DIFFRACTION

In this lab the phenomenon of diffraction will be explored. Diffraction is interference of a wave with itself. According to Huygen's Principle waves propagate such that each point reached by a wavefront acts as a new wave source. The sum of the secondary waves emitted from all points on the wavefront propagate the wave forward. Interference between secondary waves emitted from different parts of the wave front can cause waves to bend around corners and cause intensity fluctuations much like interference patterns from separate sources. Some of these effects were touched in the previous lab on interference.

In this lab the intensity patterns generated by monochromatic (laser) light passing through a single thin slit, a circular aperture, and around a opaque circle will be calculated and experimentally verified.

The intensity distributions of monochromatic light diffracted from the described objects are based on:

a) the Superposition Principle

b) the wave nature of light

Disturbance: $A = A_0 \sin(\omega t + \phi)$

Intensity: $I = (\sum A)^2$

c) Huygen's Principle -- Light propagates in such a way that each point reached by the wave acts as a point source of a new light wave. The superposition of all these waves represents the propagation of the light wave.

All calculations are based on the assumption that the distance L between the slit and the viewing screen is much larger than the slit width a:, i.e. $L >> a$. This particular case is called *Fraunhofer scattering.* The calculations of this type of scattering are much simpler than the *Fresnel scattering* in which case the $L >> a$ constraint is removed.

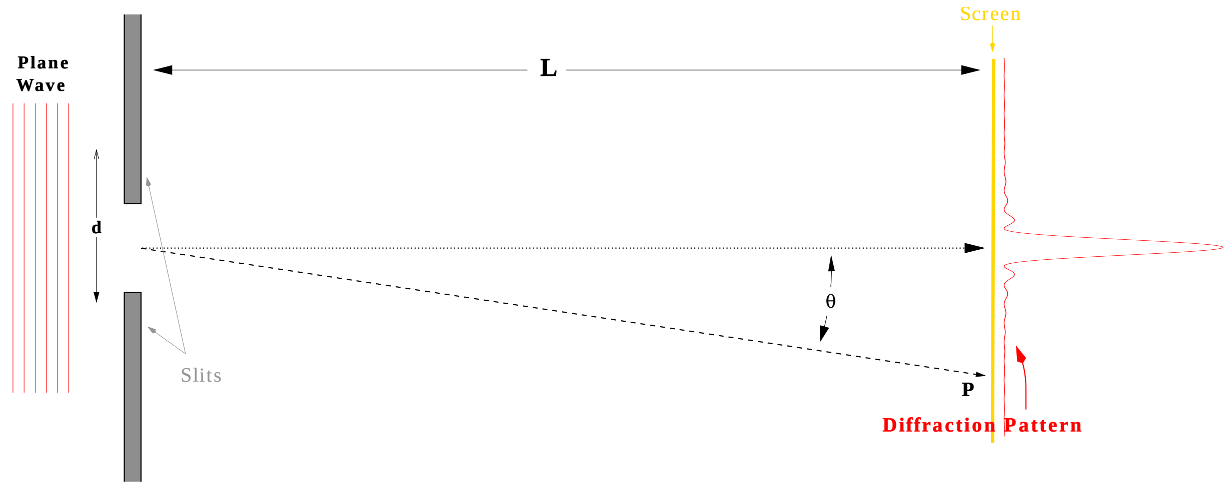


Figure 1 Intensity Profile of the Diffraction Pattern Resulting from a Plane Wave Passing Through a Single Narrow Slit

Experiment 1: Single Slit Diffraction

THEORY

A narrow slit of infinite length and width a is illuminated by a plane wave (laser beam) as as shown in Figure 1. The intensity distribution observed (on a screen) at an angle θ with respect to the incident direction is given by equation (1). This relation is derived in detail in the appendix and every student must make an effort to go through its derivation. The mathematics used to calculate this relation are very simple. The contributions from the field at each small area of the slit to the field at a point on the screen are added together by integration. Squaring this result and disregarding sinusoidal fluctuations in time gives the intensity. The main difficulty in the calculation is determining the relative phase of each small contribution. Figure 2 shows the expected shape of this distribution.

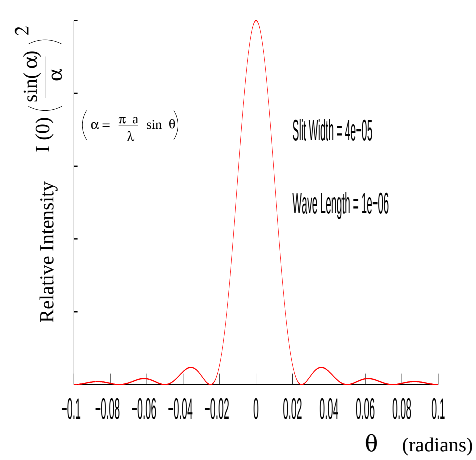


Figure 2 Single Slit Diffraction Intensity Pattern

Diffraction Single Slit
$$\frac{I(\theta)}{I(0)} = \frac{\sin^2\alpha}{\alpha^2}, \quad \alpha = \frac{\pi a}{\lambda}\sin\theta \qquad (1)$$

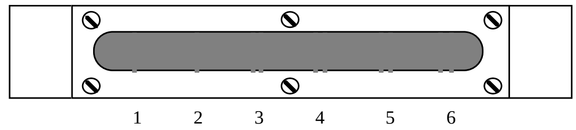
where: λ = wavelength of incident plane wave

a = slit width

θ = angle of observation (with respect to incident direction)

$I(\theta)$ = intensity in direction of observation

$I(0)$ = maximum intensity of diffraction pattern (central fringe)



Slit	Number of Slits	Width of each Slit	Space between Slits (mm)
1.	1	4×10^{-2} mm	
2.	1	8.77×10^{-2} mm	
3.	2	4.39×10^{-2} mm	0.13 mm
4.	2	8.77×10^{-2} mm	0.18 mm
5.	2	8.77×10^{-2} mm	0.35 mm
6.	2	8.77×10^{-2} mm	0.70 mm

Figure 3 Dimensions of Slits on Slide

This relation will have a value of zero each time that $\sin^2 \alpha = 0$. This occurs when,

$$\alpha = \pm m\pi \quad \text{or} \quad \frac{\pi a}{\lambda} \sin \theta = \pm m\pi$$

yielding the following condition for observing a minimum light intensity from a single slit:

$$\text{Single Slit Minima:} \qquad \sin \theta = m \frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \ldots \tag{2}$$

This relation is satisfied for integer values of m. Increasing values of m give minima at correspondingly larger angles. The first minimum will be found for $m = 1$, the second for $m = 2$ and so forth. If $\frac{a}{\lambda}\sin\theta$ is less than one for all values of θ, there will be no minima, i.e. when the

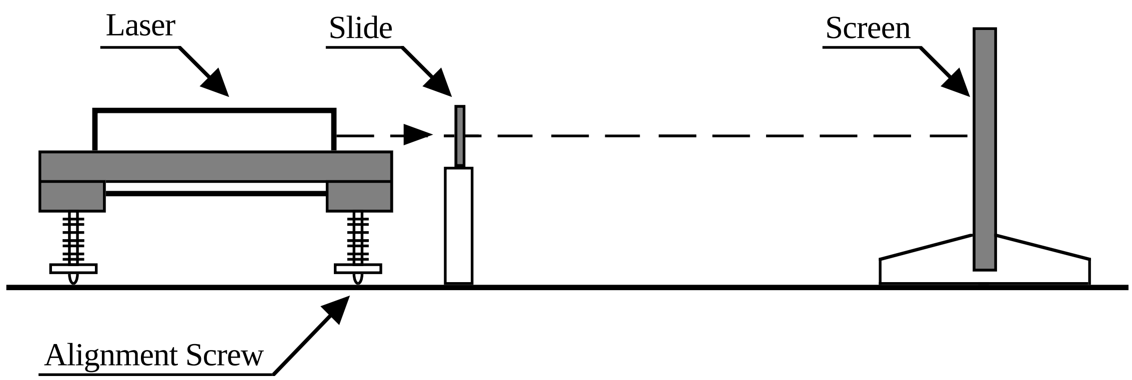


Figure 4 Diffraction Experiment Setup

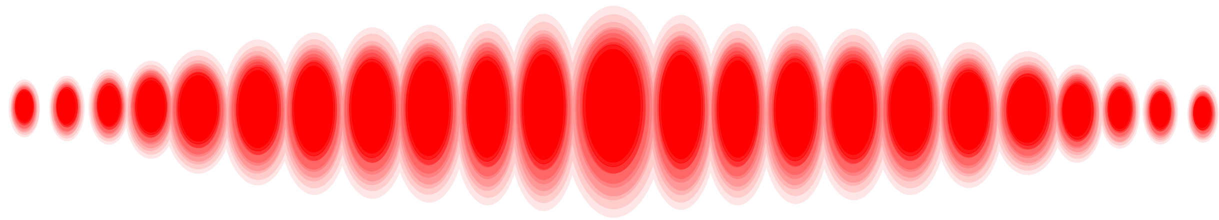


Figure 5 Observed Diffraction Pattern. The pattern observed by ones eyes does not die off as quickly in intensity as one expects when comparing the observed pattern with the calculated intensity profile given by Equation (1) and shown in Figure 2. This is because the bright laser light saturates the eye . Thus the center and nearby fringes seem to vary slightly in size but all appear to be the same brightness.

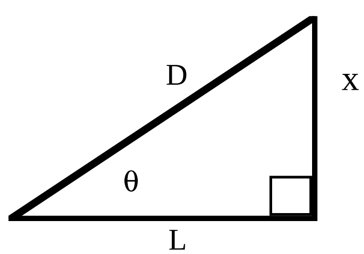
size of the aperture is smaller than a wavelength ($a < \lambda$). This indicates that diffraction is most strongly caused be perturbances with sizes that are about the same dimension of a wavelength.

PROCEDURE

Two single slits (along with some double slits) are on a slide similar to the one diagrammed in Figure 3. To observe diffraction from a single slit, align the laser beam parallel to the table, at the height of the center of the long slide, as shown in Figure 4. The diffraction pattern you are expected to observe is shown in Figure 5.

1a) **Observe** on the screen the different patterns generated by both of the single slits of this slide.

1b) **Calculate** the width of each one of the two single slits. This quantity can be calculated from Equation (2) using measurements of the spacing of the intensity minima. The wavelength of the HeNe laser is 6328Å, (1Å $\equiv 10^{-10}$ m). The quantity to be determined experimentally is $\sin\theta$. This can be done by trigonometry as shown below:



$$\sin\theta = \frac{x}{D} \quad , \quad \tan\theta = \frac{x}{L}$$

for small θ : $\sin\theta \simeq \tan\theta = x / L$.

Measure the slit width using several intensity minima of the diffraction pattern.

This measurement can be done using the screen covered with white paper. With a sharp pencil mark the position of the diffraction minima and then measure their relative distance with the ruler. To improve the accuracy of your measurements make the distance from slit to screen as large as possible. Compare your result with those given in Figure 1.

Experiment 2: Diffraction by a Circular Aperture

THEORY

For a circular hole of diameter *d* the diffraction pattern consists of concentric rings, which are analogous to the bands which we obtained for the slit. The pattern for this intensity distribution can be calculated in the same way as for the single slit (see appendix), but because the aperture here is circular, it is more convenient to use cylindrical coordinates (z, ρ, θ). The superposition principle requires us to integrate over a disk, and the result is a Bessel function.

The condition for observing a minimum of intensity is found from the zeroes of the Bessel function:

Circular Aperture Minima $$\sin\theta = k_m \frac{\lambda}{d} \tag{3}$$

For the first order (m = 1) minimum k_1 = 1.22. Higher order minima will have different k_m coefficients.

PROCEDURE

The 35 mm slide given you has four patterns of dots and openings of two different diameters. Put the slide in the laser beam and choose the configuration which gives the best diffraction pattern. It will consist of concentric rings.

2a) **Calculate** the diameter of the aperture by measuring the distance between the center of the dot pattern and the first minimum and using Equation (3). To do an accurate measurement, the distance between the object and the screen must be large; you may choose to use the wall as your screen. The quantity $\sin\theta$ can be measured in the same way as in Experiment 1.

2b) **Measure** the value of the current constant k_m in front of the ration λ/d in equation (3) for the second order minimum: $m = 2$. Do this by using as aperture diameter d the value previously obtained, and measuring $\sin\theta$ corresponding to the second minimum. In measuring the constant k_m you are determining the zeroes of a function called a Bessel function.

Experiment 6: Light as Matter or Waves: The Poisson Spot

THEORY

In 1818 Fresnel entered a competition sponsored by the French Academy. His paper was on the theory of diffraction. He showed that if light is to be described as a wave phenomenon, then a bright spot would be visible at the center of the shadow of a circular opaque obstacle, a result which he felt proved the absurdity of the wave theory of light. This surprising prediction, fashioned by Poisson as the death blow to wave theory, was almost immediately verified experimentally by Dominique Argo. The spot actually existed.

In less than 60 seconds you can now settle a controversy that has preoccupied the minds of the brightest philosophers and scientists for centuries. Is light described as a stream of particles or as a wave phenomena? Or in other words, does the Poisson spot exist?

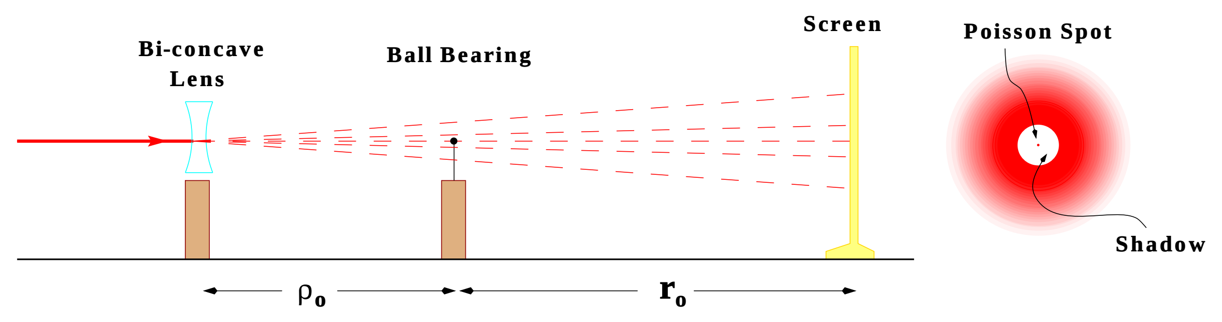


Figure 6 Poisson Spot Experiment Setup

PROCEDURE

3a) **Observe** the Poisson Spot.

Use the lens given to you to expand the beam slightly. Place the circular obstacle in the beam, (Fig. 11), and observe on the screen, at the center of the shadow generated by this obstacle, a bright spot—the Poisson spot! By varying the position of obstacle between the lens and screen, you can optimize the intensity of the Poisson spot

Late 20th Century View of Light:

The distinction between wave and particle relies on two types of experiments. Observation of *interference* phenomena demonstrates the presence of *waves*. Experiments which show discrete as opposed to continuous changes such as the photoelectric effect demonstrate *particle* phenomena (each photon of light knocks one photoelectron off a atom). Light is a wave (you observe interference) and light is also a particle called the photon (you observe scattering and absorbtion of single photons). These two descriptions of light are not mutually exclusive. The wave nature of light is observable in certain natural phenomena, whereas the particle nature becomes apparent in other natural phenomena. The surprising conclusion is that light behaves sometimes like a wave, and sometimes like a particle-depending on the particular experimental situation. This is called "wave particle duality."

Sections of this write up were taken from:
Physics Laboratory: Third Quater, Bruno Gobi, Northwestern University
Physics Part 2, D. Halliday & R. Resnick, John Wiley & Sons.
Physics Volume 2, Electricity, Magnetism, and Light, R. Blum & D. E. Roller, Holden-Day.
Optics, E. Hecht/A. Zajac, Addison-Wesley Publishing.

Following is a list of questions intended to help you prepare for this laboratory session. If you have read and understood this write up, you should be able to answer most of these questions. The TA may decide to check your degree of preparedness by asking you some of these questions:

- The relation of $\sin^2 \alpha/\alpha^2$ for $\alpha = 0$ is an undefined expression of 0/0. What is the limit of this relation for $\alpha \to 0$?
- Are the eyes sensitive to the amplitude or to the intensity of light?
- Which relation gives the position of the diffraction minima for a slit of width *a*, illuminated with light of wavelength λ?
- The distribution of light from two slits is represented by the product $\left(\frac{\sin \alpha}{\alpha}\right)^2 (\cos \beta)^2$. Which one of these two terms is called the diffraction term and which one is the interference term? Which term is responsible for the interference fringes?
- If you want to sharpen up a beam spot by inserting into the beam a narrow vertical slit, will the beam spot get more and more narrow as you close the slit? Explain.
- What is responsible for the factor 1.22 in formula (3)?
- What size object will generate an observable diffraction pattern if placed in the path of light with wavelength λ?
- Beams of particles act like waves with very short wavelengths when scattered by perturbances such as other particles. If a target of Pb and one of C are bombarded with a beam of protons, which target will show the sharpest diffraction pattern, the large size Pb target or the small C one? (Hint: The pattern due to an aperture is identical to the pattern caused by its negative: a disk in empty space.)
- What is the difference between Fraunhofer and Fresnel scattering?
- What is the Poisson spot?
- What does the existence of the Poisson spot demonstrate?
- What is our present view of light? Is it made up of waves or particles?

APPENDIX

Intensity Distribution from a Single Slit

The calculation of the intensity distribution of diffraction phenomena is based upon the superposition principle and Huygen's principle. Light is a wave, and if we choose to characterize it by the magnitude of its electric vector $\vec{E}$, then it can be represented as,

$$\vec{E} = \vec{E}_0 \sin(\omega t + \phi). \qquad \text{(a1)}$$

If two waves with amplitudes $\vec{E}_1$ and $\vec{E}_2$ reach the same point P at the same time, then the superposition principle gives the total amplitude $\vec{E}_{\mathrm{TOT}}$ as,

$$\vec{E}_{\mathrm{TOT}} = \vec{E}_1 + \vec{E}_2 \qquad \text{(a2)}$$

This total amplitude depends upon the relative phase ϕ between two waves. Looking at Figure 7, try to imagine what happens when $\vec{E}_2$ shifts with respect to $\vec{E}_1$ (this shift is given by the phase ϕ). By lining up peaks with peaks and valleys with valleys, one gets a maximum amplitude. In the opposite case, when one lines up peaks with valleys, the amplitude $\vec{E}_{\mathrm{TOT}}$ becomes zero.

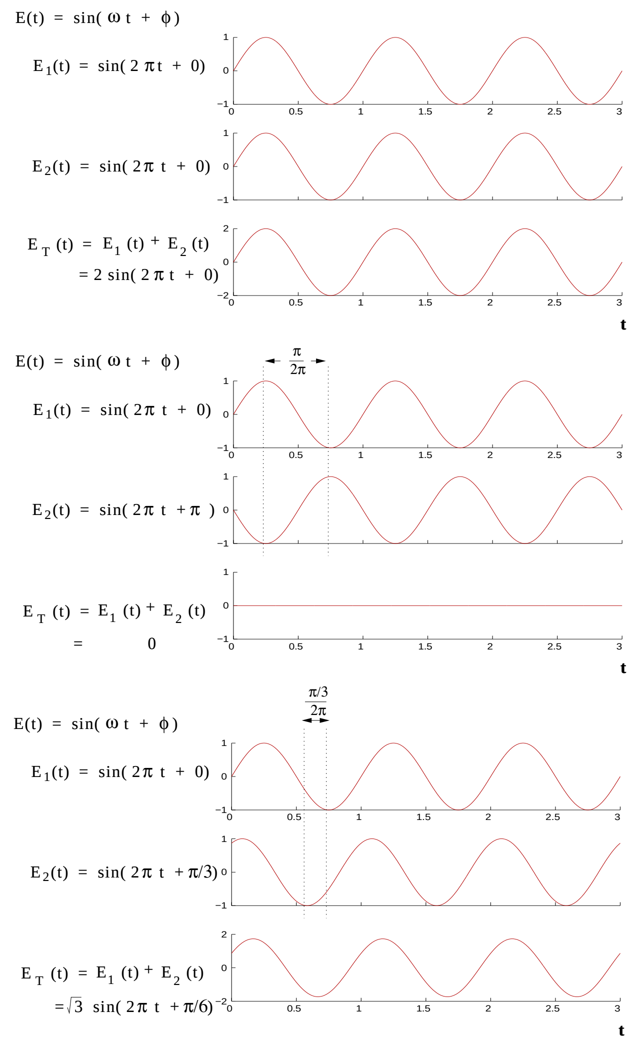


Figure 7 Interference Betweeen Two Sinusoidal Waves

The light intensity I is related to the amplitude $\vec{E}_{\mathrm{TOT}}$ by the important relation

$$I = \left(\vec{E}_{\mathrm{TOT}}\right)^2 . \qquad \text{(a3)}$$

Note that amplitudes must be summed first before the square is taken for the intensity:

$$\left(\vec{E}_{\mathrm{TOT}}\right)^2 = \left(\vec{E}_1 + \vec{E}_2\right)^2$$

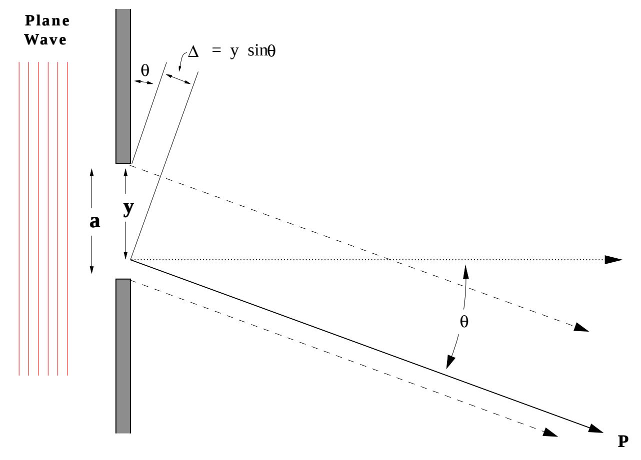


Figure 8 Diffraction by a Single Slit

Figure Figure 8 shows the quantities used to calculate the diffraction from a slit of width *a*. Each point of the slit along the *y*-direction will generate its own wavelet (Huygen's principle)—and each of these wavelets will have the same wavelength λ as the incident plane wave.

The amplitude of a wavelet generated at point *y* and reaching point *P* will be

$$E = E_0 \sin(\omega t + \phi)$$

with $\omega = 2\pi/T = 2\pi\nu$.

The phase ϕ can be arrived at by using the path difference:

$$\frac{\text{phase difference}}{2\pi} = \frac{\phi}{2\pi} = \frac{\text{path difference}}{\lambda} = \frac{\Delta}{\lambda} = \frac{y \sin\theta}{\lambda}$$

Consequently $\phi = \dfrac{2\pi}{\lambda} y \sin\theta$.

At point *P*, the amplitude of each wavelet generated by any point y along the slit will be

$$E = E_0 \sin\left(\omega t + \frac{2\pi}{\lambda} y \sin\theta\right) \qquad \text{(a4)}$$

The total amplitude at *P* will be (according to the superposition principle) the sum of the contribution of each point along *y*. This is obtained by integrating along *y*:

$$E_{\mathrm{TOT}} = \int_0^a E(y)dy . \qquad \text{(a5)}$$

This integral is straightforward, and is given in the textbook as

$$E_{TOT} \propto \sin\left(\frac{\pi a}{\lambda}\sin\theta\right)\Big/\left(\frac{\pi a}{\lambda}\sin\theta\right).$$

We are interested in the light intensity $I(=E^2_{TOT})$ rather than the amplitude E_{TOT} (see a3). Neglecting constants (which relate the intensity at P to the intensity of the incident wave) one obtains

$$I(\theta) = I(0)\frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2} \qquad \text{(a6)}$$

with $\alpha = \frac{\pi a}{\lambda}\sin\theta$, this relation is found in the literature as

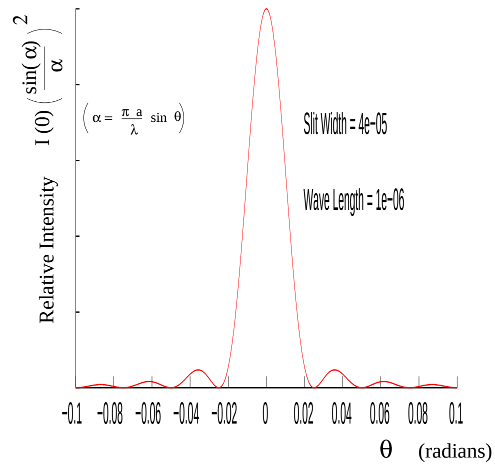


Figure 9 Intensity Profile of Single Slit Diffraction Pattern

$$\text{Single Slit Diffraction} \qquad I(\theta) = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \alpha = \frac{\pi a}{\lambda}\sin\theta \qquad \text{(a7)}$$

$I(0)$ is the intensity of the central fringe ($\theta = 0$). It is always the maximum intensity of the diffraction pattern. This relation will have a value of zero each time that $\sin^2\alpha = 0$. This occurs when $\alpha = \pm m\pi$ or $\frac{\pi a}{\lambda}\sin\theta = \pm m\pi$, yielding the following condition for observing a minimum light intensity from a single slit:

$$\text{Singe Slit Minima} \qquad \sin\theta = m\frac{\lambda}{a}, \; m = \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(a8)}$$

The shape of the distribution given by (a7) is shown in figure Figure 9.

Diffraction from a Double Slit

Figure 10 shows the quantities used to calculate the double slit diffraction. The amplitude of the light waves which pass through each slit and which reach P (for a given θ) will be the same as was calculated for the single slit case:

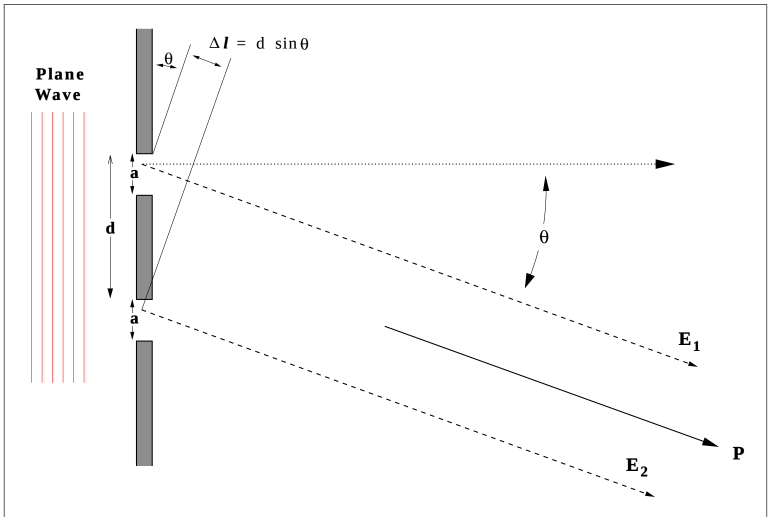


Figure 10 Double Slit Diffraction

$$E_1 = E_2 = \frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)}\sin(\omega t + \phi) \qquad \text{(a9)}$$

If we take the phase of E_1 to be $\pi = 0$, then the relative phase of E_2 is $\phi = \frac{2\pi}{\lambda} d\sin\theta$. The total amplitude at P is,

$$E_{TOT} = E_1 + E_2 = \frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)}\left[\underbrace{\sin(\omega t + 0)}_{\text{Wave }\uparrow\text{ Part of }E_1} + \underbrace{\sin\left(\omega t + \frac{\pi d}{\lambda}\sin\theta\right)}_{\text{Wave }\uparrow\text{ Part of }E_2}\right] \qquad \text{(a10)}$$

The two wave functions E_1 and E_2 can be added trigonometrically.

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$$

so that we obtain,

$$\underbrace{2\sin\left(\omega t + \frac{\pi d}{2\pi}\sin\theta\right)}_{\text{Wave part of }E_{TOT}} \times \cos\left(-\frac{\pi d}{2\pi}\sin\theta\right). \qquad \text{(a11)}$$

The amplitude of E_{TOT} is the product of two quantities. The first,

$$A_D(\theta) = \frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}$$

characterizes the intensity distribution from each slit of aperture a. The second amplitude

$$A_I(\theta) = \cos\left(\frac{\pi d}{2\pi}\sin\theta\right)$$

has its origin in the sum of the amplitude of the two slits. The total amplitude of the light reaching *P* after diffracting from the two slits is the product

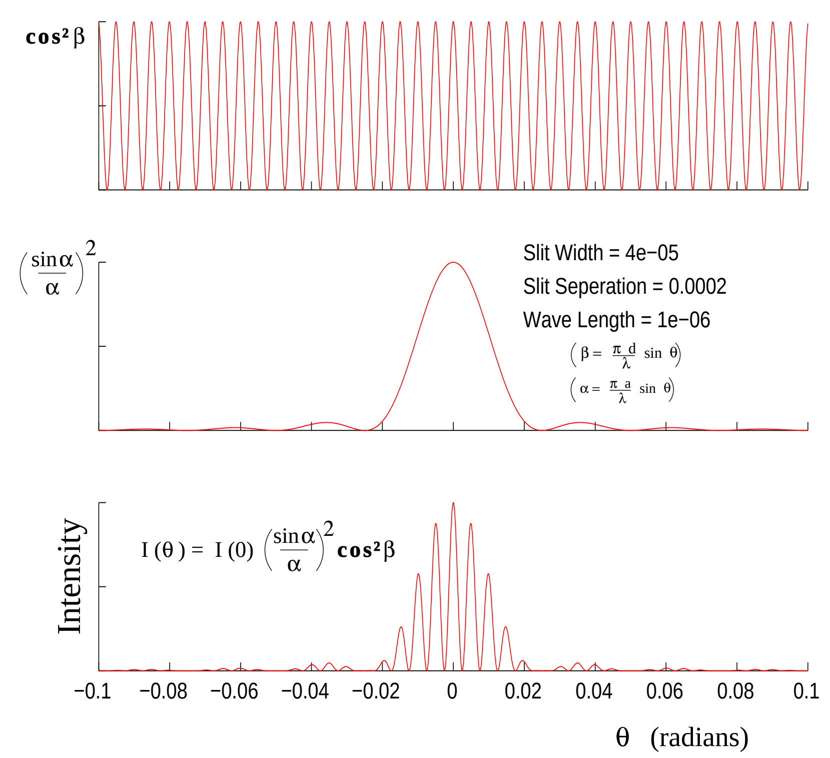


Figure 11 Diffration and Interference Terms of the Two Slit Diffraction/Interference Intensity Profile

$$A_{TOT}(\theta) = A_D(\theta)\, A_I(\theta) \qquad \text{(a12)}$$

Now that we have calculated the total amplitude, we are ready to calculate the quantity which we will observe, the light intensity, which is the square of the amplitude.

$$I(\theta) = \frac{\left(\sin\frac{\pi a}{\lambda}\sin\theta\right)^2}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}\left(\cos\frac{\pi d}{2\lambda}\sin\theta\right)^2 \qquad \text{(a13)}$$

In the textbook this relation is given by,

Double Slit Diffraction $$I(\theta) = I(0)\,\frac{\sin^2\theta}{\alpha^2}\,\cos^2\beta \qquad \text{(a14)}$$

It is the product of two factors:

Diffraction factor: $\dfrac{\sin^2\alpha}{\alpha}\,,\ \alpha \equiv \dfrac{\pi a}{\lambda}\sin\theta,$

The diffraction factor's shaped depends upon the width a of the 2 slits and has minima at:

Diffraction Factor Minima: $\alpha = \sin\theta = m\,\dfrac{\lambda}{a}\,,\ m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$

Interference factor: $\cos^2\beta\,,\ \beta \equiv \dfrac{\pi d}{2\lambda}\sin\theta\,,$

The interference factor's shape depends on the separation d between center to center of the two slits. Figure 11 shows the shape of both factors as well as their products. The interference factor has maxima at:

Interference Factor Maxima: $\beta = \sin\theta = m\,\dfrac{\lambda}{d}\,,\ m = 0,\ \pm 1,\ \pm 2,\ \ldots$