DECIBEL (dB)

The Decibel is a subunit of a larger unit called the bel. As originally used, the bel represented the power ratio of 10 to 1 between the strength or intensity i.e., power, of two sounds, and was named after Alexander Graham Bell. Thus a power ratio of 10:1 = 1 bel, 100:1 = 2 bels, and 1000:1 = 3 bels. It is readily seen that the concept of bels represents a logarithmic relationship since the logarithm of 100 to the base 10 is 2 (corresponding to 2 bels), the logarithm of 1000 to the base 10 is 3 (corresponding to 3 bels), etc. The exact relationship is given by the formula

$$\text{Bels} = \log(P_2/P_1) \qquad [1]$$

where P_2/P_1 represents the power ratio.

Since the bel is a rather large unit, its use may prove inconvenient. Usually a smaller unit, the Decibel or dB, is used. 10 decibels make one bel. A 10:1 power ratio, 1 bel, is 10 dB; a 100:1 ratio, 2 bels, is 20 dB. Thus the formula becomes

$$\text{Decibels (dB)} = 10 \log(P_2/P_1) \qquad [2]$$

The power ratio need not be greater than unity as shown in the previous examples. In equations [1] and [2], P_1 is usually the reference power. If P_2 is less than P_1, the ratio is less then 1.0 and the resultant bels or decibels are negative. For example, if P_2 is one-tenth P_1, we have

$$\text{bels} = \log(0.1/1) = -1.0 \text{ bels}$$

and

$$\text{dB} = 10 \log(0.1/1) = -10 \text{ dB.}$$

It should be clearly understood that the term decibel does not in itself indicate power, but rather is a ratio or comparison between two power values. It is often desirable to express power levels in decibels by using a fixed power as a reference. The most common references in the world of electronics are the milliwatt (mW) and the watt. The abbreviation dBm indicates dB referenced to 1.0 milliwatt. One milliwatt is then zero dBm. Thus P_1 in equations [1] or [2] becomes 1.0 mW. Similarly, The abbreviation dBW indicates dB referenced to 1.0 watt, with P_2 being 1.0 watt, thus one watt in dBW is zero dBW or 30 dBm or 60 dBμW. For antenna gain, the reference is the linearly polarized isotropic radiator, dBLI. Usually the “L” and/or “I” is understood and left out.

dBc is the power of one signal referenced to a carrier signal, i.e. if a second harmonic signal at 10 GHz is 3 dB lower than a fundamental signal at 5 GHz, then the signal at 10 GHz is -3 dBc.

THE DECIBEL, ITS USE IN ELECTRONICS

The logarithmic characteristic of the dB makes it very convenient for expressing electrical power and power ratios. Consider an amplifier with an output of 100 watts when the input is 0.1 watts (100 milliwatts); it has an amplification factor of

$$P_2/P_1 = 100/0.1 = 1000$$

or a gain of:

$$10 \log(P_2/P_1) = 10 \log(100/0.1) = 30 \text{ dB.}$$

(notice the 3 in 30 dB corresponds to the number of zeros in the power ratio)

The ability of an antenna to intercept or transmit a signal is expressed in dB referenced to an isotropic antenna rather than as a ratio. Instead of saying an antenna has an effective gain ratio of 7.5, it has a gain of 8.8 dB (10 log 7.5).

A ratio of less than 1.0 is a loss, a negative gain, or attenuation. For instance, if 10 watts of power is fed into a cable but only 8.5 watts are measured at the output, the signal has been decreased by a factor of

$$8.5/10 = .85$$

or

$$10 \log(.85) = -0.7 \text{ dB}.$$

This piece of cable at the frequency of the measurement has a gain of -0.7 dB. This is generally referred to as a loss or attenuation of 0.7 dB, where the terms "loss" and "attenuation" imply the negative sign. An attenuator which reduces its input power by factor of 0.001 has an attenuation of 30 dB. The utility of the dB is very evident when speaking of signal loss due to radiation through the atmosphere. It is much easier to work with a loss of 137 dB rather than the equivalent factor of 2×10^{-14}.

Instead of multiplying gain or loss factors as ratios we can add them as positive or negative dB. Suppose we have a microwave system with a 10 watt transmitter, and a cable with 0.7 dB loss connected to a 13 dB gain transmit antenna. The signal loss through the atmosphere is 137 dB to a receive antenna with a 11 dB gain connected by a cable with 1.4 dB loss to a receiver. How much power is at the receiver? First, we must convert the 10 watts to milliwatts and then to dBm:

10 watts = 10,000 milliwatts

and

$$10 \log(10{,}000/1) = 40 \text{ dBm}$$

Then

40 dBm - 0.7 dB + 13 dB - 137 dB + 11 dB - 1.4 dB = -75.1 dBm.

-71.1 dBm may be converted back to milliwatts by solving the formula:

$$\text{mW} = 10^{(\text{dBm}/10)}$$

giving: $10^{(-75.1/10)} = 0.00000003$ mW

Voltage and current ratios can also be expressed in terms of decibels, provided the resistance remains constant. First we substitute for P in terms of either voltage, V, or current, I. Since P=VI and V=IR we have:

$$P = I^2R = V^2/R$$

Thus for a voltage ratio we have: $\text{dB} = 10 \log[(V_2^2/R)/(V_1^2/R)] = 10 \log(V_2^2/V_1^2) = 10 \log(V_2/V_1)^2$
$= 20 \log(V_2/V_1)$

Like power, voltage can be expressed relative to fixed units, so one volt is equal to 0 dBV or 120 dBμV.

Similarly for current ratio: $\text{dB} = 20 \log(I_2/I_1)$

Like power, amperage can be expressed relative to fixed units, so one amp is equal to 0 dBA or 120 dBμA.

Decibel Formulas (where Z is the general form of R, including inductance and capacitance)

When impedances are equal:
$$dB = 10 \log\frac{P_2}{P_1} = 20 \log\frac{E_2}{E_1} = 20 \log\frac{I_2}{I_1}$$

When impedances are unequal:
$$dB = 10 \log\frac{P_2}{P_1} = 20 \log\frac{E_2\sqrt{Z_1}}{E_1\sqrt{Z_2}} = 20 \log\frac{I_2\sqrt{Z_2}}{I_1\sqrt{Z_1}}$$

SOLUTIONS WITHOUT A CALCULATOR

Solution of radar and EW problems requires the determination of logarithms (base 10) to calculate some of the formulae. Common "four function" calculators don't usually have a log capability (or exponential or fourth root functions either). Without a scientific calculator (or math tables or a Log-Log slide rule) it is difficult to calculate any of the radar equations, simplified or "textbook". The following gives some tips to calculate a close approximation without a calculator.

DECIBEL TABLE

DB	Power Ratio	Voltage or Current Ratio	DB	Power Ratio	Voltage or Current Ratio
0	1.00	1.00	10	10.0	3.16
0.5	1.12	1.06	15	31.6	5.62
1.0	1.26	1.12	20	100	10
1.5	1.41	1.19	25	316	17.78
2.0	1.58	1.26	30	1,000	31.6
3.0	2.00	1.41	40	10,000	100
4.0	2.51	1.58	50	10^5	316
5.0	3.16	1.78	60	10^6	1,000
6.0	3.98	2.00	70	10^7	3,162
7.0	5.01	2.24	80	10^8	10,000
8.0	6.31	2.51	90	10^9	31,620
9.0	7.94	2.82	100	10^{10}	10^5

For dB numbers which are a multiple of 10

An easy way to remember how to convert dB values that are a multiple of 10 to the absolute magnitude of the power ratio is to place a number of zeros equal to that multiple value to the right of the value 1.
i.e. 40 dB = 10,000 : 1 (for Power)

Minus dB moves the decimal point that many places to the left of 1.
i.e. -40 dB = 0.0001 : 1 (for Power)

For voltage or current ratios, if the multiple of 10 is even, then divide the multiple by 2, and apply the above rules. i.e. 40 dB = 100 : 1 (for Voltage)
-40 dB = 0.01 : 1

If the power in question is not a multiple of ten, then some estimation is required. The following tabulation lists some approximations, some of which would be useful to memorize.

DB RULES OF THUMB

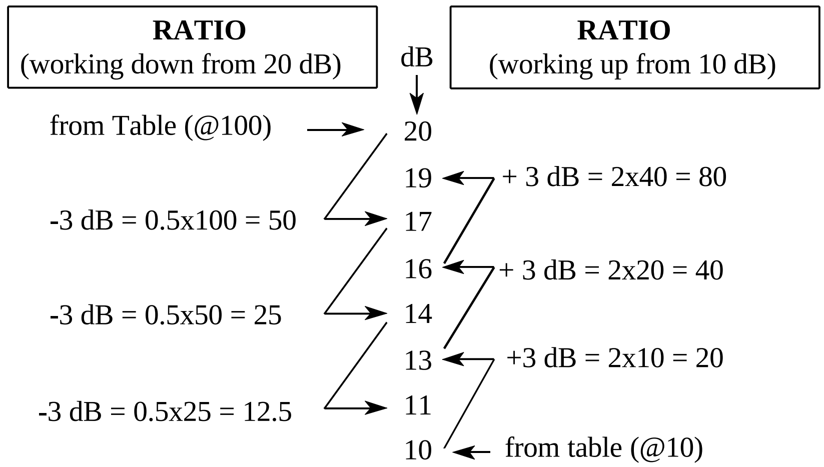
Multiply Current / Voltage By: if +dB	Multiply Current / Voltage By: if -dB	**dB**	Multiply Power By: if +dB	Multiply Power By: if -dB
1	1	0	1	1
1.12	0.89	1	1.26	0.8
1.26	0.79	2	1.58	0.63
1.4	0.707	3	2	0.5
2.0	0.5	6	4	0.25
2.8	0.35	9	8	0.125
3.16	0.316	10	10	0.1
4.47	0.22	13	20	0.05
10	0.1	20	100	0.01
100	0.01	40	10,000	0.0001

You can see that the list has a repeating pattern, so by remembering just three basic values such as one, three, and 10 dB, the others can easily be obtained without a calculator by addition and subtraction of dB values and multiplication of corresponding ratios.

Example 1:

A 7 dB increase in power (3+3+1) dB is an increase of (2 x 2 x 1.26) = 5 times whereas
A 7 dB decrease in power (-3-3-1) dB is a decrease of (0.5 x 0.5 x 0.8) = 0.2.

Example 2: Assume you know that the ratio for 10 dB is 10, and that the ratio for 20 dB is 100 (doubling the dB increases the power ratio by a factor of ten), and that we want to find some intermediate value.



We can get more intermediate dB values by adding or subtracting one to the above, for example, to find the ratio at 12 dB we can:

work up from the bottom; 12 = 1+11 so we have 1.26 (from table) x 12.5 = 15.75
alternately, working down the top 12 = 13-1 so we have 20 x 0.8 (from table) = 16

The resultant numbers are not an exact match (as they should be) because the numbers in the table are rounded off. We can use the same practice to find any ratio at any other given value of dB (or the reverse).

dB AS ABSOLUTE UNITS

Power in absolute units can be expressed by using 1 Watt (or 1 milliwatt) as the reference power in the denominator of the equation for dB. We then call it dBW or dBm. We can then build a table such as the adjoining one.

dB AS ABSOLUTE UNITS

dBμW	dBm	POWER	dBW
120	90	1 MW	60
90	60	1 kW	30
80	50	100 W	20
70	40	10 W	10
60	30	1 W (1000 mW)	0
50	20	100 mW	-10
40	10	10 mW	-20
33	3	2 mW	-27
32	2	1.58 mW	-28
31	1	1.26 mw	-29

From the above, any intermediate value can be found using the same dB rules and memorizing several dB values i.e. for determining the absolute power, given 48 dBm power output, we determine that 48 dBm = 50 dBm - 2 dB so we take the value at 50 dB which is 100W and divide by the value 1.58 (ratio of 2 dB) to get:

100 watts/1.58 = 63 W or 63,291 mW.

Because dBW is referenced to one watt, the Log of the power in watts times 10 is dBW. The Logarithm of 10 raised by any exponent is simply that exponent. That is: $\text{Log}(10)^4 = 4$. Therefore, a power that can be expressed as any exponent of 10 can also be expressed in dBW as that exponent times 10. For example, 100 kW can be written 100,000 watts or 10^5 watts. 100 kW is then +50 dBW. Another way to remember this conversion is that dBW is the number of zeros in the power written in watts times 10. If the transmitter power in question is conveniently a multiple of ten (it often is) the conversion to dBW is easy and accurate.