: C. Nguyen, “Micromechanical Signal Processors”, Ph.D. thesis, UC Berkeley, EECS, 1994.

Simulation

- Electrostatic Solvers:

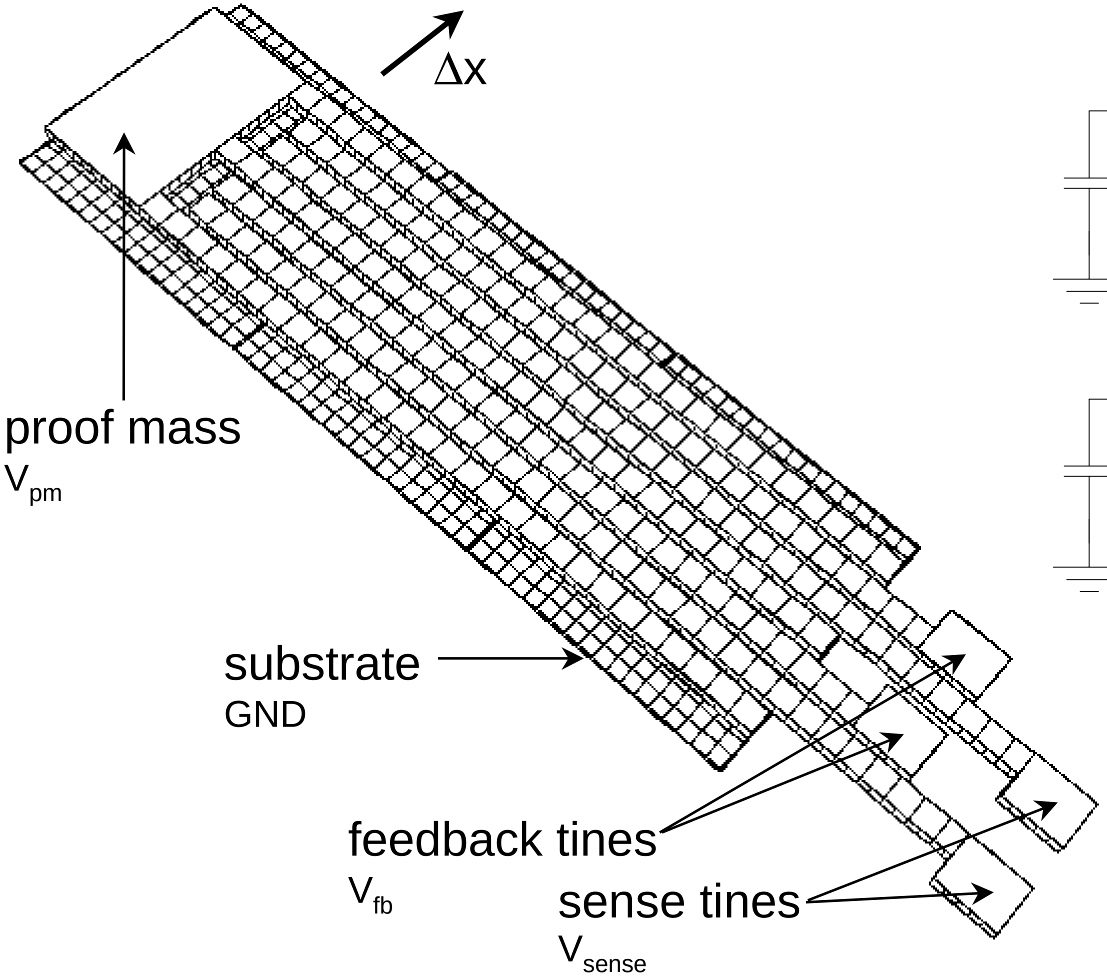
Fastcap	http://rle-vlsi.mit.edu/projects.html
Maxwell	Ansoft, Pittsburgh, Pennsylvania, 1991.
General Text	H. H. Woodsen and J. R. Melcher: Electromechanical Dynamics, Part I: Discrete Systems, R. E. Krieger Publishing, Malabar, Florida 32950, 1990. Reprinted from J. Wiley edition of 1968.

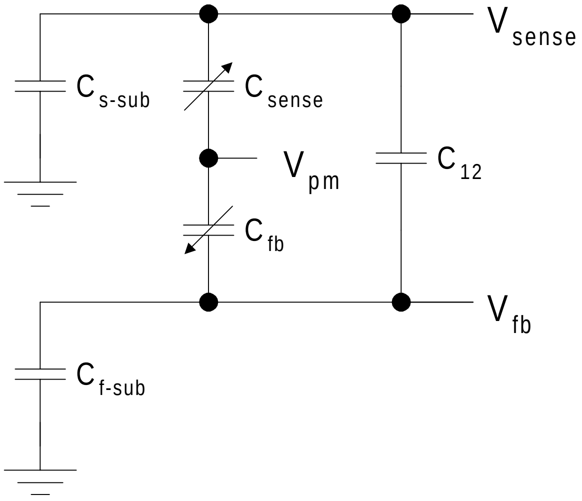
- Self-Consistent Electromechanical Simulation:

Simultaneous solution of electrostatic and mechanical equations required for large displacements *x:*

MEMCAD 2.0	http://www-mems.mit.edu/groups/SenturiaGroup.html

Fastcap Example: Transverse Comb





Boundary Element (BEM) Generation

```
cubegen -n1 -xo0 -yo0 -zo0 -xh4 -yh115 -zh2 -natine_pm > tine_pm.qui
cubegen -n1 -xo0 -yo0 -zo0 -xh4 -yh144 -zh2 -natine_sense > tine_sense.qui
cubegen -n1 -xo0 -yo0 -zo0 -xh4 -yh124 -zh2 -natine_drive > tine_drive.qui
cubegen -n1 -xo0 -yo0 -zo0 -xh8 -yh10 -zh2 -naendtine > endtine.qui

cubegen -n1 -xo0 -yo0 -zo0 -xh34 -yh20 -zh2 -naproofmass > proofmass.qui
capgen -p1 -n20 -w45 -nasubstrate > subst.qui
```

BEM Assembly

```
* PROOF MASS
C tine_pm.qui 1 0.020000 0 0+
C tine_pm.qui 1 15.020000 0 0+
C tine_pm.qui 1 30.020000 0 0+
C proofmass.qui 1 0.020000 -19.99 0
*
* SENSE TINES
C tine_sense.qui 1 10 5 0 +
C tine_sense.qui 1 25 5 0 +
C endtine.qui 1 10 148.99 0 +
C endtine.qui 1 25 148.99 0
*
* FEEDBACK TINES
C tine_drive.qui 1 5 5 0 +
C tine_drive.qui 1 20 5 0 +
C endtine.qui 1 1 128.99 0 +
C endtine.qui 1 16 128.99 0
*
* SUBSTRATE
C subst.qui 1 -5.5 -20 -1.6 +
C subst.qui 1 -5.5 25 -1.6 +
C subst.qui 1 -5.5 70 -1.6
```

File `lsensor_0.020000.bem`

$\Delta x = 0.02\ \mu m$

(one file for each x)

Run Fastcap

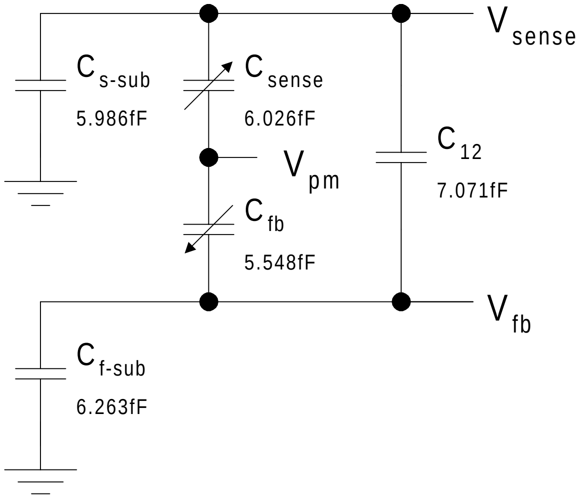
One simulation run for each Δx = -0.05µm to +0.05µm:

```
fastcap -lsensor_-0.050000.bem > out-0.050000
fastcap -lsensor_-0.040000.bem > out-0.040000
fastcap -lsensor_-0.030000.bem > out-0.030000
fastcap -lsensor_-0.020000.bem > out-0.020000
fastcap -lsensor_-0.010000.bem > out-0.010000
fastcap -lsensor_0.000000.bem  > out0.000000
fastcap -lsensor_0.010000.bem  > out0.010000
fastcap -lsensor_0.020000.bem  > out0.020000
fastcap -lsensor_0.030000.bem  > out0.030000
fastcap -lsensor_0.040000.bem  > out0.040000
fastcap -lsensor_0.050000.bem  > out0.050000
```

Fastcap Output

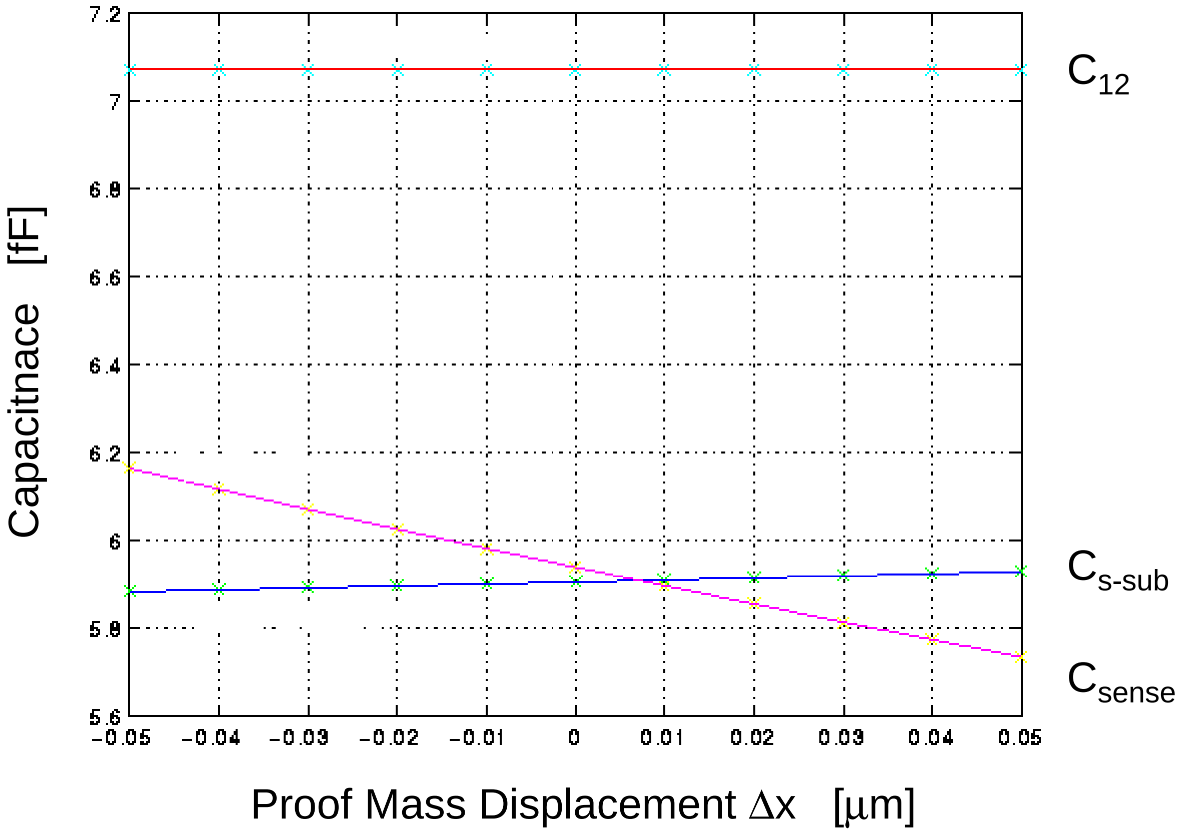
`out0.020000`   ($\Delta x = 0.02\mu m$)

```
Running fastcap 2.0 (15Jul92)
  Input: sensor_-0.020000.lst
  Input surfaces:
   GROUP1
    tine_pm.qui, conductor
      title: `4mX115mX2m cube (n=1 e=0.1)'
      outer permittivity: 1
      number of panels: 174
      number of extra evaluation points: 0
      translation: (0.02 0 0)
    tine_pm.qui, conductor
      title: `4mX115mX2m …
```



```
CAPACITANCE MATRIX, femtofarads
                      Vpm         Vsense        Vfb        GND
Vpm                  28.81        -6.026       -5.548     -16.54
Vsense               -6.026       19.9         -7.071      -5.896
Vfb                  -5.548       -7.071       19.29       -6.263
GND                 -16.54        -5.896       -6.263      30.58
```

BEM Analysis Summary (Matlab)



(levitation!)

$$\frac{\Delta C_{sense}}{\Delta x} \approx 4.1\ fF/\mu m$$

Summary

- capacitive interfaces:
  - position sensing
  - electrostatic forcer

- interface types:

– parallel plate:	large C, negative spring, asymmetric
– transverse comb:	symmetric, good position sense, ω_r tuning
– lateral comb:	linear forcer, small dC/dx