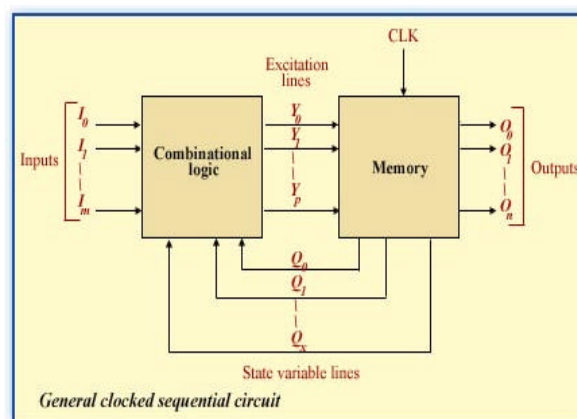


4.0 Design of Synchronous Counters

This section begins our study of designing an important class of clocked sequential logic circuits-synchronous finite-state machines. Like all sequential circuits, a finite-state machine determines its outputs and its next state from its current inputs and current state. A synchronous finite-state machine changes state only on the clocking event.

4.1 General Model of a sequential Circuit

The following diagram shows the general sequential circuit that consists of a combinational logic section and a memory section (flip-flops). The Combinational logic module is for us, as the designer, to match the design specifications.



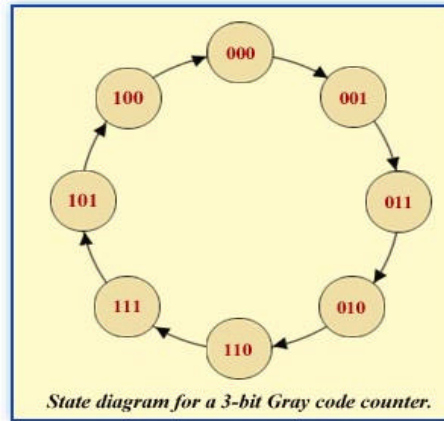
4.2 Counter Design Procedure

- ?? Describe a general sequential circuit in terms of its basic parts and its input and outputs.
- ?? Develop a state diagram for a given sequence.
- ?? Develop a next-state table for a specific counter sequence.
- ?? Create a FF transition table.
- ?? Use K-map to derive the logic equations.
- ?? Implement a counter to produce a specified sequence of states.

4.3. Design the 3-bit Gray code counter

Step 1: State Diagram

State Diagram for a 3-bit Gray code counter:



Step 2: Next-State Table

Next state table for a 3-bit Gray code counter

Present State			Next State		
Q_2	Q_1	Q_0	Q_2	Q_1	Q_0
0	0	0	0	0	1
0	0	1	0	1	1
0	1	1	0	1	0
0	1	0	1	1	0
1	1	0	1	1	1
1	1	1	1	0	1
1	0	1	1	0	0
1	0	0	0	0	0

Step 3: Flip-Flop Transition Table

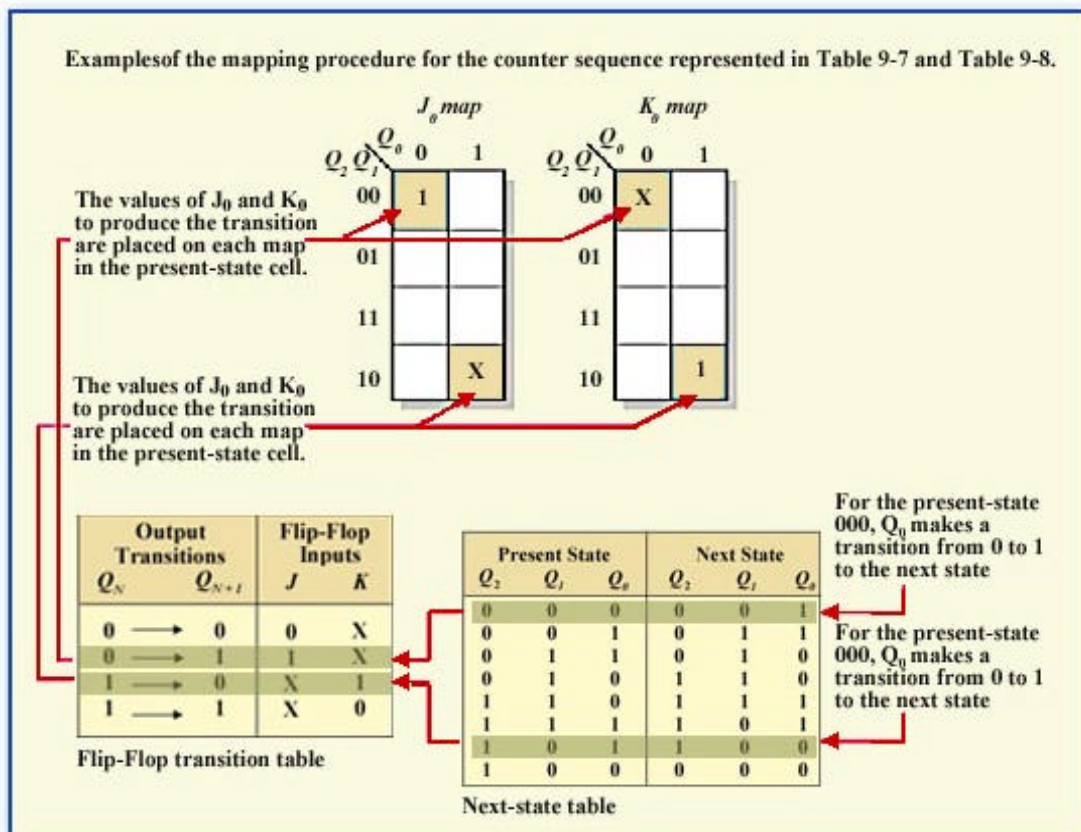
Transition table for a J-K Flip-Flop

Output Transitions		Flip-Flop Inputs	
Q_N	Q_{N+1}	J	K
0	→ 0	0	X
0	→ 1	1	X
1	→ 0	X	1
1	→ 1	X	0

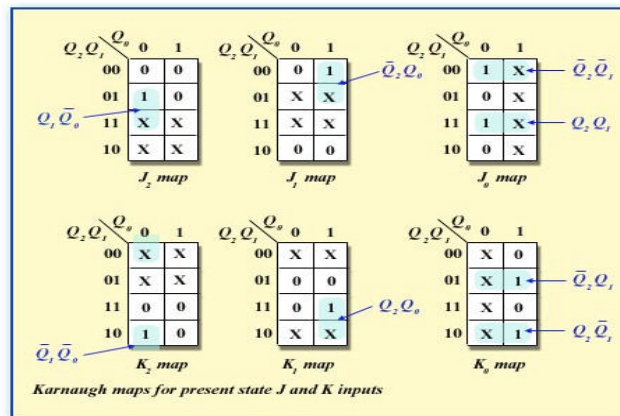
Q_N : present state, Q_{N+1} : next state
X: "don't care"

Step 4: Karnaugh Maps

The following diagram shows the steps to create separate next states of separate J and K from the current states of J and K.



Karnaugh maps for present-state J and K inputs for the 3-bit Gray code counter.



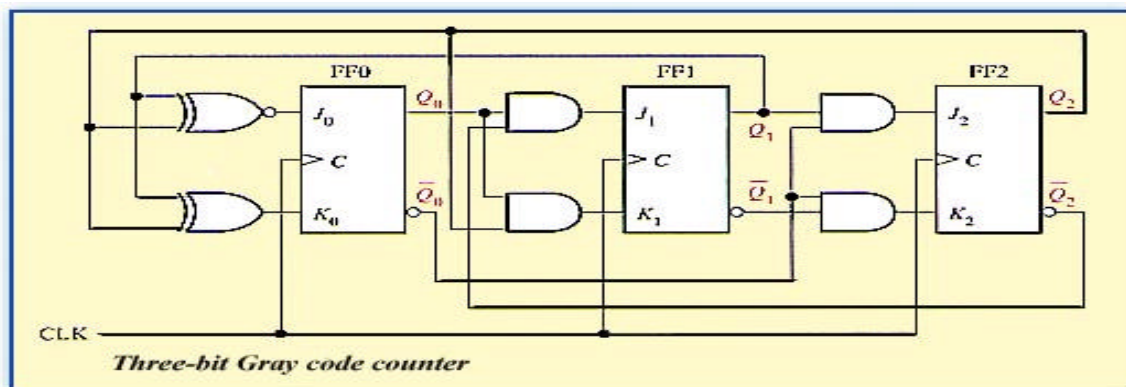
Step 5: Logic Expressions for Flip-flop Inputs

The next-state J and K outputs for a 3-bit Gray code counter.

$$\begin{aligned}
 J_0 &= Q_2 Q_1 + \overline{Q_2} Q_1 = \overline{Q_2} + Q_1 \\
 K_0 &= Q_2 \overline{Q_1} + \overline{Q_2} Q_1 = Q_2 + Q_1 \\
 J_1 &= \overline{Q_2} Q_0 \\
 K_1 &= Q_2 Q_0 \\
 J_2 &= Q_1 \overline{Q_0} \\
 K_2 &= \overline{Q_1} \overline{Q_0}
 \end{aligned}$$

Step 6: Counter Implementation

The hardware diagram of the 3-bit Gray code counter

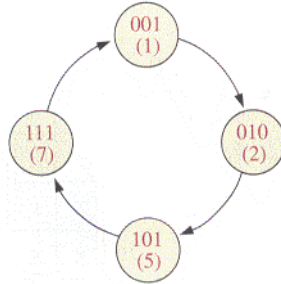


?? There are many more examples for the design of synchronous counter. These can be found in any digital network related textbooks.

4.4. Design – Example 1

Design a counter with the irregular binary count sequence shown in the state diagram of Figure 4.1.

Step 1: State Diagram



Step 2: Next-State Table

Present State			Next State		
Q_2	Q_1	Q_0	Q_2	Q_1	Q_0
0	0	1	0	1	0
0	1	0	1	0	1
1	0	1	1	1	1
1	1	1	0	0	1

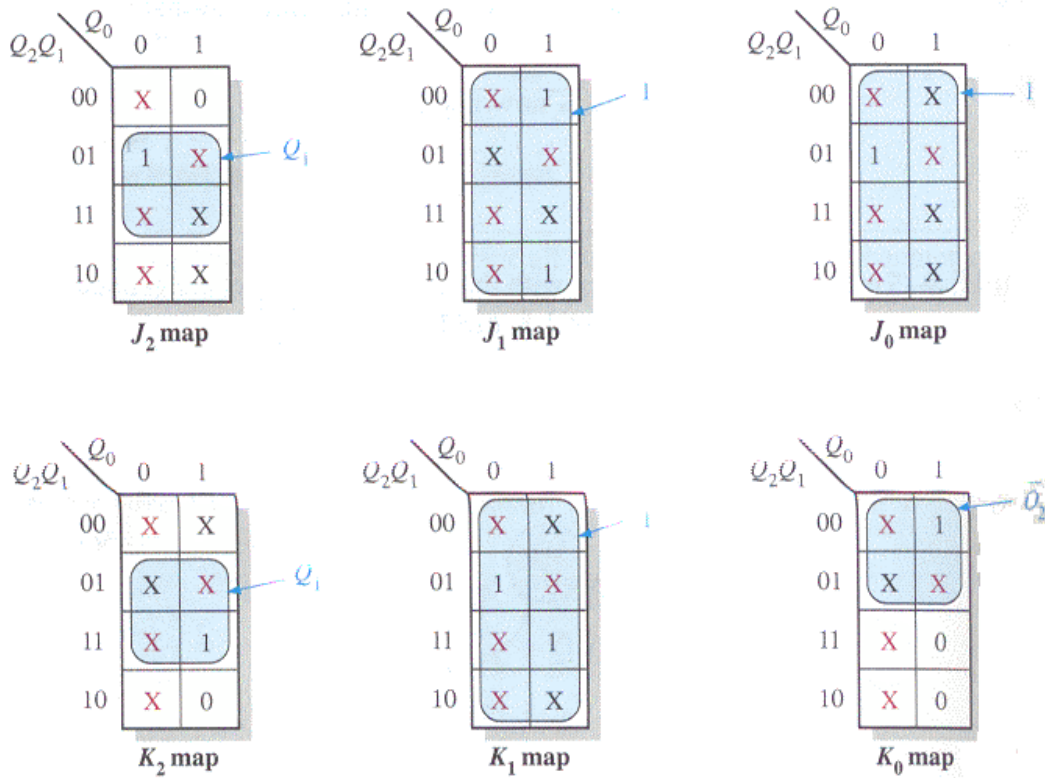
Step 3: Flip-Flop Transition Table

Transition table for a J-K Flip-Flop

Output Transitions		Flip-Flop Inputs	
Q_n	Q_{n+1}	J	K
0	→ 0	0	X
0	→ 1	1	X
1	→ 0	X	1
1	→ 1	X	0

Q_n : present state, Q_{n+1} : next state
X: "don't care"

Step 4: Karnaugh Maps



Step 5: Logic Expressions for Flip-flop Inputs

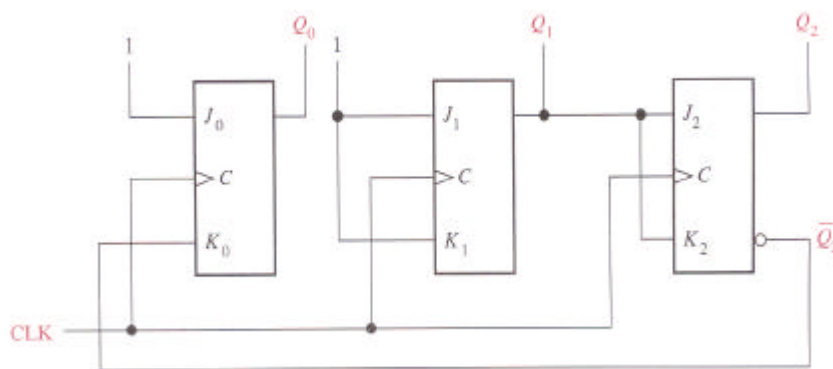
The expression for each J and K input taken from the maps is as follows:

$$J_0 = 1, K_0 = Q_2$$

$$J_1 = K_1 = 1$$

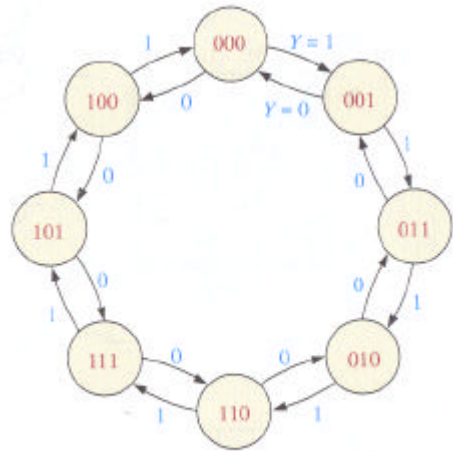
$$J_2 = K_2 = Q_1$$

Step 6: Counter Implementation



4.5. Example 2 - Design the 3 Up/down counter (Gray code sequence)

Step 1: State Diagram



Step 2: Next-State Table

Next-state table for 3-bit up/down Gray code counter.

Present State			Next State					
			Y = 0 (DOWN)			Y = 1 (UP)		
Q_2	Q_1	Q_0	Q_2	Q_1	Q_0	Q_2	Q_1	Q_0
0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1	1
0	1	1	0	0	1	0	1	0
0	1	0	0	1	1	1	1	0
1	1	0	0	1	0	1	1	1
1	1	1	1	1	0	1	0	1
1	0	1	1	1	1	1	0	0
1	0	0	1	0	1	0	0	0

$Y = \text{UP}/\overline{\text{DOWN}}$ control input.

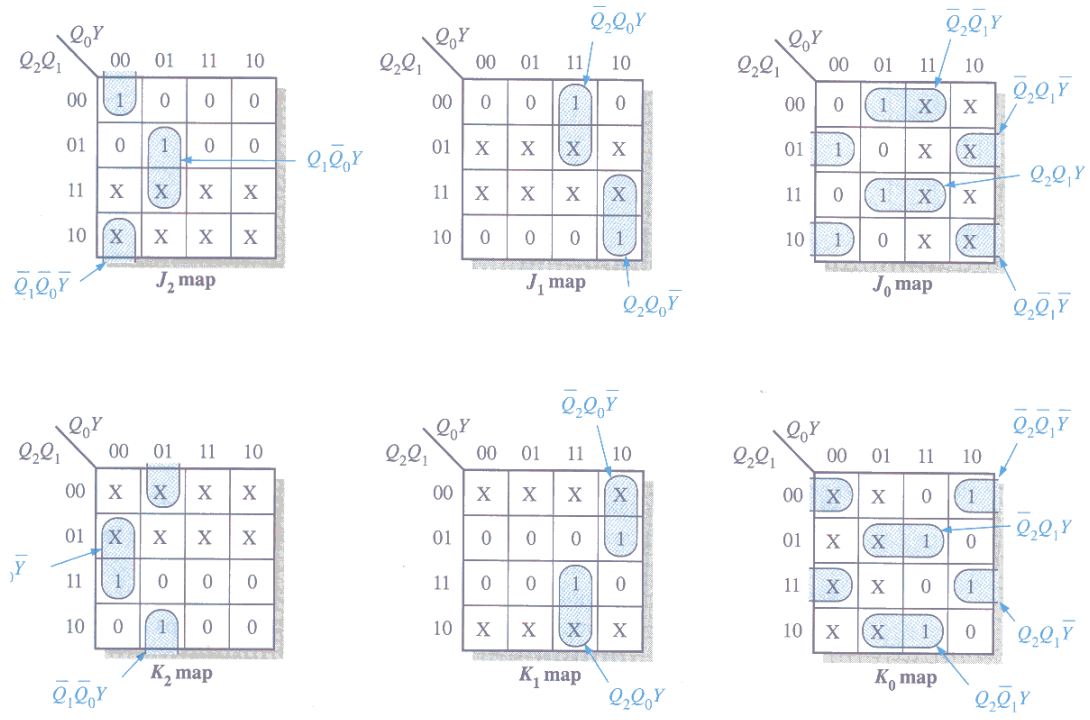
Step 3: Flip-Flop Transition Table

Transition table for a J-K Flip-Flop

Output Transitions		Flip-Flop Inputs	
Q_n	Q_{n+1}	J	K
0	→ 0	0	X
0	→ 1	1	X
1	→ 0	X	1
1	→ 1	X	0

Q_n : present state, Q_{n+1} : next state
X: "don't care"

Step 4: Karnaugh Maps



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Step 5: Logic Expressions for Flip-flop Inputs

$$J_0 = Q_2Q_1Y + Q_2\bar{Q}_1\bar{Y} + \bar{Q}_2Q_1\bar{Y} + \bar{Q}_2\bar{Q}_1Y$$

$$J_1 = \bar{Q}_2Q_0Y + Q_2Q_0\bar{Y}$$

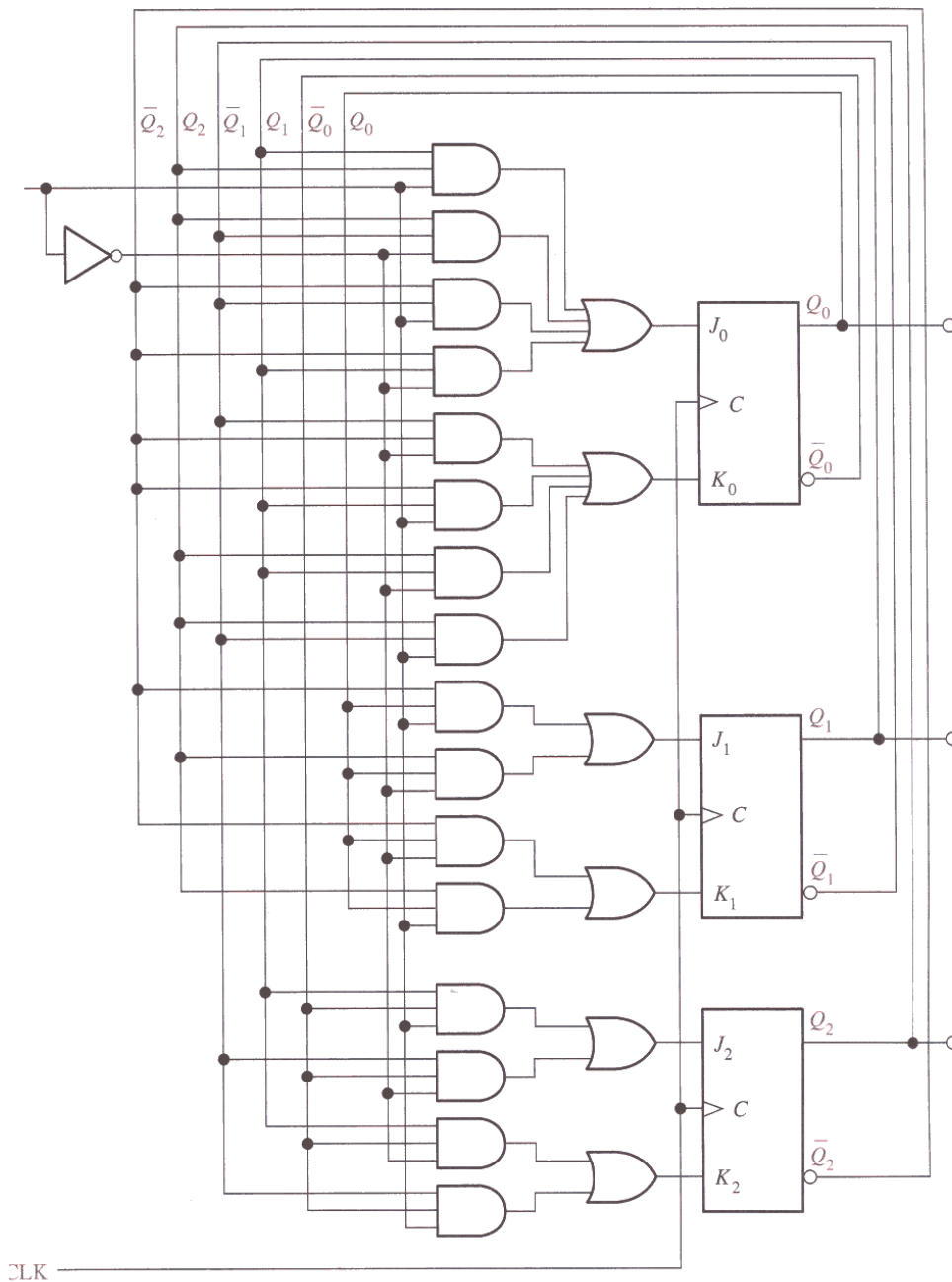
$$J_2 = Q_1\bar{Q}_0Y + \bar{Q}_1\bar{Q}_0\bar{Y}$$

$$K_0 = \bar{Q}_2\bar{Q}_1\bar{Y} + \bar{Q}_2Q_1Y + Q_2Q_1\bar{Y} + Q_2\bar{Q}_1Y$$

$$K_1 = \bar{Q}_2Q_0\bar{Y} + Q_2Q_0Y$$

$$K_2 = Q_1\bar{Q}_0\bar{Y} + \bar{Q}_1\bar{Q}_0Y$$

Step 6: Counter Implementation



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5.0 Cascaded Counters

Counters can be connected in cascade to achieve higher-modulus operation. Figure 5.1 shows an example of two counters, modulus-4 and modulus-8 connected in cascade.

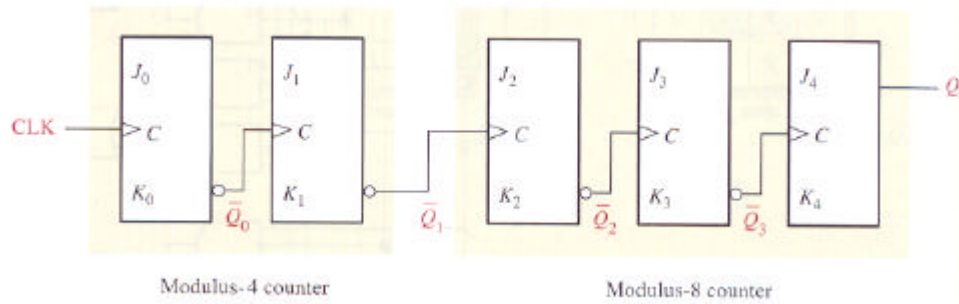


Figure 5.1: 2 cascaded counters

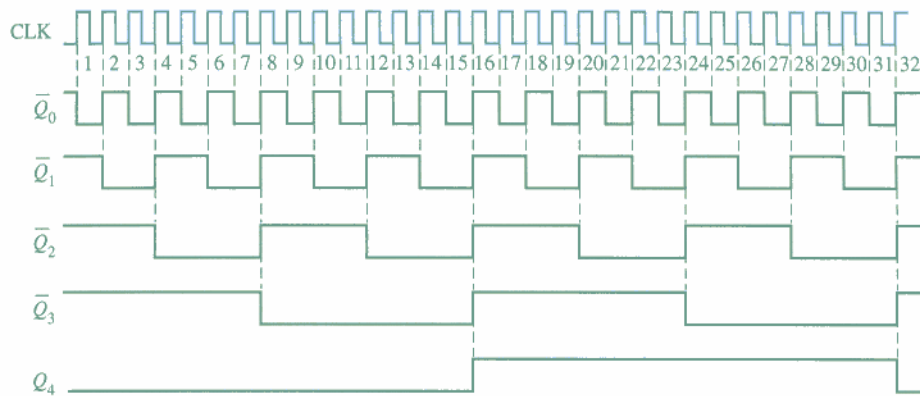


Figure 5.2: Timing diagram for the cascaded counters

5.1 Example 1: A modulus-100 counter

Figure 5.3 illustrates a modulus-100 counter using 2 cascaded decade counters. This counter can be viewed as a frequency divider. It divides the input clock frequency by 100.

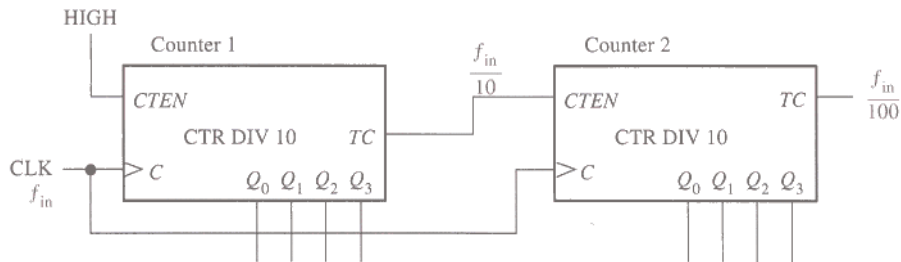


Figure 5.3: A modulus-100 counter using 2 cascaded decade counters

5.2 Example 2: A modulus-1000 counter

If you have a basis clock frequency of 1 MHz and you wish to obtain 100kHz, 10kHz, and 1kHz, a series of cascaded decade counters can be used. If 1 MHz signal is divided by 10, the output is 100kHz. Then if the 100 kHz signal is divided by 10, the output is 10kHz. Further division by 10 gives the 1 kHz frequency. The implementation is shown in Figure 5.4.

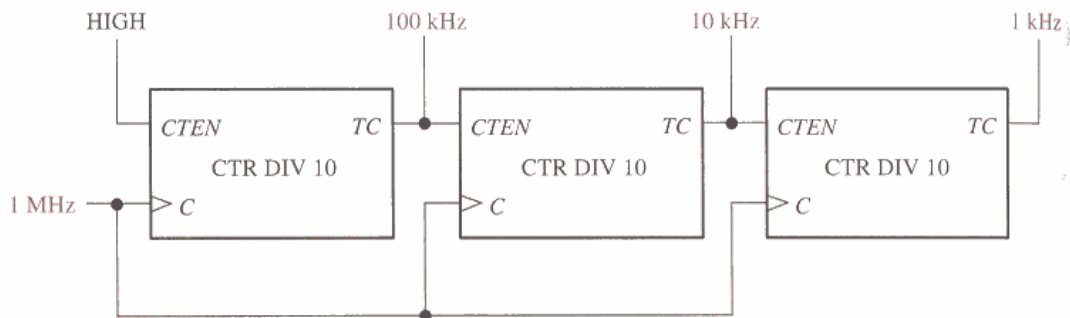


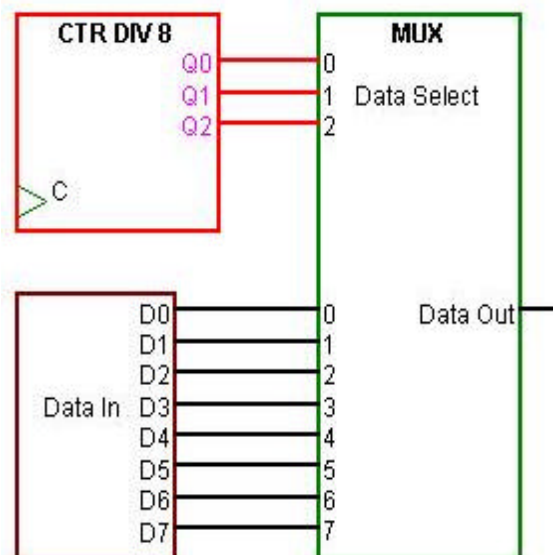
Figure 5.4: A modulus-1000 counter using 3 cascaded decade counters

6.0 Applications

Digital counters are very useful in many applications. They can be easily found in digital clocks and parallel-to-serial data conversion (multiplexing). In this section, we these two examples on how counters are being used.

Example 1:

A group of bits appearing simultaneously on parallel lines is called *parallel data*. A group of bits appearing on a single line in a time sequence is called *serial data*. *Parallel-to-serial conversion* is normally accomplished by the use of a counter to provide a binary sequence for the data-select inputs of a multiplexer, as illustrated in the circuit below.



Parallel-to-serial data conversion logic.

The Q outputs of the modulus-8 counter are connected to the data-select inputs of an eight-bit multiplexer. The first byte (eight-bit group) of parallel data is applied to the multiplexer inputs. As the counter goes through a binary sequence from 0 to 7, each bit beginning with D0, is sequentially selected and passed through the multiplexer to the output line.

After eight clock pulses, the data byte has been converted to a serial format and sent out on the transmission line. Then, the counter recycles back to 0 and converts another parallel byte sequentially again by the same process.

Example 2:

The following diagram shows the simplified logic diagram for a 12-hour digital clock.

