THREE-PHASE TRANSFORMERS

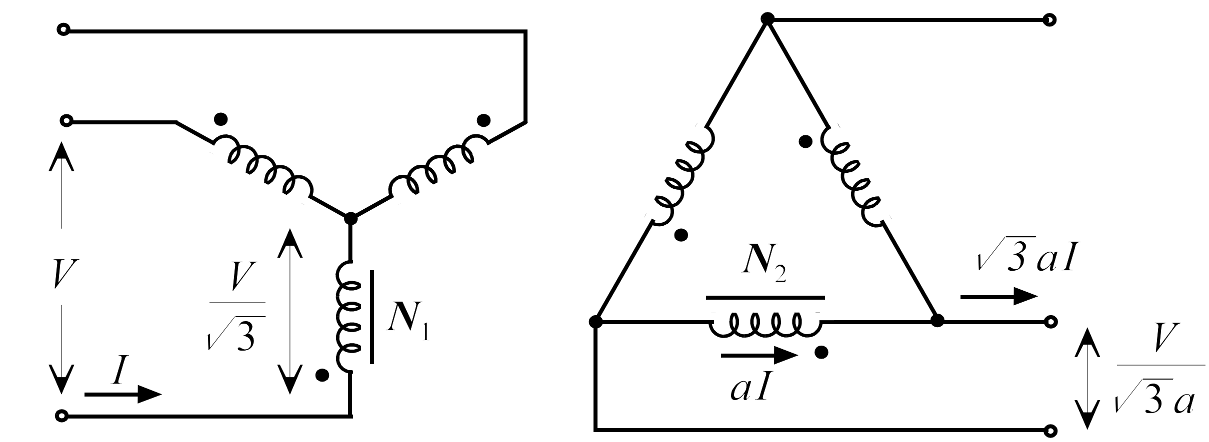
Transformers used in three-phase systems may consist of a bank of three single-phase transformers or a single three-phase transformer which is wound on a common magnetic core. A three-phase transformers wound on a common core offers advantages over a bank of single-phase transformers. A three-phase transformers wound on a common core is lighter, smaller and cheaper than the bank of three single-phase transformers. The common core three-phase transformer also requires much less external wiring than the bank of single-phase transformers and can typically achieve a higher efficiency.

The bank of three single-phase transformers does offer the advantage of flexibility. In the case of an unbalanced load, one or more transformer in the bank can be replaced by a larger or smaller kVA-rated transformer. In terms of maintenance, a malfunctioning transformer in the bank of transformers can be easily replaced while the entire common core three-phase transformer would require replacement.

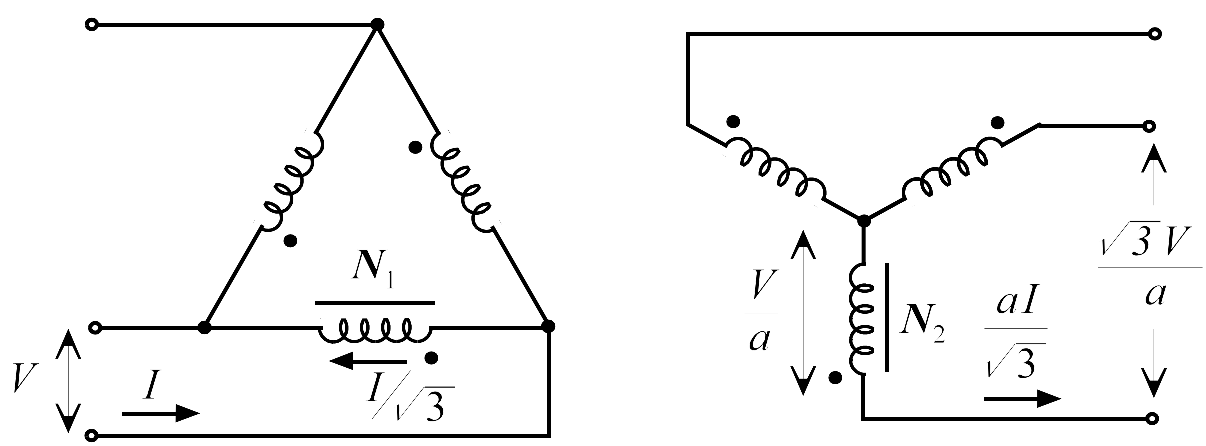
The bank of single-phase transformers or the common core three-phase transformer can be connected in one of four combinations relative to the primary and secondary connections.

Wye-Delta:	Commonly used in a step-down transformer, wye connection on the HV side reduces insulation costs, the neutral point on the HV side can be grounded, stable with respect to unbalanced loads.
Delta-Wye:	Commonly used in a step-up transformer for the same reasons as above.
Delta-Delta:	Offers the advantage that one of the transformers can be removed while the remaining two transformers can deliver three-phase power at 58% of the original bank.
Wye-Wye:	Rarely used, problems with unbalanced loads.

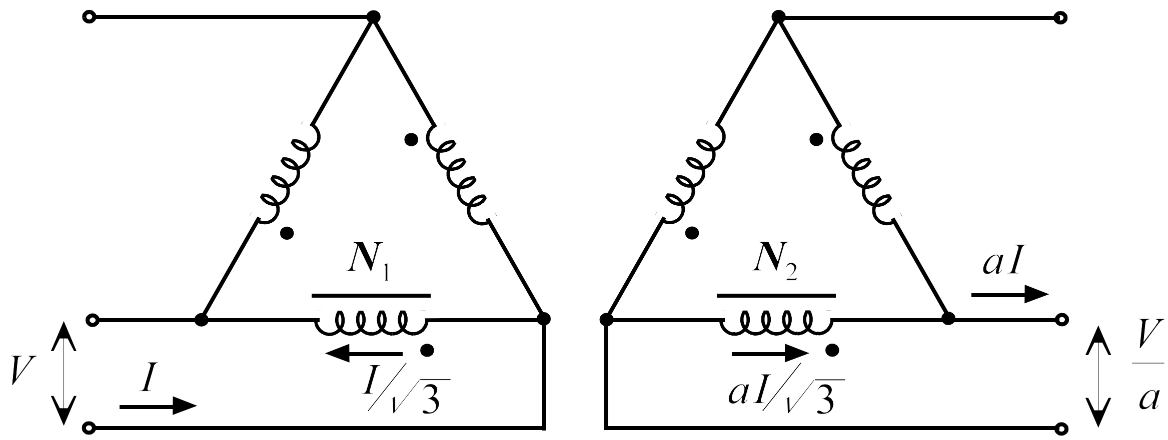
Wye-Delta Connection



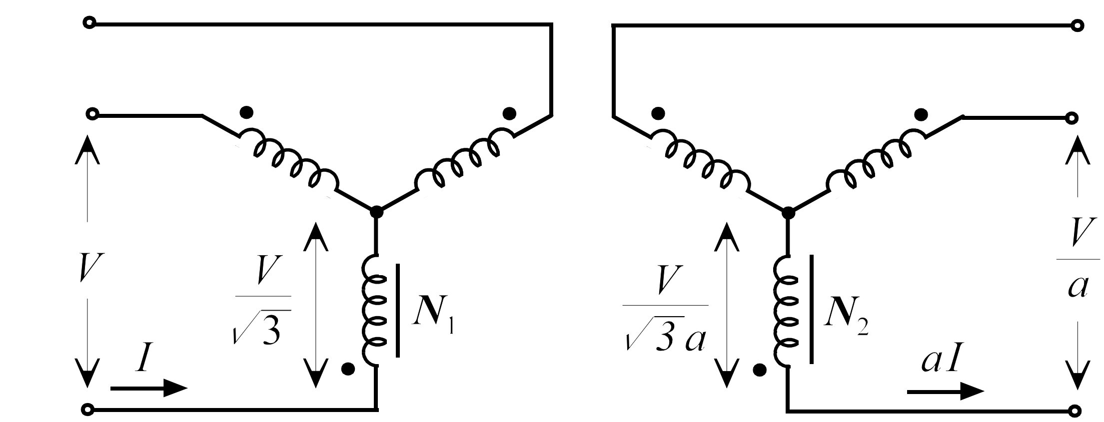
Delta-Wye Connection



Delta-Delta Connection



Wye-Wye Connection



Note that the voltage across a wye-connected primary or secondary winding is the line-to-neutral voltage while the voltage across a delta-connected primary or secondary winding is the line-to-line voltage. The magnitude of the complex three-phase power into or out of a three-phase transformer in a balanced system may be written as

$$S = 3\,V_w I_w$$

where V_w is the magnitude of the voltage across each winding and I_w is the magnitude of the current through each winding. In the wye-configuration, the winding voltage is the line-to-neutral voltage (V_{LN}) while the winding current is the line current (I_L). In the delta-configuration, the winding voltage is the line-to-line voltage (V_{LL}) while the winding current is the delta current (I_Δ). Thus, for a wye-connected winding, the magnitude of the complex power is

$$S_Y = 3\,V_{LN} I_L = 3\,\frac{V_{LL}}{\sqrt{3}}\,I_L = \sqrt{3}\,V_{LL}\,I_L$$

The magnitude of the complex power for the delta-connected winding is

$$S_\Delta = 3\,V_{LL}\,I_\Delta = 3\,V_{LL}\,\frac{I_L}{\sqrt{3}} = \sqrt{3}\,V_{LL}\,I_L$$

so that the equation for the complex power for either transformer winding connection is the same given the line-to-line voltage and the line current.

$$S = \sqrt{3}\,V_{LL}\,I_L$$

Per-Phase Analysis of Three-Phase Transformers

Assuming the three transformers in the three-phase transformer are identical and the sources and loads in the three-phase problem are balanced, circuits involving a the three-phase transformer can be analyzed on a per-phase basis as illustrated in our study of three-phase circuits. As previously discussed, the easiest three-phase topology to analyze is the wye-wye connection. Thus, given any other configuration for the three-phase transformer other than wye-wye, one should transform the circuit into wye-wye form.

The equivalent turns ratio for the transformed wye-wye per-phase equivalent circuit for the transformer is the ratio of the primary line-to-line voltage to the secondary line-to-line voltage for the original configuration.

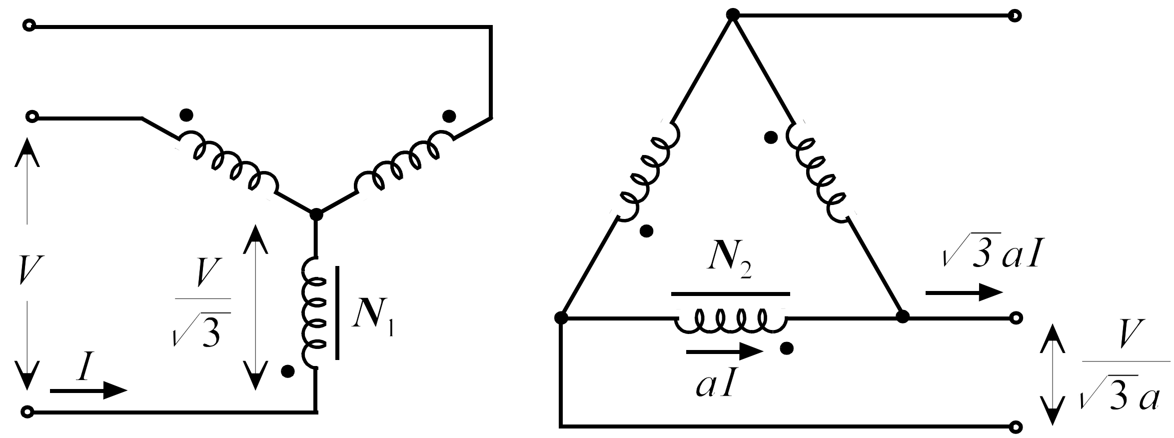
$$a' = \frac{\text{Primary line}-\text{to line voltage}}{\text{Secondary line}-\text{to line voltage}}$$

The concept of the equivalent turns ratio can be illustrated by an example transformation of a transformer configuration.

The wye-delta and delta-wye configurations of three-phase transformers result in 30° phase shifts between the primary and secondary line-to-line voltages. The industry standard is such that the lower voltages in these configurations should lag the higher voltages by 30°. The wye-wye or delta-delta configurations produce line-to-line voltages in the primary and secondary that are in phase.

Example

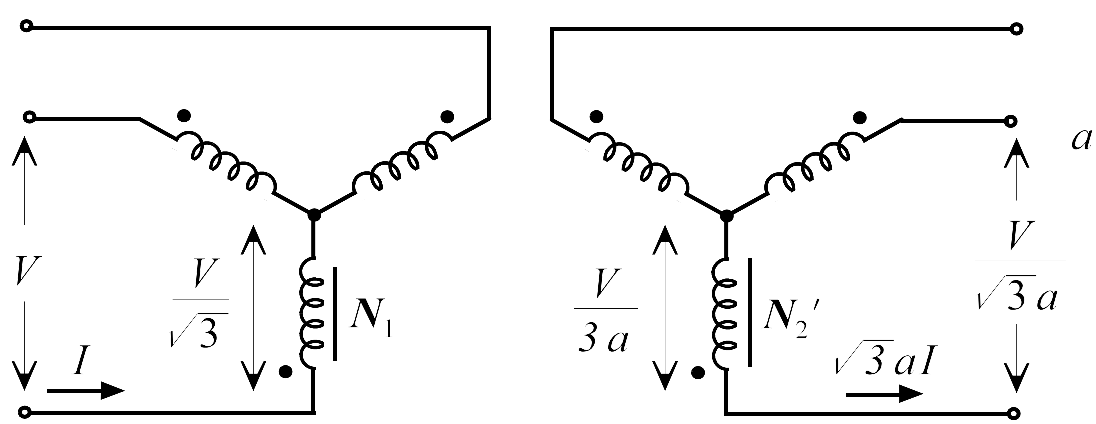
Transform a wye-delta three-phase transformer into the wye-wye configuration and determine the equivalent turns ratio a' of the resulting wye-wye transformer. Draw the per-phase equivalent circuit for the resulting wye-wye transformer.



The line-to-neutral voltages across the windings of the equivalent wye-connected secondary are found by dividing the line-to-line voltages across the windings of the of the delta-connected secondary by $\sqrt{3}$.

$$V_{\Delta,LL} = \frac{V}{\sqrt{3}a}$$

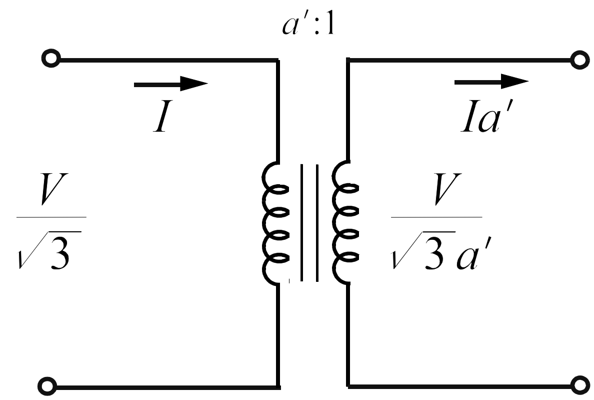
$$V_{Y,LN} = \frac{V_{\Delta,LL}}{\sqrt{3}} = \frac{V}{3a}$$



Comparing the voltages and currents of the primary and secondary windings, we see the that the equivalent turns ratio of the wye-wye configuration is

$$a' = \frac{N_1}{N_2'} = \sqrt{3}\,a = \frac{\sqrt{3}\,N_1}{N_2} \qquad \text{(Wye–Delta)}$$

The equivalent wye-wye model for the wye-delta connected three-phase transformer is



In a similar fashion, if we consider the transformation of the the delta-wye and delta-delta configurations to the wye-wye configurations, we find equivalent turns ratios of

$$a' = \frac{N_1'}{N_2} = \frac{a}{\sqrt{3}} = \frac{N_1}{\sqrt{3}\,N_2} \qquad \text{(Delta–Wye)}$$

$$a' = \frac{N_1'}{N_2'} = a = \frac{N_1}{N_2} \qquad \text{(Delta–Delta)}$$

Example (Per-phase equivalent circuit / three-phase transformer)

Three single-phase 50 kVA, 2300/230 V 60 Hz transformers are connected to form a three-phase 4000/230 V transformer bank (these voltages are line to line) which supplies a 120 kVA, 230 V, three-phase load with a power factor of 0.85 lagging. The equivalent impedance for each transformer referred to the LV winding is (0.012 + j 0.016) Ω.

(a.) Determine the transformer configuration required and draw the per-phase equivalent circuit.
(b.) Determine the transformer winding currents.
(c.) Determine the primary voltage required to produce the rated output.
(d.) Determine the voltage regulation.

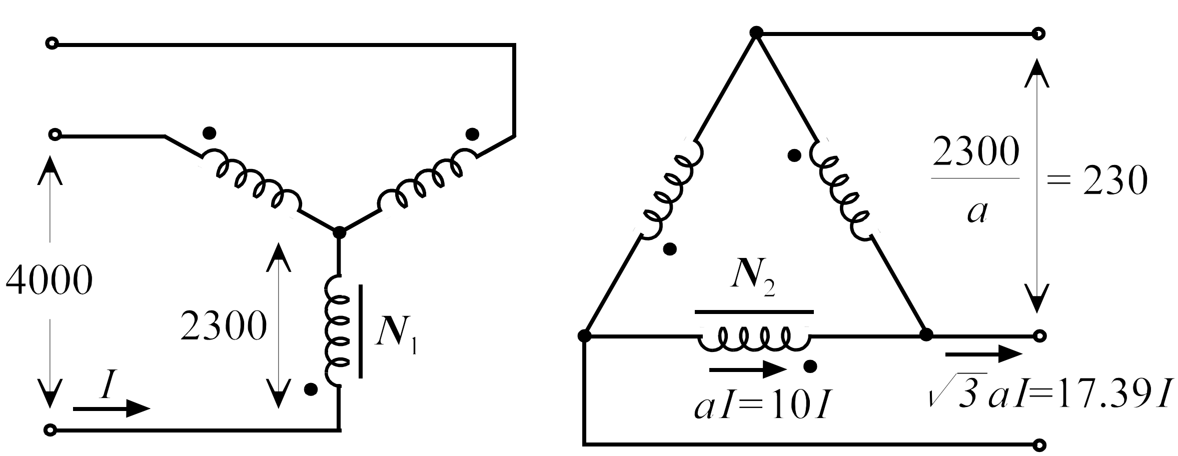
(a.) Individual transformers:

Primary winding rated voltage $V_{1,rated} = 2300$ V
Secondary winding rated voltage $V_{2,rated} = 230$ V
Turns ratio $a = N_1/N_2 = V_{1,\text{rated}}/V_{2,rated} = 2300/230 = 10$

3ϕ transformer:

Primary line to line voltage $V_{1,LL} = 4000$ V $\approx 2300 \times \sqrt{3}$
Secondary line to line voltage $V_{2,LL} = 230$ V
Y-Y equivalent turns ratio $a' = V_{1,LL}/V_{2,LL} = 4000/230 = 17.39$

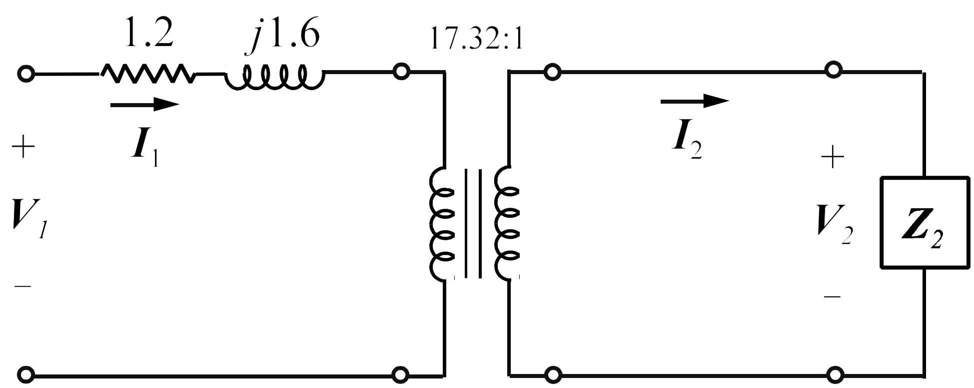
The required transformer connection is Wye-Delta.



The given equivalent impedance for each transformer is referred to the LV winding (secondary). This impedance referred to the HV input winding is

$$Z_{eq1} = a^2 Z_{eq2} = 10^2 (0.012 + j0.016) = (1.2 + j1.6)\ \Omega$$

Note that the turns ratio of the individual transformer is used to reflect the impedance between the primary and the secondary. The resulting wye-wye per-phase equivalent circuit is shown below.



(b.) The line current delivered to the three-phase load can be found from the complex power equation:

$$S = \sqrt{3}\, V_{LL} I_L \qquad \Rightarrow \qquad I_L = \frac{S}{\sqrt{3}\, V_{LL}} = \frac{120000}{\sqrt{3}\,(230)} = 301.23\ \text{A}$$

The actual current in the delta-connected secondary winding of this transformer is

$$I_\Delta = \frac{I_L}{\sqrt{3}} = \frac{301.23}{\sqrt{3}} = 173.92\ \text{A}$$

The corresponding current in the wye-connected primary is

$$I_Y = \frac{I_\Delta}{a} = \frac{173.92}{10} = 17.39\ \text{A}$$

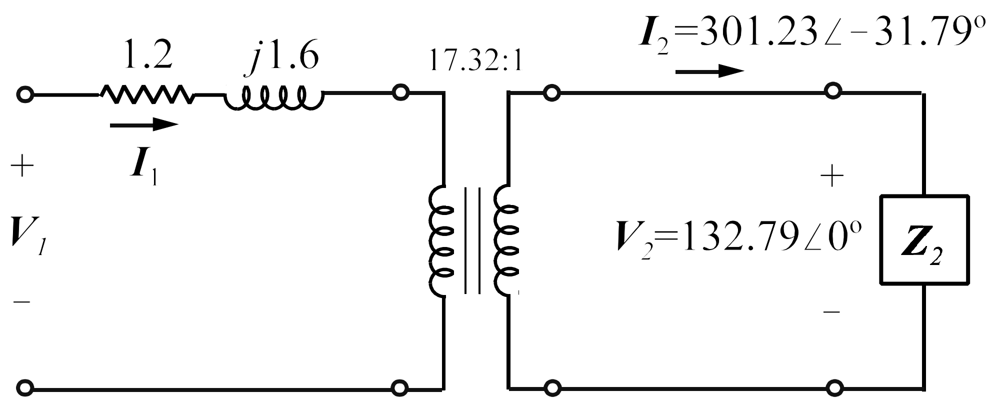
(c.) To determine the line-to-line voltage on the primary required to produce a secondary line-to-line voltage of 230 V, we must analyze the per-phase equivalent circuit. In the per-phase equivalent circuit, the current $\boldsymbol{I}_2$ is the secondary line current (magnitude = 301.23 A) while the voltage $\boldsymbol{V}_2$ is the secondary line-to-neutral voltage (magnitude = $230/\sqrt{3}$ = 132.79 V). The power factor of the load gives the phase angle difference between $\boldsymbol{V}_2$ and $\boldsymbol{I}_2$.

$$PF = 0.85 \text{ lagging} \quad \Rightarrow \quad (\theta_v - \theta_i)_{\text{Load}} = \cos^{-1}(0.85) = 31.79^o$$

Using the secondary line-to-neutral voltage as our reference gives

$$\boldsymbol{V}_2 = 132.79\angle 0^o \text{ V}$$

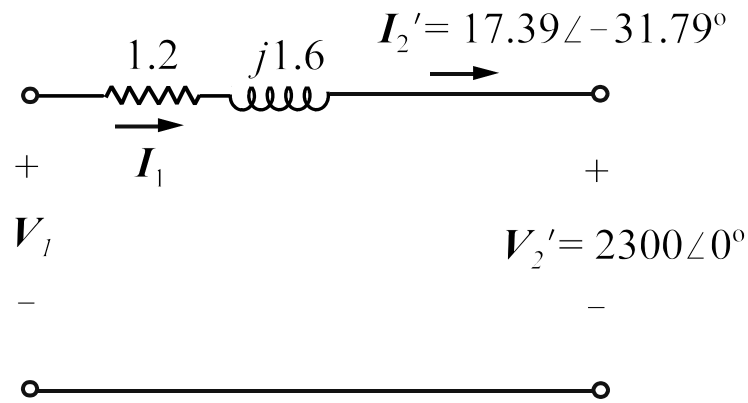
$$\boldsymbol{I}_2 = 301.23\angle -31.79^o \text{ V}$$



The secondary values can be reflected back to the primary according to the modified turns ratio a'.

$$\boldsymbol{V}_2' = a'\,\boldsymbol{V}_2 = (17.32)(132.79\angle 0^o) = 2300\angle 0^o \text{ V}$$

$$\boldsymbol{I}_2' = \frac{\boldsymbol{I}_2}{a} = \frac{301.23\angle -31.79^o}{17.32} = 17.39\angle -31.79^o \text{ V}$$



The resulting primary voltage $\boldsymbol{V}_1$ (line-to-neutral) is

$$\begin{aligned}\boldsymbol{V}_1 &= \boldsymbol{V}_2' + \boldsymbol{I}_2'\boldsymbol{Z}_{eq1} \\ &= 2300\angle 0^o + (17.39\angle -31.79^o)(2\angle 53.13^o) \\ &= 2300\angle 0^o + 34.78\angle 21.34^o \\ &= 2332.4\angle 0.31^o\end{aligned}$$

The magnitude of the primary line-to-line voltage is

$$\sqrt{3}\,(2332.4) = 4039.8\ \text{V}$$

(d.) The voltage regulation of this transformer is given by

$$\begin{aligned}VR &= \frac{|\boldsymbol{V}_1| - |\boldsymbol{V}_2'|_{rated}}{|\boldsymbol{V}_2'|_{rated}} \times 100 \\ &= \frac{2332.4 - 2300}{2300} \times 100 \\ &= 1.41\ \%\end{aligned}$$