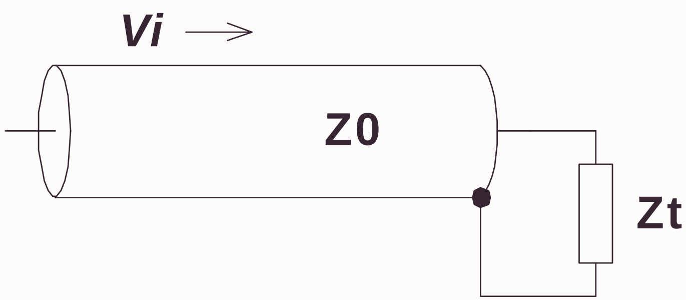
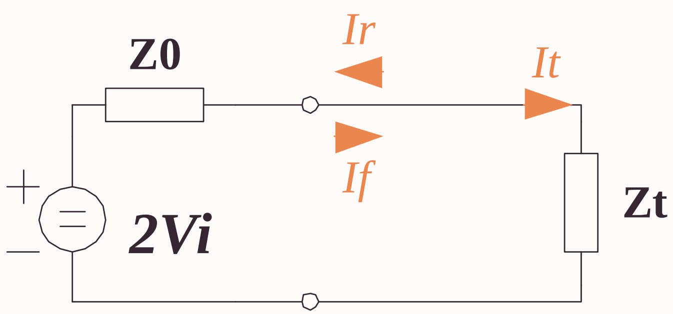
TL/Finite Length

What happens when a traveling wave reaches the end of a transmission line?

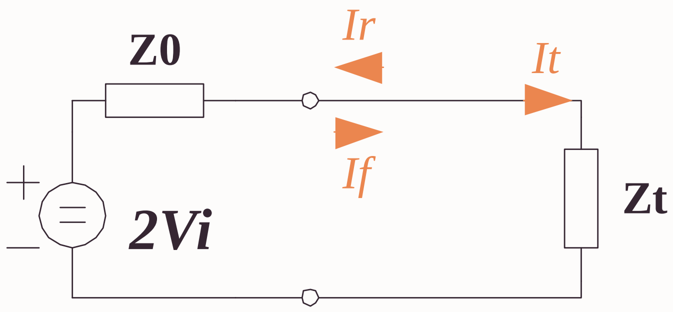




Thought Process:

- Transmission line with impedance Z0
- Cut open
- Develop TEC for open transmission line
- Terminate with termination impedance Zt
- Determine current **It** in termination impedance
- Split up current into two components:
    - Current **If (=Ii)** due to forward traveling wave
    - Current **Ir** due to reflection

TL/Finite Length/Reflection Coefficient



$$It = \frac{2Vi}{Z0 + Zt} \qquad If = \frac{Vi}{Z0} = Ii$$

$$Ir = If - It = \frac{Vi}{Z0} - \frac{2Vi}{Z0 + Zt}$$

Termination current is the superposition of the current due to the forward traveling wave, and the current due to the reflection

$$Ir = \frac{Vi}{Z0} \cdot \frac{Zt - Z0}{Zt + Z0}$$

Famous Telegrapher's Equation:

- Reflection coefficient kr
- kr can be complex (and f-dependent)
- but in practice it is desirable to keep kr constant and real

$$kr = \frac{Ir}{Ii} = \frac{Vr}{Vi} = \frac{Zt - Z0}{Zt + Z0}$$

TL/Finite Length/TL/Coefficients

Definitions:

- TLIA(p): TL Input Acceptance Coefficient
- TLT(p): TL Output Transmission Coefficient
- TLRl(p): TL Load-End Reflection Coefficient
- TLRs(p): TL Source-End Reflection Coefficient

TL/Finite Length/Coefficients/TLRl & TLRs

Load-end reflection coefficient TLRl(p):

$$TLRl(p) = \frac{Zl(p) - Z0(p)}{Zl(p) + Z0(p)}$$

Definition of reflection coefficient kr applied to both load-end and source-end of transmission line…

Source-end reflection coefficient TLRs(p):

$$TLRs(p) = \frac{Zs(p) - Z0(p)}{Zs(p) + Z0(p)}$$

Reflection coefficients

•***No reflection*** if Zt=Z0

•If Zt and Z0 are real:

  •kr in a range [-1..+1]

•if Zt=0 $\rightarrow$ kr=-1

•if Zt=∞ $\rightarrow$ kr=+1

TL/Finite Length/Coefficients/TLT

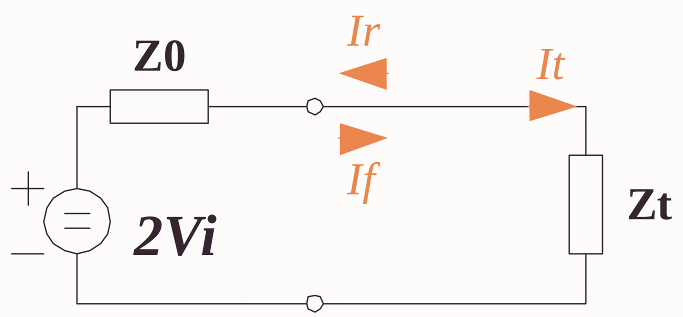
Output Transmission Coefficient TLT(p):

$$TLT(p) = \frac{Vt}{Vi} = 1 + TLRl(p)$$ *because…*

$$1 + TLRl = 1 + \frac{Zt - Z0}{Zt + Z0} = \frac{Zt + Z0 + Zt - Z0}{Zt + Z0} = \frac{2Zt}{Zt + Z0}$$

$$Vt = 2Vi \frac{Zt}{Z0 + Zt}$$

$$Vt = Vi(1 + TLRl) \quad \text{q.e.d.}$$



TL/Finite Length/Coefficients/TLIA

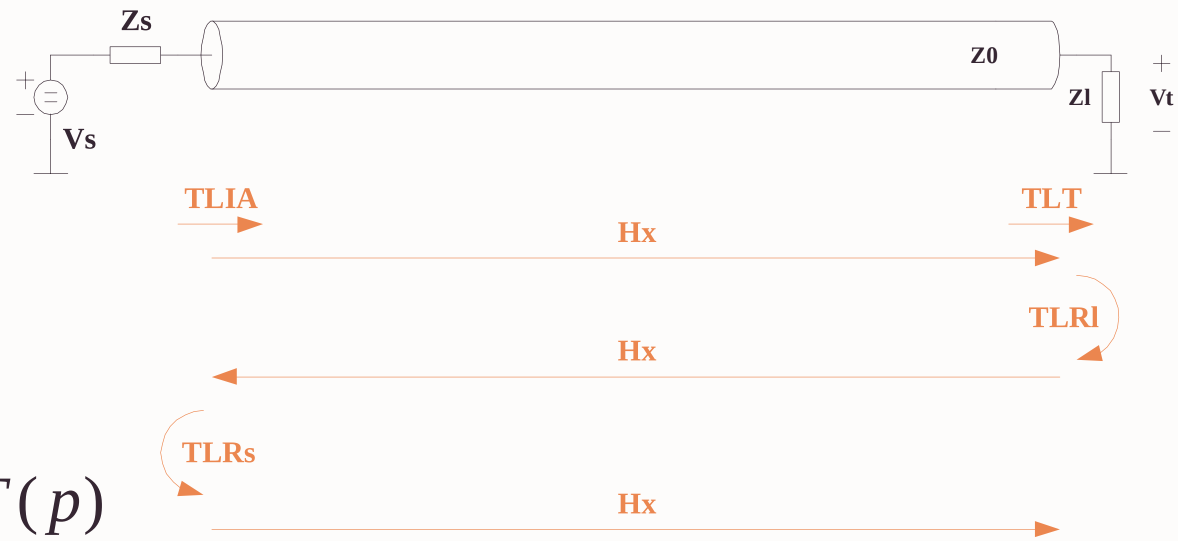
Input acceptance coefficient TLIA(p):

$$TLIA(p) = \frac{Z0(p)}{Zs(p) + Z0(p)}$$

Fraction of the input voltage accepted by the transmission line…

TL/Finite Length/Transfer Function

Transfer function $S_\infty(p)$ for signals emerging from the transmission line:



$$S_0(p) = TLIA(p) \cdot Hx(p) \cdot TLT(p)$$

$$S_1(p) = TLIA(p) \cdot Hx(p)\left(TLRl(p) \cdot Hx(p) \cdot TLRs(p) \cdot Hx(p)\right)TLT(p)$$

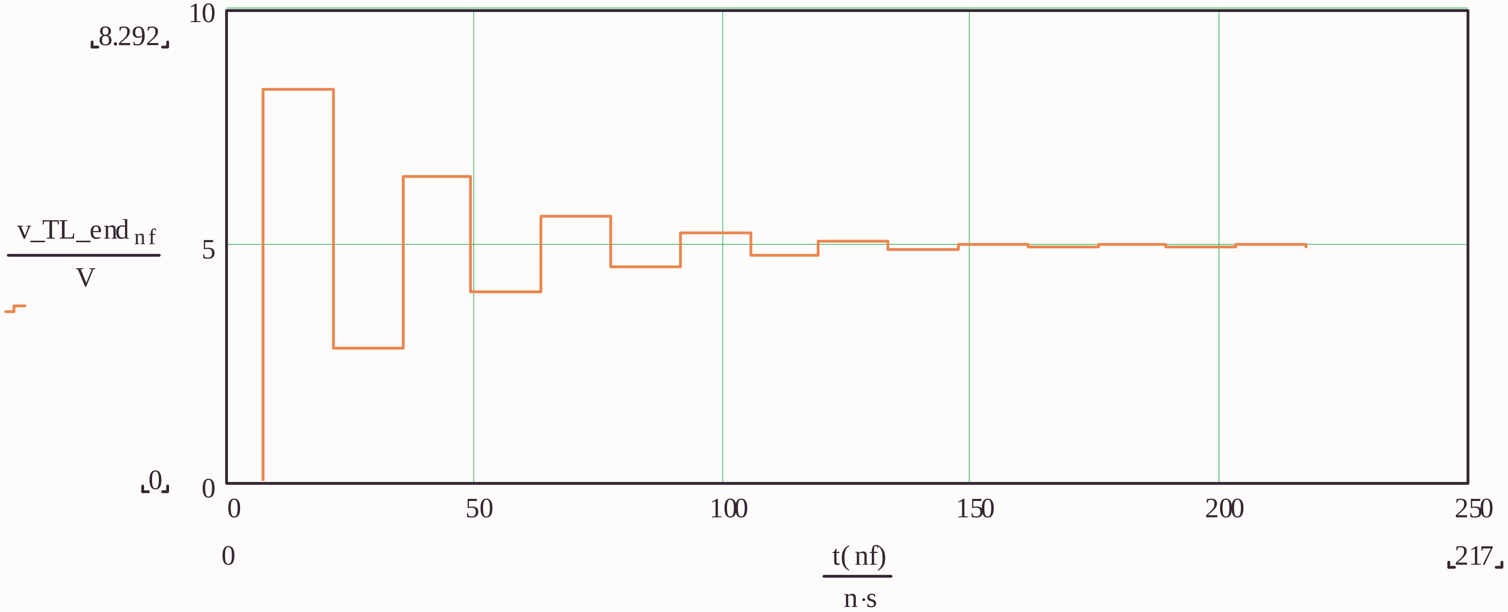
$$S_N(p) = TLIA(p) \cdot Hx(p)\left(TLRl(p) \cdot Hx(p)^2 \cdot TLRs(p)\right)^N TLT(p)$$

$$S_\infty(p) = \sum_{N=0}^{\infty} S_N(p) = \frac{TLIA(p) \cdot Hx(p) \cdot \left(1 + TLRl(p)\right)}{1 - TLRl(p) \cdot Hx(p)^2 \cdot TLRs(p)} = \frac{Vt(p)}{Vs(p)}$$

TL/Finite Length/Transfer Function/Example

Example: Reflections on a transmission line

- C=140pF/m, L=350nH/m. R=G=negligible. Length=1m.
- Z0=50Ω. Tp_pul=7ns/m.
- Vcc=5V. Zs=10Ω. Zl=10kΩ.
- TLIA=0.833. TLRl=0.99. TLRs=-0.667. S_{∞}=99.8%



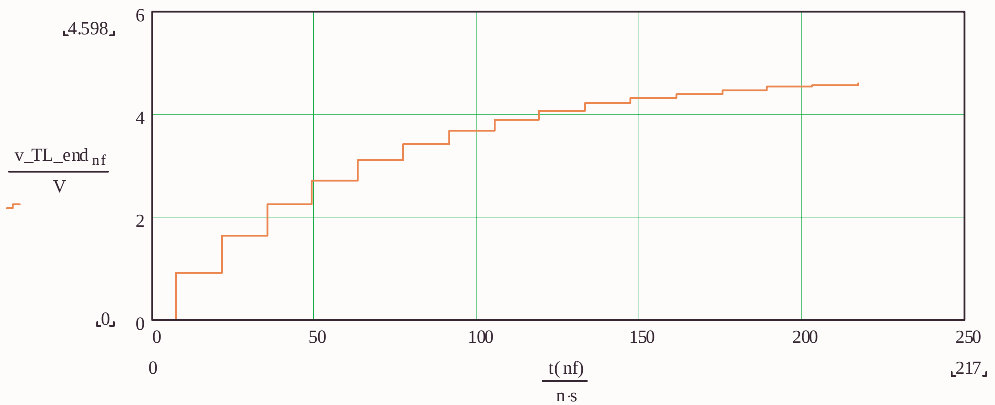
S =

	0
0	1.658
1	-1.095
2	0.722
3	-0.477
4	0.315
5	-0.208
6	0.137
7	-0.09
8	0.06
9	-0.039
10	0.026
11	-0.017
12	0.011
13	-0.007
14	0.005
15	-0.003

$\frac{V}{V}$

Case 1: Low source impedance with unterminated transmission line

TL/Finite Length/Transfer Function/Example

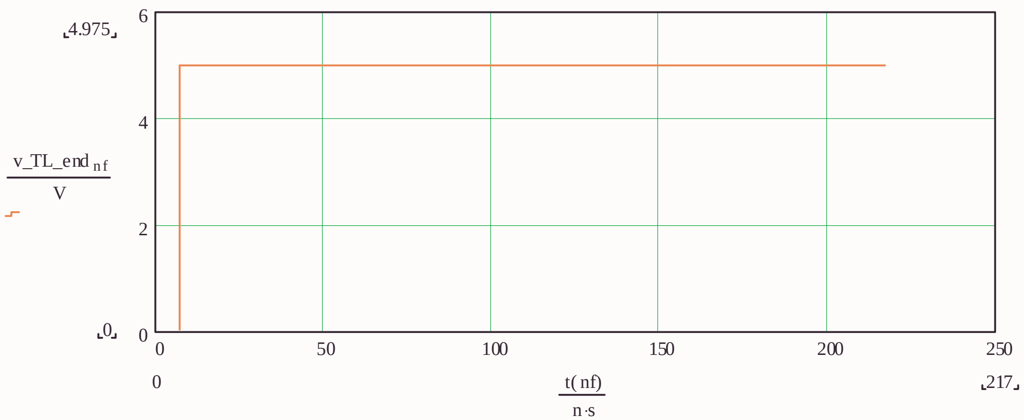


Case 2: High source impedance with unterminated transmission line

Same example… different termination resistors…

•top: Zs=500Ω. Zl=10kΩ. (TLIA=0.091. TLRl=0.99. TLRs=0.818. S_∞=92%)

•bottom: Zs=50Ω. Zl=10kΩ. (TLIA=0.5. TLRl=0.99. TLRs=0. S_∞=99.5%)



Case 3: Source-end terminated

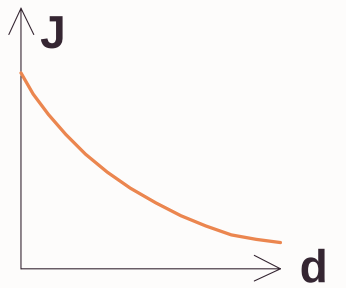
TL Part 2/Overview

- Transmission Lines
  - High Frequency Mechanisms in Transmission Lines
    - Skin-Effect
    - Proximity Effect
  - Terminations
  - Transmission Lines on PCBs
    - Equations

TL/Skin Effect

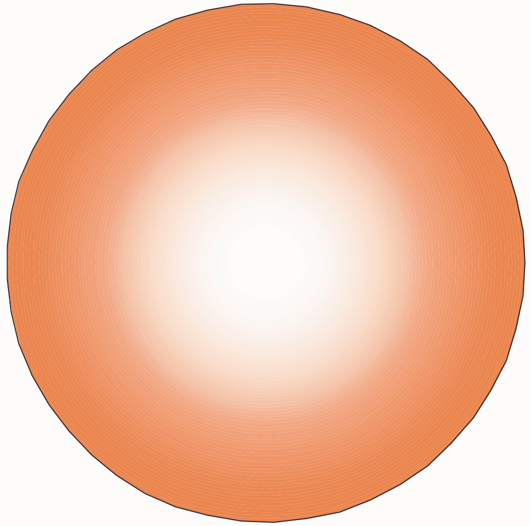
At low frequencies, current density inside a conductor is uniform. At high frequencies, it isn't.

Conductor carrying high frequency currents:
•Current flow primarily on the surface of a conductor
•Phenomena is called ***skin effect***
•Current density falls off exponentially with depth into the conductor



$$J(d) = J_0 \cdot e^{-\frac{d}{\delta}}$$

with skin depth $\delta = \sqrt{\dfrac{\rho}{\pi \cdot f \cdot \mu}}$



ρ : material resistivity

μ : permeability

TL/Skin Effect

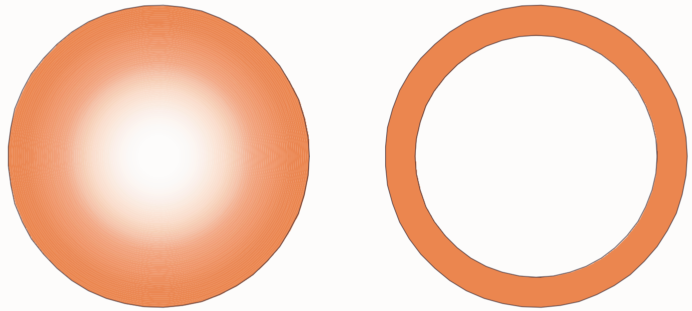
Skin effect

•Current density falls off exponentially with depth into the conductor

•Modelling: Current flows uniformly in an outer shell of the conductor with thickness δ.

•Skin depth is a material property (not a function of conductor shape)

•For most transmission lines, skin effect is the reason for their lossy nature



$$fs = \frac{\rho}{\pi \mu r^2}$$

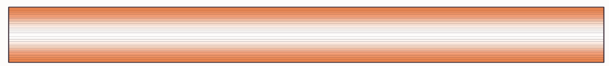
•for f<fs skin effect negligible. R=Rdc

•for f>fs skin effect. Resistance increases with square root of frequency

How to tackle skin effect problems

•Litz wire

•Planar conductors



TL/Skin Effect

conducting area:

$$A = r^2\pi - (r-\delta)^2\pi$$

$$A = \pi(2r\delta - \delta^2)$$

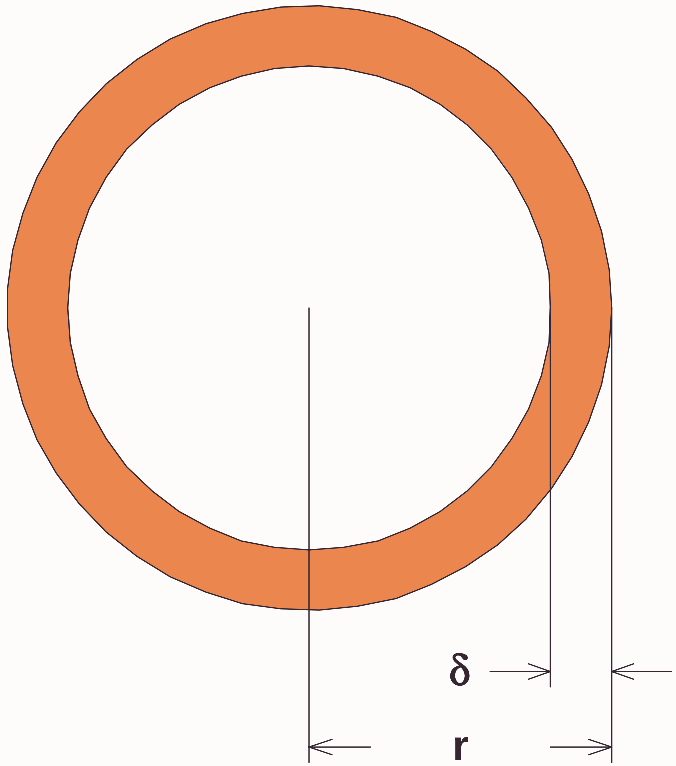
for high frequencies δ<<r:

$$A \approx \pi 2r\delta$$

Resistance (per unit length):

$$R_{pul} = \frac{\rho}{A}$$

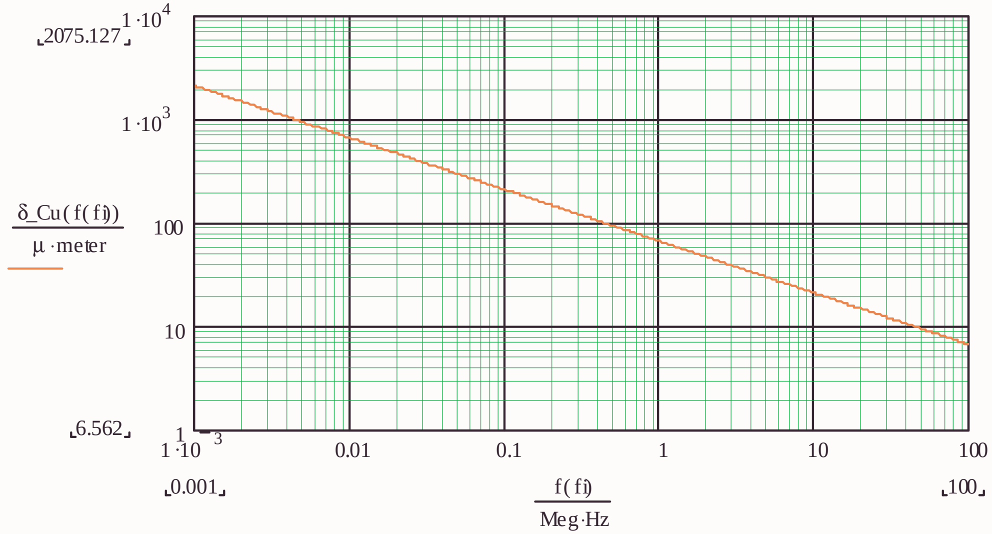
$$Rhf_{pul} = \frac{1}{2r}\sqrt{\frac{f\mu\rho}{\pi}}$$



TL/Skin Effect/Example

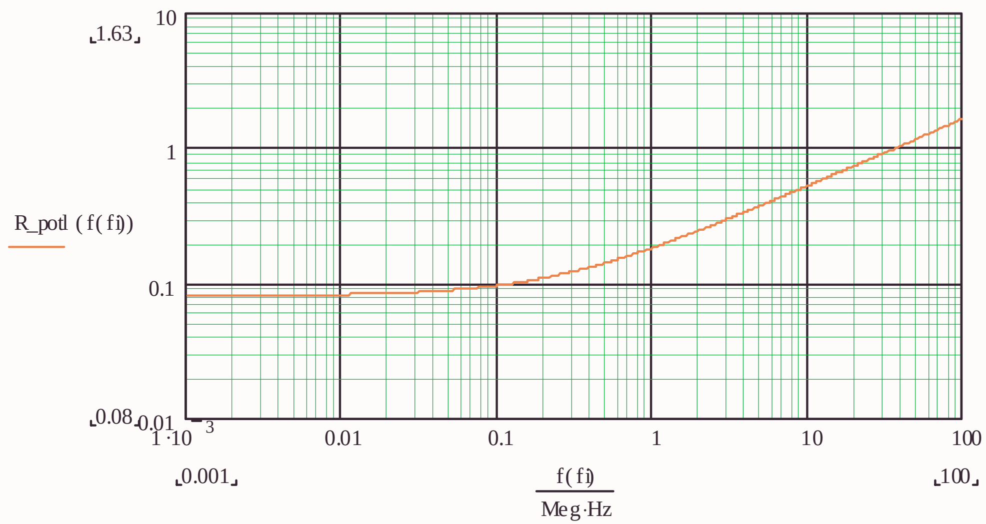
Example: AWG24 Transmission Line

- C=40pF/m, L=400nH/m, Rdc=80mΩ/m
- wire radius AWG24: r=253μm
- skin effect frequency fs=67kHz



Plots

- top right: skin depth
- bottom right: effective resistance per unit length



$$R_{pul} \approx \sqrt{Rdc_{pul}^{\ 2} + Rhf_{pul}^{\ 2}}$$

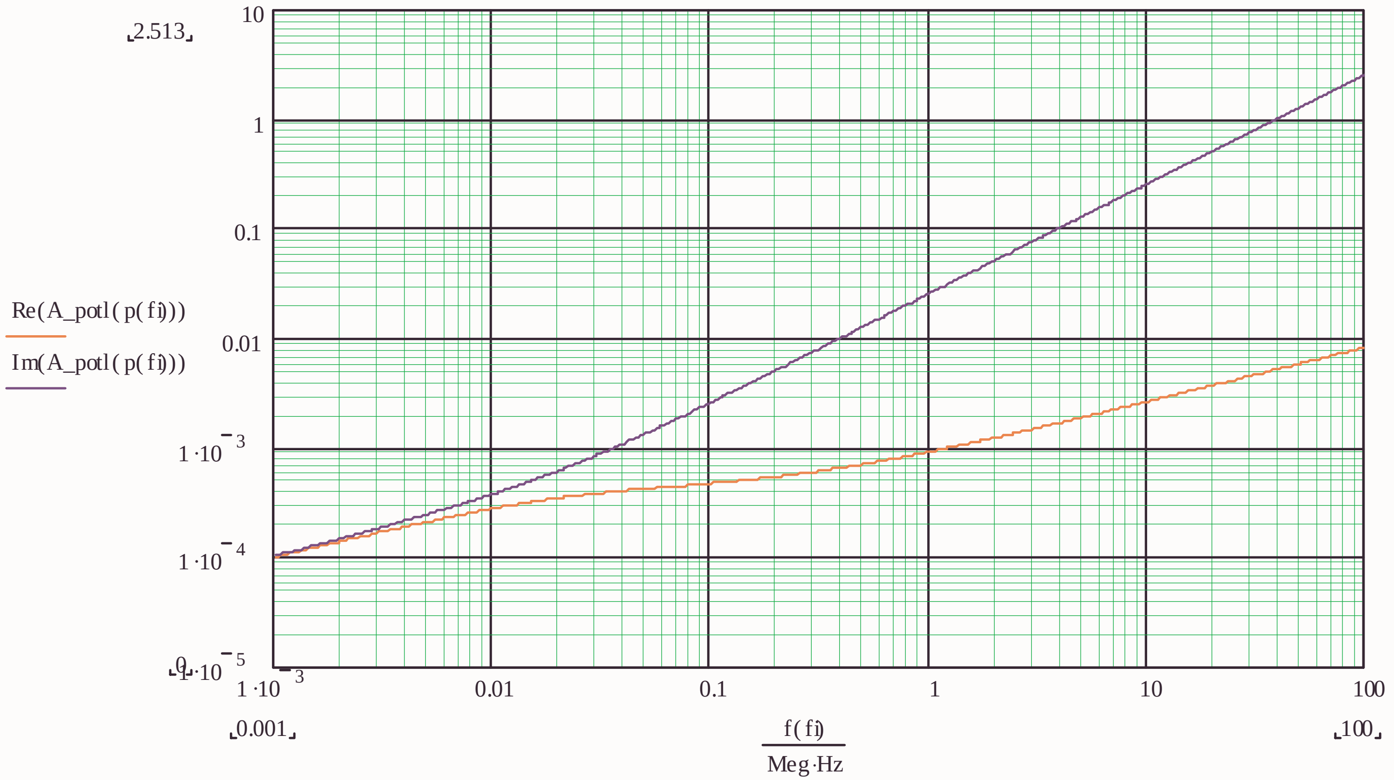
TL/Skin Effect/Propagation Constant A

Example: AWG24 Transmission Line. Three regions:

•Low frequency: RC TL behaviour (distortion)

•Mid frequency: LC TL behaviour (no distortion, just delay)

•High frequency: Distortion because of skin effect



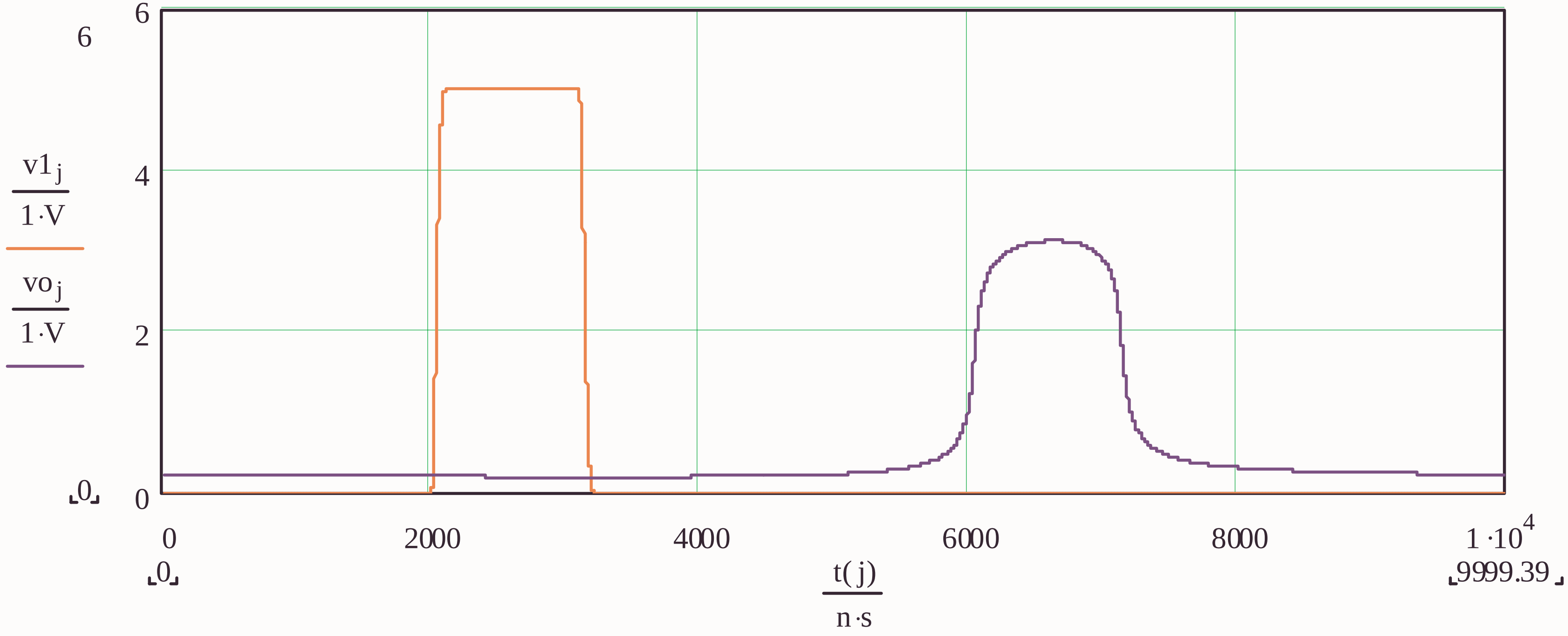
TL/Skin Effect/Example

Example: Transmission of a pulse over a long AWG24 Telephone Line

•C=40pF/m, L=400nH/m, R=80mΩ/m

•Tp=4µs/km

•Length of transmission line: 1km

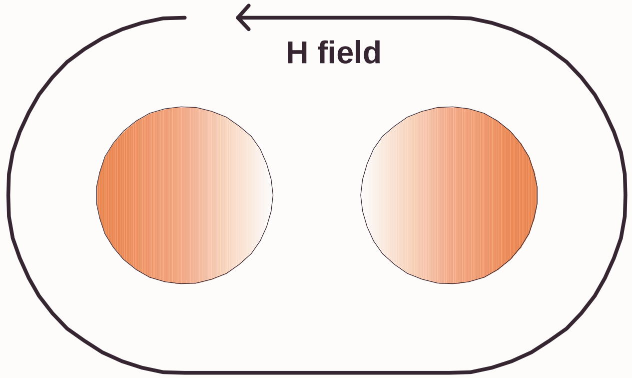


Signal distortion on an RLC Transmission Line due to skin effect

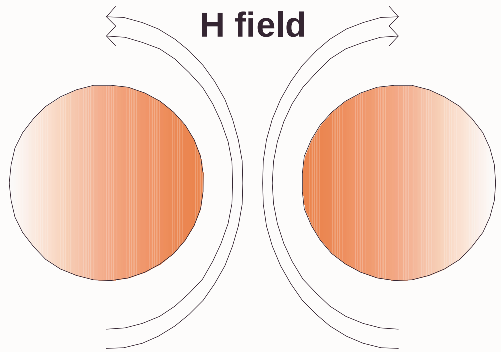
TL/Proximity Effect

Proximity Effect

- Current distribution in a conductor is affected by currents in adjacent conductors
- Like the skin effect, the proximity effect leads to a larger effective resistance at high frequencies
- Much harder to quantify… (use tables, graphs, field solvers)
- For same current direction increase in resistance is modest (even if conductors almost touch)
- For opposite current direction proximity effect can be many times higher than skin effect (depending on distance of conductors)
- Take proximity effect into account whenever conductors are brought closer together than about 3 times their diameter

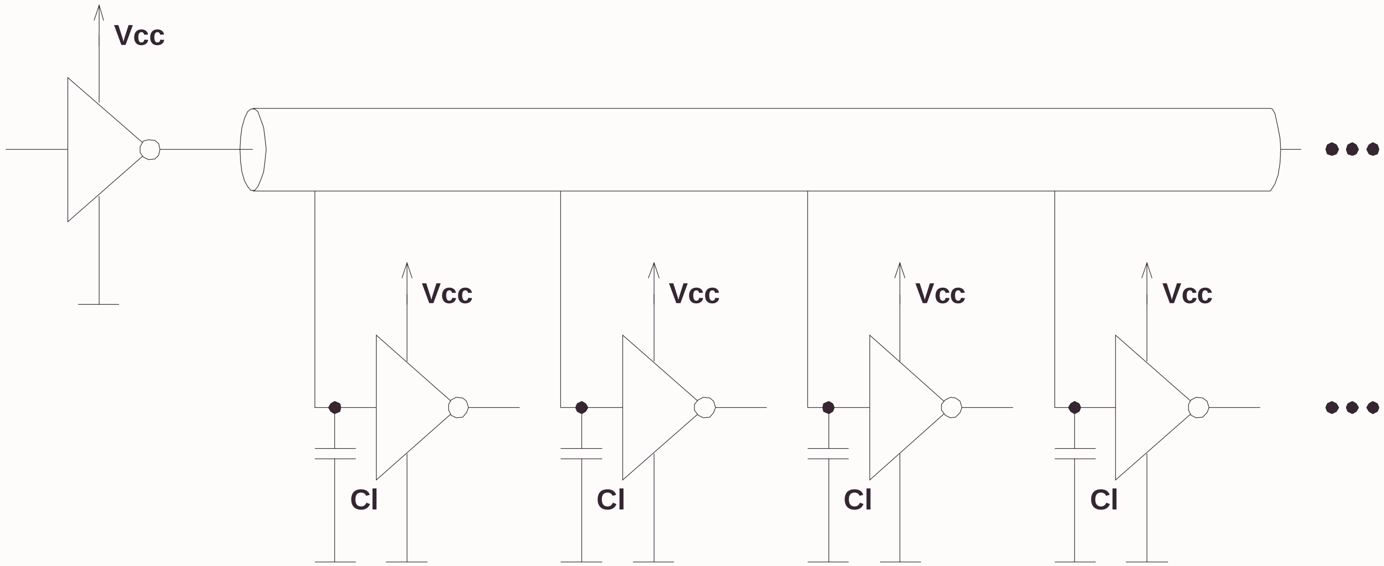


Same Current Direction



Opposite Current Direction

TL/Special Case



Equally Spaced Capacitive Loads

•Frequently encountered in large bus formations (e.g. memory modules)

•n capacitive loads are of equal value and spaced evenly over the length of the transmission line

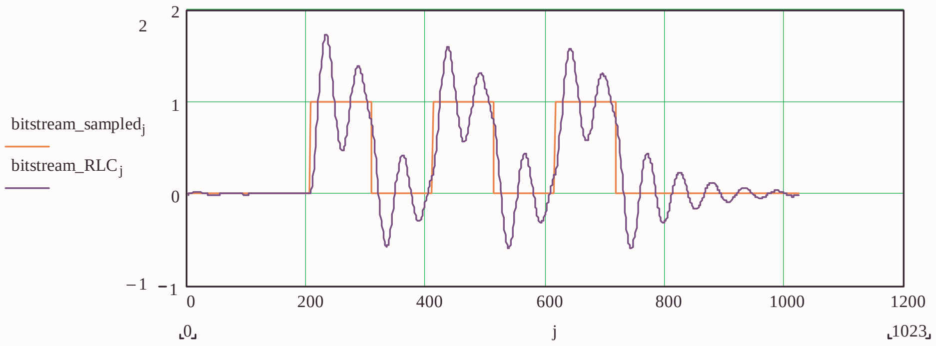
•applicable if effective length of rising edge exceeds spacing between capacitive loads…

$$Z0' = \sqrt{\frac{L}{C + \frac{n \cdot Cl}{length}}}$$

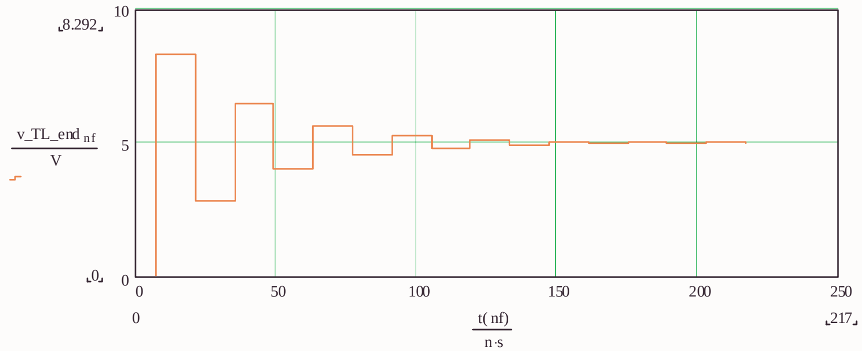
$$Tp' = \sqrt{L\left(C + \frac{n \cdot Cl}{length}\right)}$$

TL/Termination

- Short Lines (l<lr/6)
  - Termination required for damping



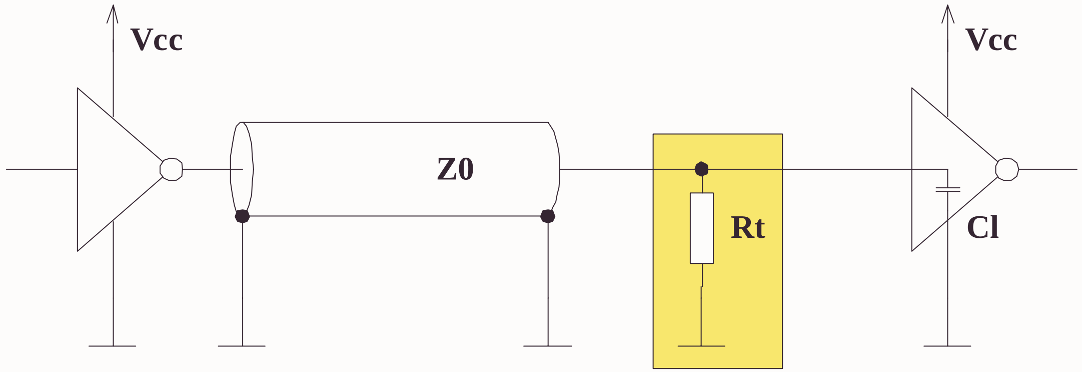
- Transmission Lines
  - Termination to eliminate reflections



TL/Terminations/End Termination

End termination

- Driver connects directly to TL
- All reflections damped by termination resistor Rt=Z0 (TLRl=0)
- Received voltage is equal to the transmitted voltage ($S_\infty \approx 100\%$)
- Short rise time
- Drawbacks:
  - •High power dissipation
  - •Imbalanced load (difficult to drive)



Assumptions:

•Rt=Z0. Hx≈1. TLIA≈1. TLRl=0.

$$Tr_{TL_{end}} = \sqrt{Tr_{driver}^{\ 2} + \left(\frac{2.2 \cdot Z0 \cdot Cl}{2}\right)^2}$$

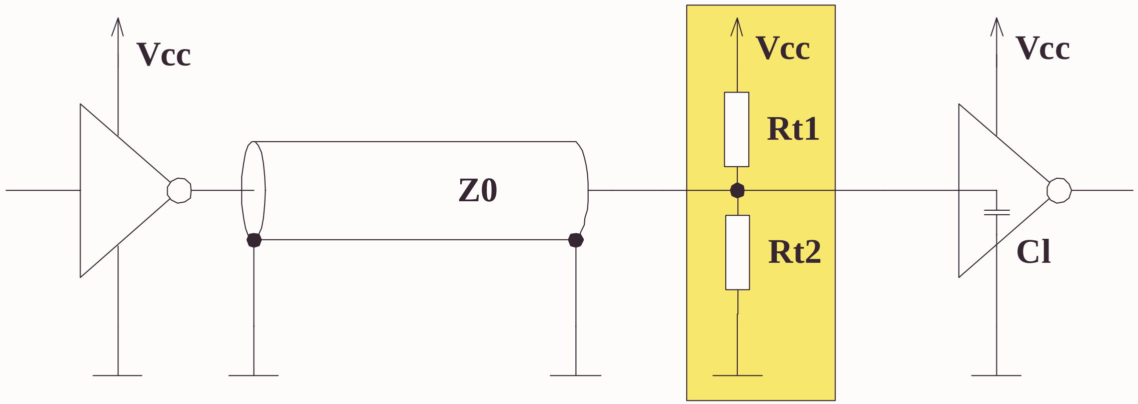
$$S_\infty(p) = \frac{TLIA(p) \cdot Hx(p) \cdot (1 + TLRl(p))}{1 - TLRl(p) \cdot Hx(p) \cdot TLRs(p)}$$

$$S_\infty(p) = TLIA(p) \cdot Hx(p) \cdot (1 + TLRl(p)) = 1$$

TL/Terminations/End Termination/Split

Split End termination

- Z0=(Rt1 || Rt2)
- Advantages:
  - Balanced power dissipation
  - Easier to drive
- For CMOS, HCMOS…
  - Rt1=Rt2=2Z0



Assumptions:

- Signal is dc-balanced (equal 1's and 0')

$$Pd_{Rt1} = Pd_{Rt2} = \frac{Vcc^2}{4 \cdot Z0}$$

$$Pd_{Rt1} + Pd_{Rt2} = \frac{Vcc^2}{2 \cdot Z0}$$

Worst case: Static signal (1 or 0)...

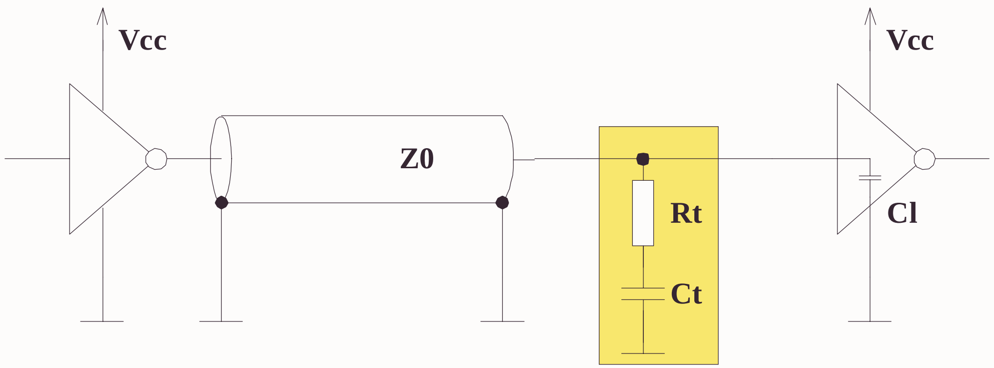
$$Pd_{Rt1\max} = Pd_{Rt2\max} = \frac{Vcc^2}{2 \cdot Z0}$$

(assuming that resistance of TL is negligible)

TL/Terminations/End Termination/AC Biased

AC Biased End termination

- Time constant large vs signal period
- Rt=Z0
- Advantages:
  - Lower average power consumption
  - Lower static power consumption
- Disadvantage
  - Difficult to drive if signal is not dc-balanced



Assumptions:

- Signal is dc-balanced (equal 1's and 0')

$$Pd_{Rt1} = \frac{Vcc^2}{4 \cdot Z0}$$

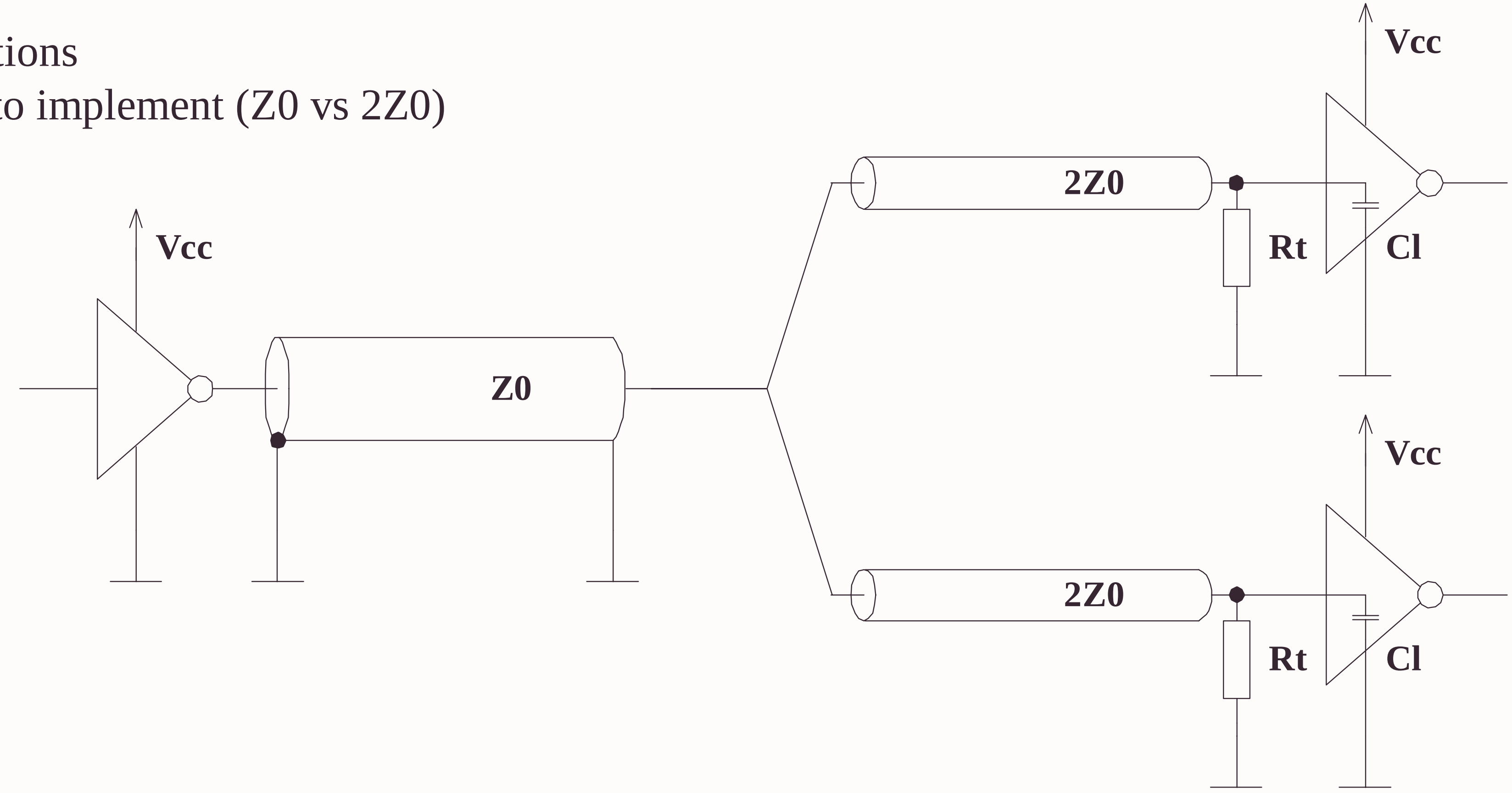
Static signal (1 or 0)...

$$Pd_{Rt1} = 0$$

TL/Terminations/End Termination/Bifurcation

Bifurcation

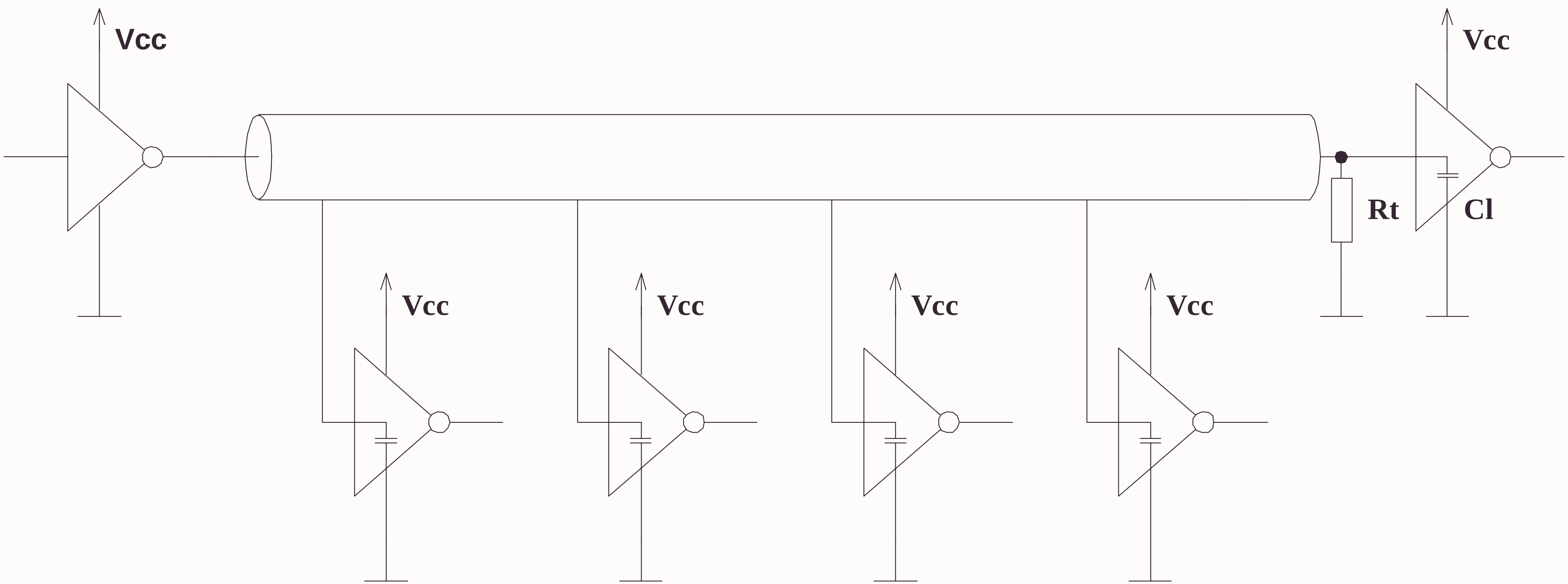
- Rt=2Z0
- No reflections
- Difficult to implement (Z0 vs 2Z0)



TL/Terminations/End Termination/Daisy Chain

Diasy Chain Configuration

•Keep stubs as short as possible
•Minimise capacitive load
•Multiple stubs: Space equally



TL/Terminations/Source Termination

Source termination

•All reflections damped at the source side by source termination resistor Rt=Z0 (TLRs=0)

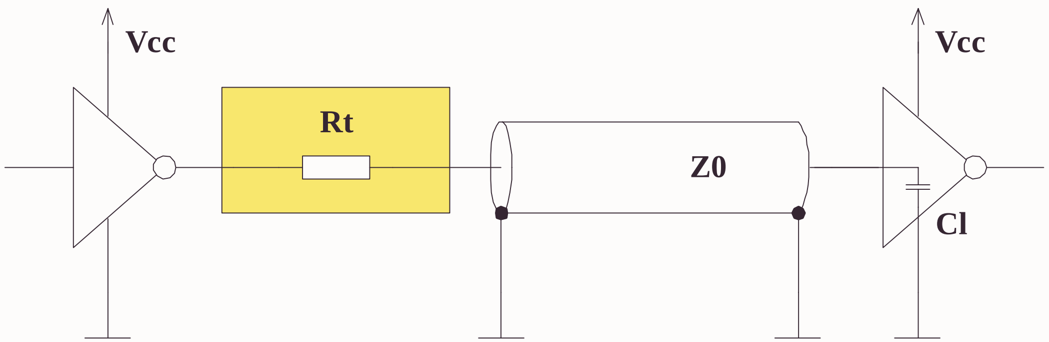
•Advantages:

  •Lower average drive currents

•Disadvantages

  •Output impedance of driver often not tightly specified

  •Daisy-chaining not recommended



Assumptions:

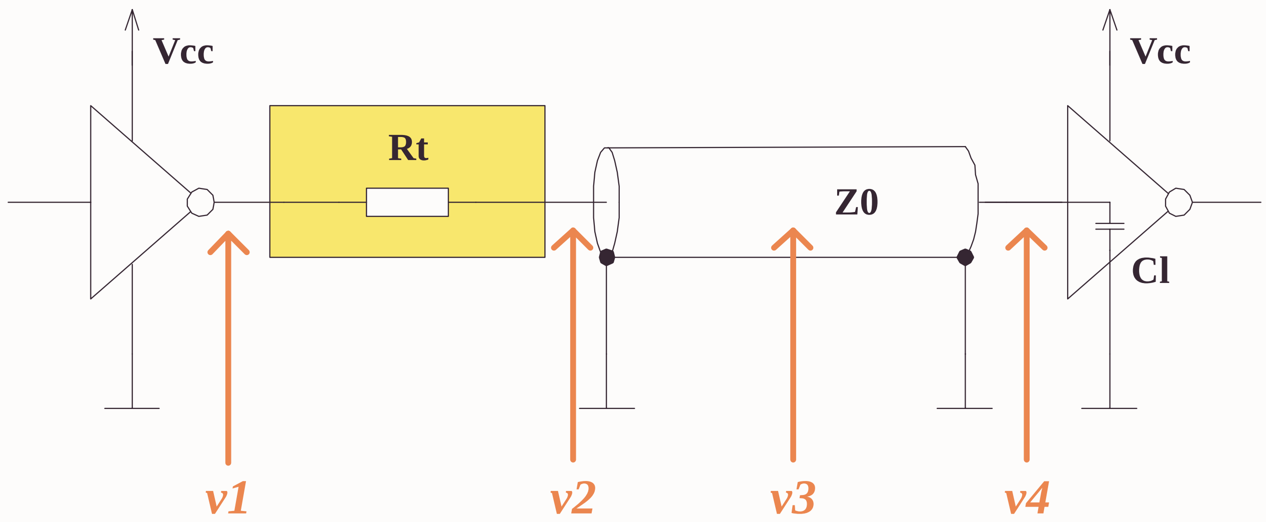
•Rt=Z0. Hx≈1. TLIA≈0.5. TLRl=1

$$S_{\infty}(p) = \frac{TLIA(p) \cdot Hx(p) \cdot (1 + TLRl(p))}{1 - TLRl(p) \cdot Hx(p) \cdot TLRs(p)}$$

$$Tr_{TL_{end}} = \sqrt{Tr_{driver}{}^{2} + (2.2 \cdot Z0 \cdot Cl)^{2}}$$

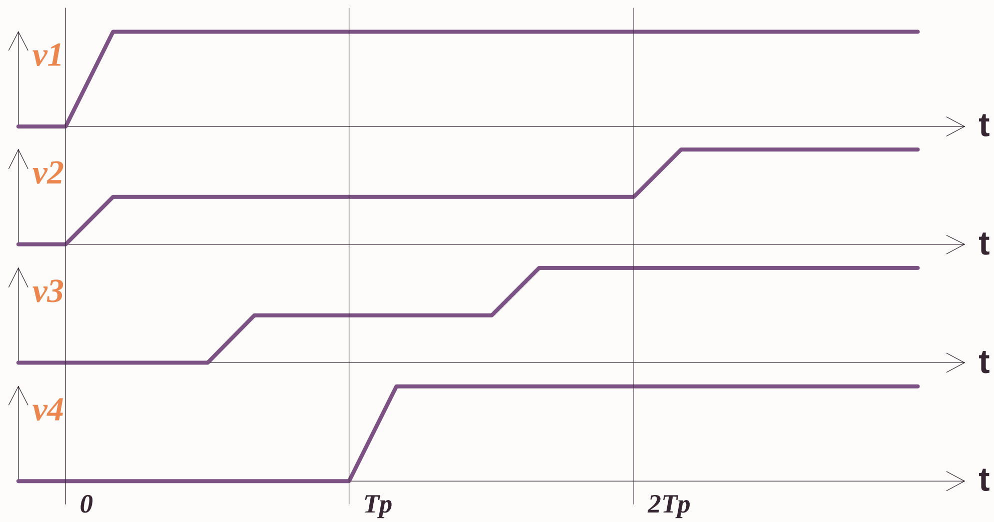
$$S_{\infty}(p) = TLIA(p) \cdot Hx(p) \cdot (1 + TLRl(p)) = 1$$

TL/Terminations/Source Termination

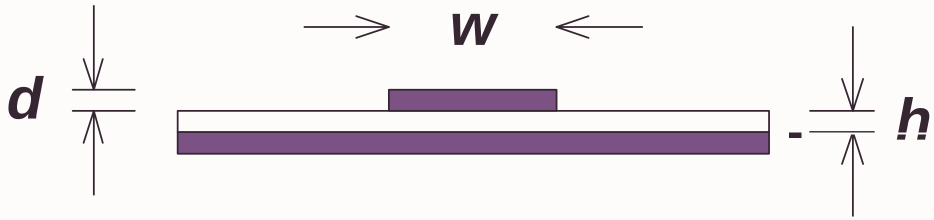


Source termination

- Driving signal cut in half (TLIA=0.5)
- Driving signal propagates down TL
- Reflection at load side (TLRl=1)
- Reflected signal travels back
- Reflected signal damps out at the source termination (TLRs=0)

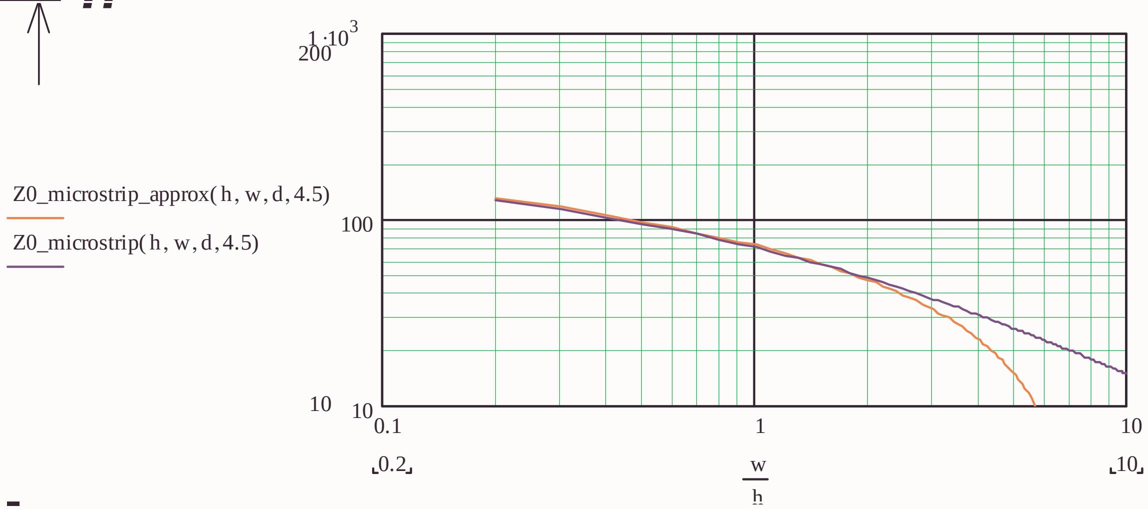


TL/Terminations/Microstrip Equations



Example: Microstrip on FR4

• εr=4.5. 2oz copper

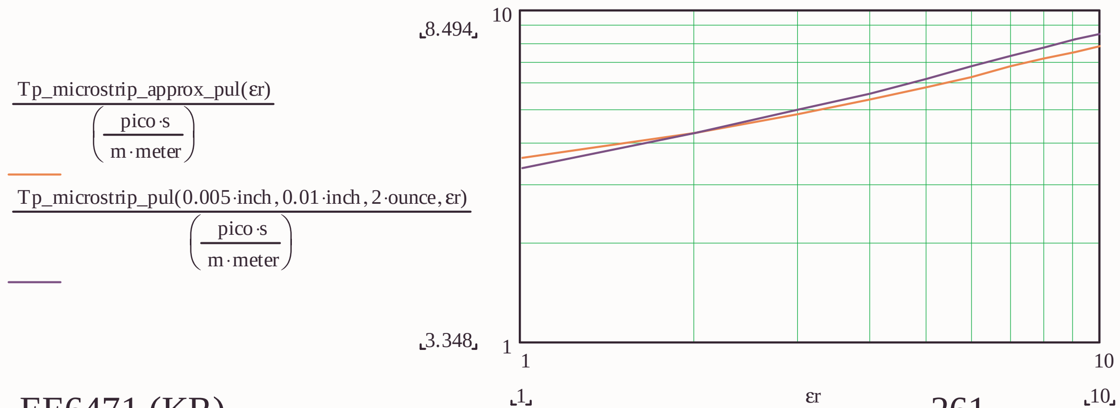


Microstrip Equations

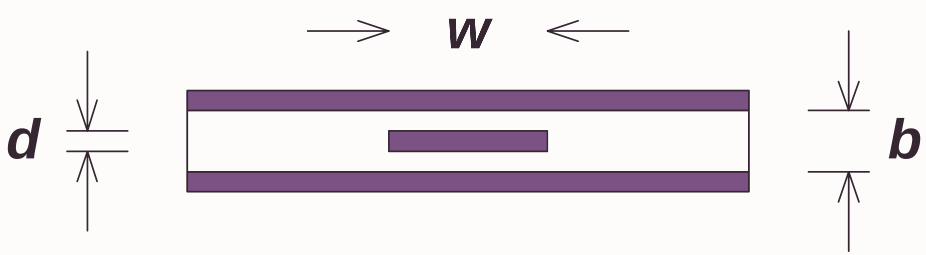
- Useful approximations
- Use numeric TL tools for improved accuracy

$$Z0 \approx \frac{87\Omega}{\sqrt{\varepsilon r + 1.41}} \ln\left(\frac{5.98h}{0.8w + d}\right)$$

$$Tp_{pul} \approx 3.35 \frac{ns}{meter} \sqrt{0.475\varepsilon r + 0.67}$$



TL/Terminations/Stripline Equations



Stripline Equations

•Useful approximations

•Use numeric TL tools for improved accuracy

$$Z0 \approx \frac{60\Omega}{\sqrt{\varepsilon r}} \ln\left(\frac{1.9b}{0.8w + d}\right)$$

$$Tp_{pul} \approx 3.35 \frac{ns}{meter} \sqrt{\varepsilon r}$$