Analytical Foundations of Current-Feedback Amplifiers

Sergio Franco
San Francisco State University
San Francisco, CA 94132 USA

Abstract--*Current-feedback amplifiers* (CFAs), also called *transimpedance amplifiers*, are a special breed of operational amplifiers (op amps) that exploit clever circuit topologies in conjunction with high-speed complementary bipolar processes to achieve extremely fast dynamics, such as gain-bandwidth products in the gigahertz range and settling times to 0.1% in the 10 ns range. This tutorial[1] addresses the analytical foundations of CFAs.

I. THE CURRENT-FEEDBACK AMPLIFIER

As shown in Fig. 1a, the CFA architecture is centered around an input buffer and a transimpedance stage [1].

The input buffer is a *unity-gain voltage buffer* connected across the inputs. Its function is to force V_n to follow V_p, very much like a conventional op amp does via negative feedback. However, because of the buffer's low output impedance, current can easily flow in or out of the inverting input, though we shall see that in the steady state (non-slewing) condition this current approaches zero.

Amplification is provided by a *transimpedance* stage which senses the current I_n delivered by the buffer to the external feedback network, and produces an output voltage

$$V_o = z(jf)\, I_n \tag{1}$$

where $z(jf)$ is the *transimpedance gain* of the amplifier, in V/A or Ω, and I_n is the current out of the inverting input.

To fully appreciate the inner workings of the CFA, it is instructive to examine the diagram if Fig. 2, top. The input buffer is made up of transistors Q_1 through Q_4. Q_1 and Q_2 form a low output-impedance push-pull stage, and Q_3 and Q_4 provide V_{BE} compensation as well as a Darlington function to raise the input impedance. Summing currents at the inverting node yields $I_1 - I_2 = I_n$, where I_1 and I_2 are the push-pull transistor currents. A pair of Wilson current mirrors, made up of transistors Q_9-Q_{10}-Q_{11} and Q_{13}-Q_{14}-Q_{15}, reflect these currents and recombine them at a common node, whose equivalent capacitance to ground is denoted as C. By mirror action, the current through this capacitance is $I_1 - I_2$, or I_n. The voltage developed by C in response to I_n is then conveyed to the output via a second buffer, made up of Q_5 through Q_8. Figure 2, bottom, summarizes the CFA features in block-diagram form.

When the feedback loop is closed as in Fig. 1b, any attempt to imbalance the inputs will cause the input buffer to source (or sink) an imbalance current I_n to the external network. This imbalance is then conveyed by the Wilson mirrors to C, causing V_o to swing in the positive (or negative) direction until the original imbalance current is neutralized via the negative feedback loop. Clearly, I_n is the *error signal* in this feedback system.

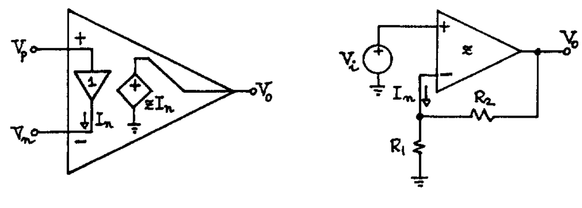
To obtain the closed-loop transfer characteristic we exploit the fact that the input buffer keeps $V_n = V_p = V_i$. Applying the superposition principle yields

$$I_n = \frac{V_i}{R_1 /\!/ R_2} - \frac{V_o}{R_2} \tag{2}$$

indicating that the feedback signal, V_o/R_2, is a *current*. Substituting into (1) and collecting, we obtain

$$A(jf) = \frac{V_o}{V_i} = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + 1/T(jf)} \tag{3}$$

$$T(jf) = z(jf)/R_2 \tag{4}$$



(a) (b)
Fig. 1. Circuit model of the CFA, and the noninverting amplifier configuration.

where $A(jf)$ is the *closed-loop gain* of the circuit, and $T(jf)$ is the *loop gain*. The designation *loop gain* stems from the fact that if we break the loop as in Fig. 3a, and inject a test signal V_x with V_i suppressed, the circuit will first convert V_x to the current $I_n = -V_x/R_2$, and then convert I_n to the voltage $V_o = zI_n$. The gain experienced by V_x is thus $V_o/V_x = -z/R_2$. Its *negative* is the loop gain, or (4).

The loop gain gives a measure of how close $A(jf)$ is to the ideal value $1 + R_2/R_1$. By (3), the larger T, the better. To ensure a substantial loop gain over a wide range of closed-loop gains, the manufacturer strives to make $z(jf)$ as large as possible relative to the expected range of values of R_2. Consequently, since $I_n = V_o/z$, the inverting-input current will be very small, even though this input is a low-impedance node because of the buffer. In the limit

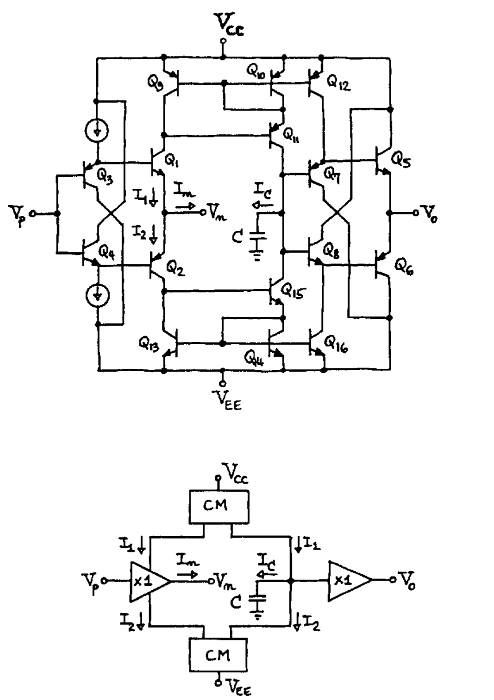


Fig. 2. Simplified circuit and block diagrams of the CFA. (*Courtesy of Comlinear Corporation.*)

[1] Based on an article published in EDN Magazine (Jan. 5, 1989), Copyright by Cahners Publishing Company 1993.

0-7803-1254-6/93$03.00 © 1993 IEEE

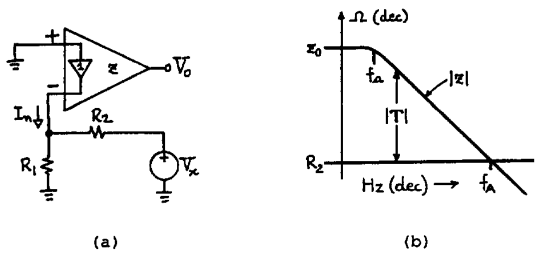


Fig. 3. Test circuit to find the loop gain $T(jf)$, and graphical method to find the closed-loop bandwidth f_A.

$z \to \infty$ we obtain $I_n \to 0$, indicating that *ideally a* CFA *will provide whatever output is needed to drive* I_n *to zero.* Thus, the familiar op amp conditions $V_n \to V_p$, $I_n \to 0$, and $I_p \to 0$ hold also for CFAs, though for different reasons.

II. CFA DYNAMICS

The transimpedance gain of a practical CFA is dominated by a single pole and it rolls off with frequency as

$$z(jf) = \frac{z_0}{1 + jf/f_a} \tag{5}$$

where z_0 is the *dc value* of the transimpedance gain, and f_a is the frequency at which rolloff begins. The popular CLC401 CFA (Comlinear Co.) has $z_0 \simeq 710$ kΩ and $f_a \simeq 350$ kHz. Moreover, since $f_a = 1/(2\pi z_0 C)$, it follows that $C = 1/(2\pi z_0 f_a) \simeq 0.64$ pF.

Substituting (5) into (4) and then into (3), and exploiting the fact that $R_2/z_0 \ll 1$, we obtain

$$A(jf) = \frac{A_0}{1 + jf/f_A} \tag{6}$$

$$A_0 = 1 + R_2/R_1 \tag{7}$$

$$f_A = z_0 f_a / R_2 \tag{8}$$

where A_0 is the *closed-loop dc gain*, and f_A the *closed-loop bandwidth*. With R_2 in the kΩ range, f_A is typically in the 100 MHz range. We can visualize T and f_A graphically as follows. If we define $|T|_{dec} = \log_{10} |T|$, then we have, by (4), $|T|_{dec} = \log_{10} |z| - \log_{10} |R_2|$, or

$$|T|_{dec} = |a|_{dec} - |R_2|_{dec}$$

indicating that we can visualize the loop gain graphically as the *difference* between the decade plot of $|z|$ and that of R_2. The frequency at which the two curves meet is called the *crossover frequency*. Since at this frequency $T = 1\underline{/-90^\circ} = -j$, then, by (3), $|A| = A_0/|1 + j| = A_0/\sqrt{2}$. Consequently, the crossover frequency is also the -3 dB frequency of $A(jf)$, or the closed-loop bandwidth f_A.

Equation (8) shows that for a given CFA the bandwidth depends only on R_2. We can thus use R_2 to select the bandwidth f_A, and R_1 to select the gain A_0. The ability *to control gain independently of bandwidth* constitutes the first major advantage of CFAs over conventional op amps. As we know, the latter exhibit a *gain-bandwidth tradeoff*.

The other major advantage of CFAs over conventional op amps is the *absence of slew-rate limiting*. Indeed, if we apply an input step V_I, the resulting current imbalance I_n yields an output V_o such that $I_n = C dV_o/dt$. Substituting into (2) and rearranging yields

$$R_2 C \frac{dV_o}{dt} + V_o = A_0 V_I$$

indicating an exponential output transient with time constant $\tau = R_2 C$. Like the frequency response, the transient response is governed by R_2, regardless of A_0. For instance, a CLC401 CFA with $R_2 = 1.5$ kΩ has $\tau = R_2 C = 1.5 \times 10^3 \times 0.64 \times 10^{-12} \simeq 1$ ns. The rise time is $t_r = 2.2\tau \simeq 2.2$ ns, and the settling time within 0.1% of the final value is $t_s \simeq 7\tau \simeq 7$ ns, in reasonable agreement with the data-sheet values $t_r = 2.5$ ns and $t_s = 10$ ns.

III. SECOND-ORDER EFFECTS

The above analysis indicates that once R_2 has been set, the dynamics of the amplifier are unaffected by the closed-loop gain setting. In practice it is found that bandwidth and rise time do vary with gain somewhat, though not as drastically as with conventional op amps. The main cause is the nonzero *output impedance* R_0 of the input buffer, whose effect is to alter the loop gain and, hence, the closed-loop dynamics. We shall refer to Fig. 4 to investigate the effect of R_0 and, subsequently, the effect of any capacitances that may be present either at the input or in the feedback path.

Consider first the case of purely resistive feedback ($C_1 = C_2 = 0$). The circuit first converts V_x to the current $I_x = V_x/(R_2 + R_1 /\!/ R_0)$, then it divides I_x to yield $I_n = -I_x R_1/(R_1 + R_0)$, and finally it converts I_n to $V_o = zI_n$. Eliminating I_x and I_n and letting $T = -V_o/V_x$,

$$T(jf) = z(jf)/Z_2 \tag{9}$$

$$Z_2 = R_2 + A_0 R_0 \tag{10}$$

Clearly, the effect of R_0 is to *increase* the baseline curve from R_2 to $Z_2 = R_2 + A_0 R_0$ (see curve 0 of Fig. 4, bottom.) Consequently, f_A will decrease and t_r will increase in proportion. Replacing R_2 with Z_2 in (8), the closed-loop bandwidth becomes

$$f_A = \frac{z_0 f_a}{R_2 + A_0 R_0} \tag{11}$$

Example 1. A CFA has $R_0 = 50$ Ω, $R_2 = 1.5$ kΩ, and $z_0 f_a / R_2 = 100$ MHz. Find f_A for $A_0 = 1$, 10, and 100 V/V.

Solution. By (11), $f_A = 10^8/(1 + A_0/30)$. The bandwidths for the given values of A_0 are, respectively,

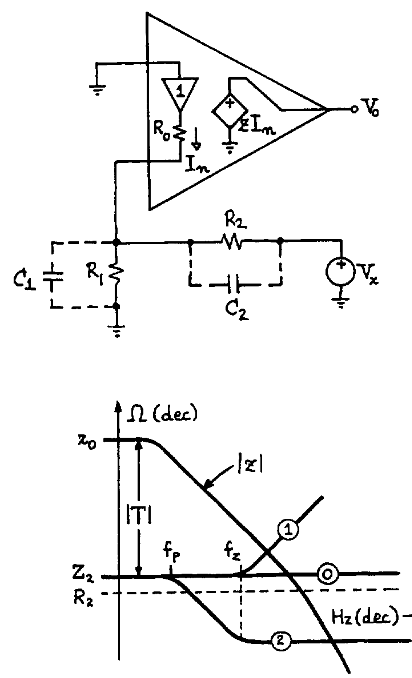


Fig. 4. Top: test circuit to investigate the effect of R_0; bottom: decade plots of $|Z_2|$ for (0) purely resistive feedback, (1) a capacitance C_1 in parallel with R_1, and (2) a capacitance C_2 in parallel with R_2.

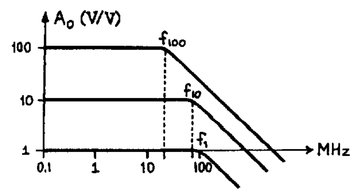


Fig. 5. Effect of R_0 upon f_A as a function of A_0.

f_1 = 96.8 MHz, f_{10} = 75.0 MHz, and f_{100} = 23.1 MHz, and are shown in Fig. 5. These values still compare favorably with a conventional op amp, whose bandwidth would be reduced, respectively, by 1, 10, and 100.

The values of R_1 and R_2 can be predistorted to compensate for the reduction in bandwidth. Using (11) we find R_2 for a given bandwidth f_A and dc gain A_0,

$$R_2 = \frac{z_0 f_a}{f_A} - A_0 R_0 \tag{12}$$

and using (7) we find R_1 for the given dc gain A_0,

$$R_1 = \frac{R_2}{A_0 - 1} \tag{13}$$

Example 2. Redo Example 1 so that f_{10} = 100 MHz.

Solution. Since with R_2 = 1.5 kΩ we have $z_0 f_a / R_2$ = 100 MHz, it follows that $z_0 f_a = 1.5 \times 10^{11}$ Ω × Hz. For A_0 = 10 and f_{10} = 100 MHz we need $R_2 = 1.5 \times 10^{11}/10^8 - 10 \times 50$ = 1 kΩ, and R_1 = 1000/(10 − 1) = 111 Ω.

IV. NOISE IN CFA'S

The noise characteristics of CFAs are specified in terms of *three* input noise densities: the *voltage density* e_n, the *inverting-input current density* i_{nn}, and the *noninverting-input current density* i_{np}. Since the BJTs of CFAs are usually biased at substantial current levels for speed, CFAs tend to exhibit lower voltage noise but higher current noise than ordinary op amps. Moreover, i_{nn} and i_{np} are dissimilar due to the presence of the input buffer.

Figure 6 shows the noise model of a CFA with resistive feedback. The *overall input noise power* is the sum of the individual noise powers,

$$e_{ni}^2 = e_n^2 + R_3^2 i_{np}^2 + (R_1 \parallel R_2)^2 i_{nn}^2 + 4kT[R_3 + (R_1 \parallel R_2)]$$

and the *total rms output noise* above a given frequency f_L is

$$E_{no} = \left(\int_{f_L}^{\infty} |A(jf)|^2 e_{ni}^2 \, df\right)^{1/2}$$

where $A(jf) = A_0/(1 + jf/f_A)$. Expressing the noise powers in terms of the white-noise floors and corner frequencies as

$$e_n^2 = e_{nw}^2\left(\frac{f_{ce}}{f} + 1\right),\ i_{nn}^2 = i_{nnw}^2\left(\frac{f_{cin}}{f} + 1\right),\ i_{np}^2 = i_{npw}^2\left(\frac{f_{cip}}{f} + 1\right)$$

substituting, and integrating yields [2]

$$\begin{aligned} E_{no} = A_0 \Big[& e_{nw}^2 (f_{ce} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L) + \\ & + R_3^2 i_{npw}^2 (f_{cip} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L) \\ & + (R_1 \parallel R_2)^2 i_{nnw}^2 (f_{cin} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L) \\ & + 4kT[R_3 + (R_1 \parallel R_2)](1.57 f_A - f_L) \Big]^{1/2} \end{aligned} \tag{14}$$

Example 3. A CLC401 CFA is configured for A_0 = 10 V/V with R_1 = 166.7 Ω and R_2 = 1.5 kΩ. Assuming R_3 = 100 Ω, find E_{no} if noise is observed over a 10 s interval.

Solution. f_L = 1/10 = 0.1 Hz. Using the data-sheet values z_0 = 710 kΩ and f_a = 350 kHz, we find $z_0 f_a / R_2$ = 165.7 MHz. Substituting into (11), along with the data-sheet value R_0 = 50 Ω yields f_A = 124 MHz. Substituting the data-sheets values $e_{nw} \simeq 2.4$ nV/√Hz, $f_{ce} \simeq 30$ kHz, $i_{npw} \simeq 2.6$ pA/√Hz, $f_{cip} \simeq 30$ kHz, $i_{nnw} \simeq 17$ pA/√Hz, and $f_{cin} \simeq 40$ kHz into (14) yields $E_{no} \simeq 0.57$ mV rms, or $E_{no} = 6 \times 0.57$ = 3.4 mV peak-to-peak.

V. APPLICATIONS CONSIDERATIONS

Though we have focused on the noninverting configuration, the CFA can be used in a variety of other familiar op amp applications [3]. However, special attention merit the cases when the external network contains reactive elements, whether intentional or parasitic.

Consider first the effect of a *feedback capacitance* C_2 in parallel with R_2 in the basic circuit of Fig. 4, top. Replacing R_2 with $R_2 \parallel (1/sC_2)$ in (7) and (10) and expanding, one readily finds that Z_2 now has a pole at $f_p = 1/(2\pi R_2 C_2)$ and a zero at $f_z = 1/[2\pi(R_0 \parallel R_1 \parallel R_2)C_2]$ (see curve 2 of Fig. 4, bottom.) The crossover frequency is now pushed into a region of substantial phase shift due to higher-order poles of $z(jf)$ that were neglected in (5). If the shift reaches −180° at this frequency, then $T = 1\angle{-180^\circ} = -1$, making $A \to \infty$, by (3). When this condition is met, the circuit will oscillate. Even if the phase shift fails to reach −180°, the closed-loop response may still exhibit intolerable peaking and ringing, indicating that *capacitive feedback must be avoided with CF amps*. The familiar Miller integrator is not suitable to CFA implementation, and other topologies must be used instead [3].

Next, we examine the effect of the *input capacitance* C_1 in parallel with R_1 in the basic circuit of Fig. 4, top. Replacing R_1 with $R_1 \parallel (1/sC_1)$ in (7), and substituting into (10), one readily finds that Z_2 now has a zero at $f_z = 1/[2\pi(R_0 \parallel R_1 \parallel R_2)C_1]$ (see curve 1 of Fig. 4, bottom.) If C_1 is sufficiently large, the phase of T at the crossover frequency will again approach −180°, bringing the circuit on the verge of instability. As with conventional op amps, the CFA can be stabilized by using a feedback capacitance C_2 to introduce sufficient phase lead around the loop so as to compensate for the phase lag due to C_1.

In the typical application of Fig. 7a the CFA is used to buffer a current-mode DAC, and C_1 is the stray output capacitance of the DAC. The use of C_2 creates a pole for Z_2 at $f_p = 1/(2\pi R_2 C_2)$. For a 45° phase margin, C_2 is chosen to make f_p coincide with the crossover frequency

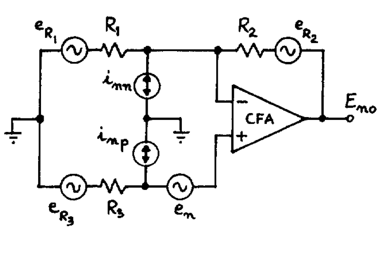
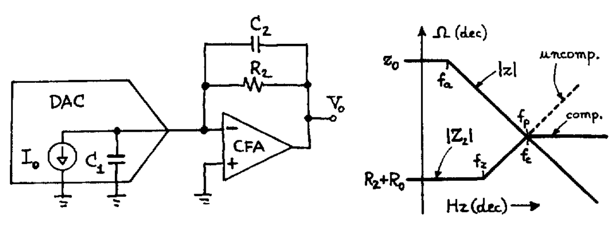


Fig. 6. Noise model of a resistive CFA circuit.



(a) (b)

Fig. 7. DAC output capacitance compensation.

f_c [2]. Referring to Fig. 7b, one can show that if f_z is sufficiently lower than f_c, then $f_c \approx \sqrt{[z_0 f_a f_z / (R_0 + R_2)]}$. Letting $f_z = 1/2\pi(R_0 /\!\!/ R_2)C_1$ and imposing $f_p = f_c$ yields

$$C_2 = \sqrt{R_0 C_1 / 2\pi R_2 z_0 f_a} \tag{15}$$

Example 4. A DAC with $C_1 = 100$ pF feeds a CFA with $R_2 = 1.5$ kΩ, $z_0 f_a / R_2 = 150$ MHz, and $R_0 = 50\ \Omega$. Find C_2 for a phase margin of 45°, and estimate the bandwidth.

Solution. $z_0 f_a = R_2 \times 150 \times 10^6 = 2.25 \times 10^{11}\ \Omega \times$ Hz, $C_2 = \sqrt{[50 \times 100 \times 10^{-12} / (2\pi \times 1.5 \times 10^3 \times 2.25 \times 10^{11})]} = 1.54$ pF, and $f_{-3\,db} \approx 1/2\pi R_2 C_2 \approx 69$ MHz. C_2 may be increased for a greater phase margin, but this will also reduce the bandwidth of the amplifier.

Most CFA manufacturers provide SPICE macro-models for the simulation of their products. Conversely, the user can readily create simplified models for a quick test of such characteristics as stability and noise. For instance, we can use the one-pole equivalent of Fig. 8, top, to verify both the frequency response and the transient response of the DAC buffer of Example 4. The input file is

```
CFA DAC
IDAC 2 0 AC 1M PULSE 0 1M 0 0 0 60NS 60NS
CDAC 2 0 100P
R2 2 6 1.5K
C2 2 6 1.54P
*CFA MODEL: Z0=750 KOHMS, FA=300 KHZ, R0=50 OHMS
R0 2 1 50
VSENSE 0 1 DC 0
F1 0 4 VSENSE 1
RPOLE 4 0 750K
CPOLE 4 0 0.7074P
EOUT 6 0 4 0 1
.AC DEC 10 1MEG 1G
.TRAN 1N 60N
.PROBE
.END
```

The transient response is shown at the bottom of the same figure. If desired, one can easily display also the frequency plots of $z(jf)$ and $Z_2(jf)$ to observe the effect of compensation upon the loop gain $T(jf)$.

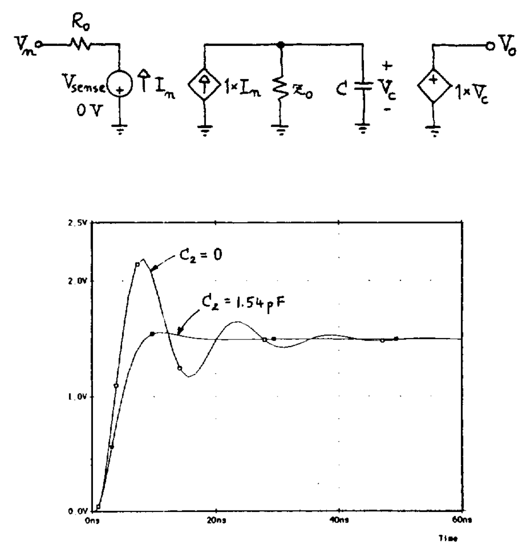


Fig. 8. Simple SPICE model for the CFA of Example 4, and PSpice™ display of the transient response.

REFERENCES

[1] D. Nelson and S. Evans, "A New Approach to Op Amp Design," Comlinear Co. App. Note 300-1, March 1985.

[2] Sergio Franco, *Design with Operational Amplifiers and Analog ICs*, McGraw-Hill Book Company, 1988.

[3] D. Potson, "Current-Feedback Op Amp Applications Circuit Guide," Comlinear Co. App. Note OA-07, May 1988.