Chapter 6.

Synchronous Machines

Topics to cover:

1) *Introduction*
2) *Synchronous machine structures*
3) *Rotating magnetic field*
4) *Equivalent circuit model*
5) *Performance as a generator*
6) *Performance as a motor*

Introduction

A synchronous machine is an ac rotating machine whose speed under steady state condition is proportional to the frequency of the current in its armature. The magnetic field created by the armature currents rotates at the same speed as that created by the field current on the rotor, which is rotating at the synchronous speed, and a steady torque results.

Synchronous machines are commonly used as generators especially for large power systems, such as turbine generators and hydroelectric generators in the grid power supply. Because the rotor speed is proportional to the frequency of excitation, synchronous motors can be used in situations where constant speed drive is required. Since the reactive power generated by a synchronous machine can be adjusted by controlling the magnitude of the rotor field current, unloaded synchronous machines are also often installed in power systems solely for power factor correction or for control of reactive kVA flow. Such machines, known as ***synchronous condensers***, may be more economical in the large sizes than static capacitors.

With power electronic variable voltage variable frequency (VVVF) power supplies, synchronous motors, especially those with permanent magnet rotors, are widely used for variable speed drives. If the stator excitation of a permanent magnet motor is controlled by its rotor position such that the stator field is always 90^o (***electrical***) ahead of the rotor, the motor performance can be very close to the conventional brushed dc motors, which is very much favored for variable speed drives. The rotor position can be either detected by using rotor position sensors or deduced from the induced *emf* in the stator windings. Since this type of motors do not need brushes, they are known as brushless dc motors.

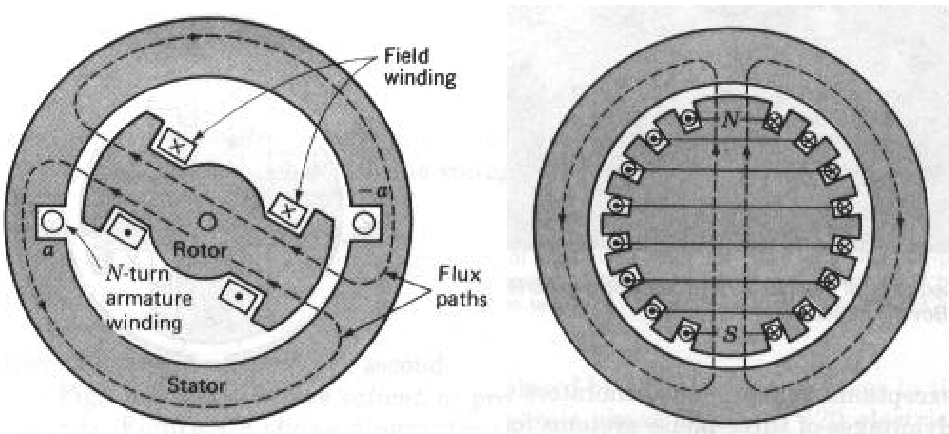
In this chapter, we concentrate on conventional synchronous machines whereas the brushless dc motors will be discussed later in a separate chapter.

Synchronous Machine Structures

Stator and Rotor

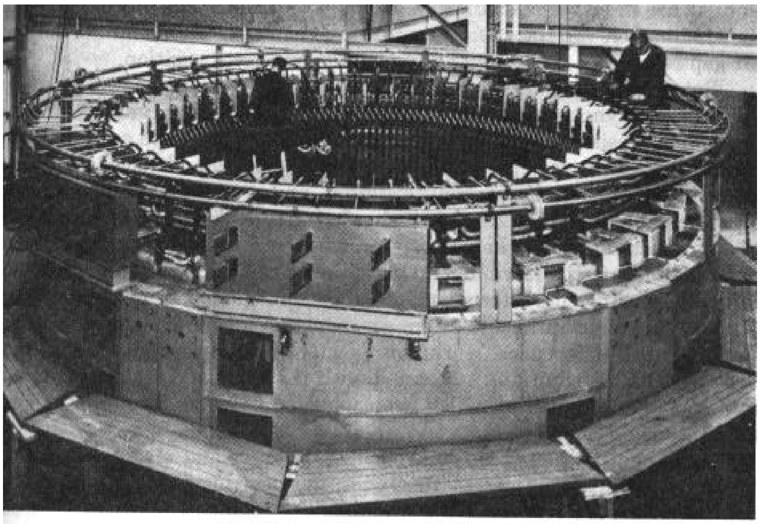
The armature winding of a conventional synchronous machine is almost invariably on the stator and is usually a three phase winding. The field winding is usually on the rotor and excited by dc current, or permanent magnets. The dc power supply required for excitation usually is supplied through a dc generator known as exciter, which is often mounted on the same shaft as the synchronous machine. Various excitation systems using ac exciter and solid state rectifiers are used with large turbine generators.

There are two types of rotor structures: ***round or cylindrical rotor and salient pole rotor*** as illustrated schematically in the diagram below. Generally, round rotor structure is used for high speed synchronous machines, such as steam turbine generators, while salient pole structure is used for low speed applications, such as hydroelectric generators. The pictures below show the stator and rotor of a hydroelectric generator and the rotor of a turbine generator.

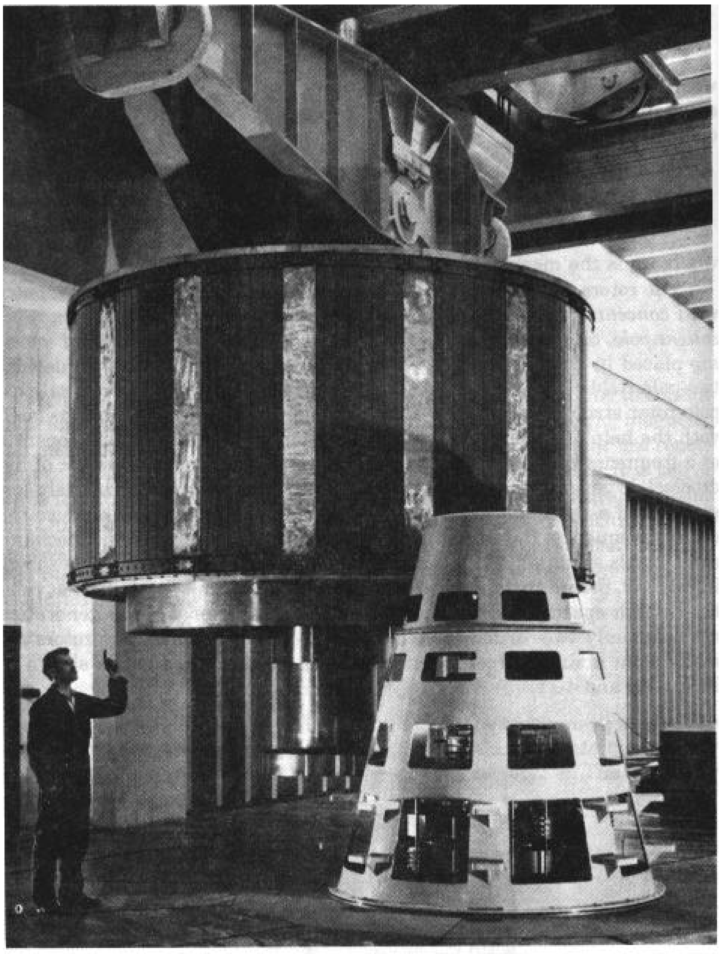


(a) (b)

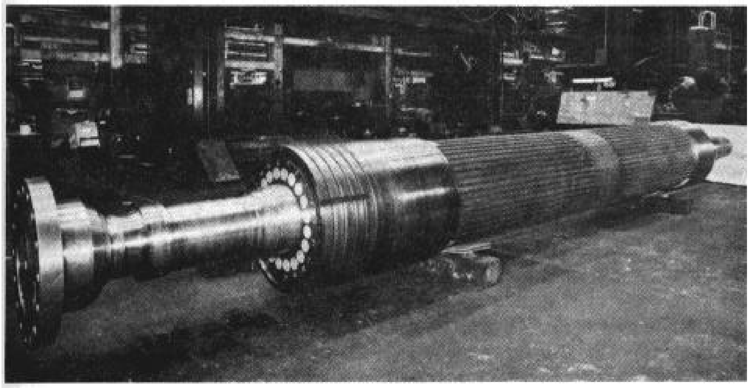
Schematic illustration of synchronous machines of
(a) round or cylindrical rotor and (b) salient rotor structures



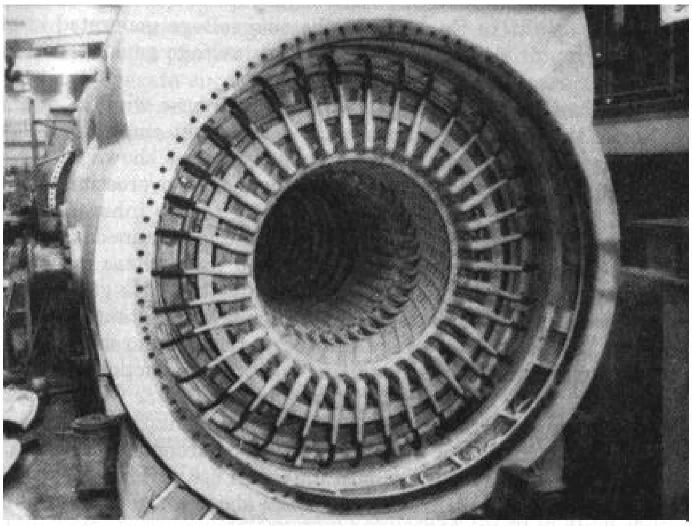
Stator of a 190-MVA three-phase 12-kV 375-r/min hydroelectric generator. The conductors have hollow passages through which cooling water is circulated. (*Brown Boveri Corporation.*)



Water-cooled rotor of the 190-MVA hydroelectric generator (*Brown Boveri Corporation.*)



Rotor of a two-pole 3600 r/min turbine generator. (*Westinghouse Electric Corporation.*)



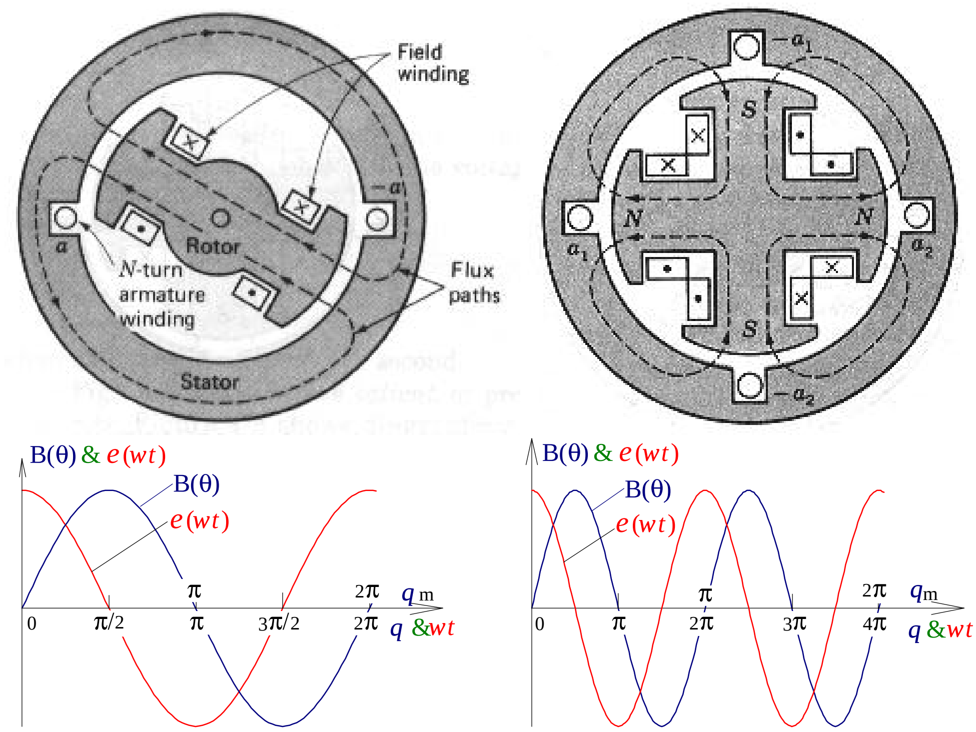
End view of the stator of a 26-kV 908-MVA 3600 r/min turbine generator with water-cooled windings. Hydraulic connections for coolant flow are provided for each winding end turn. (*General Electric Company.*)

Angle in Electrical and Mechanical Units

Consider a synchronous machine with two magnetic poles. The idealized radial distribution of the air gap flux density is sinusoidal along the air gap. When the rotor rotates for one revolution, the induced *emf*, which is also sinusoidal, varies for one cycle as illustrated by the waveforms in the diagram below. If we measure the rotor position by physical or mechanical degrees or radians and the phase angles of the flux density and emf by ***electrical degrees or radians***, in this case, it is ready to see that the angle measured in mechanical degrees or radians is equal to that measured in electrical degrees or radians, i.e.

$$q = q_m$$

where q is the angle in electrical degrees or radians and q_m the mechanical angle.



(a) (b)

Flux density distribution in air gap and induced *emf* in the phase winding of a (a) two pole and (b) four pole synchronous machine

A great many synchronous machines have more than two poles. As a specific example, we consider a four pole machine. As the rotor rotates for one revolution (q_m=2π), the induced *emf* varies for two cycles (q = 4π), and hence

$$q = 2q_m$$

For a general case, if a machine has *P* poles, the relationship between the electrical and mechanical units of an angle can be readily deduced as

$$q = \frac{P}{2} q_m$$

Taking derivatives on the both side of the above equation, we obtain

$$w = \frac{P}{2} w_m$$

where w is the angular frequency of *emf* in electrical radians per second and w_m the angular speed of the rotor in mechanical radians per second. When w and w_m are converted into cycles per second or Hz and revolutions per minute respectively, we have

$$f = \frac{P}{2}\frac{n}{60}$$

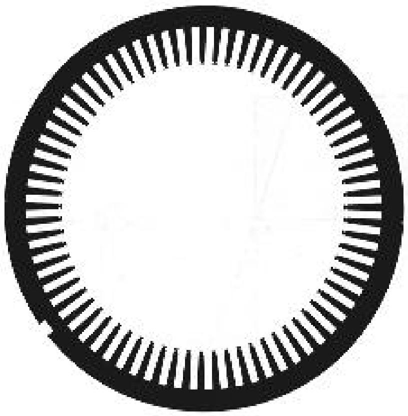
or

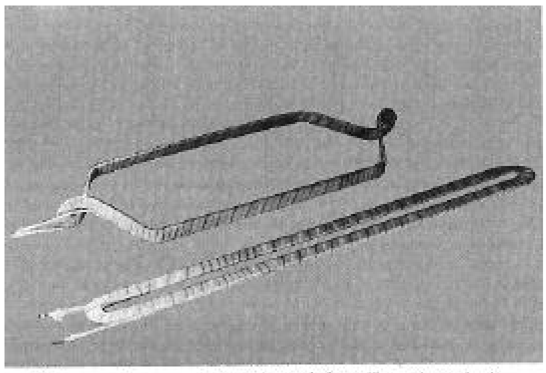
$$n = \frac{120 f}{P}$$

where $w=2pf$, $w_m=2pn/60$, and n is the rotor speed in rev/min. It can be seen that the frequency of the induced *emf* is proportional to the rotor speed.

Distributed Three Phase Windings

The stator of a synchronous machine consists of a laminated electrical steel core and a three phase winding. Fig.(a) below shows a stator lamination of a synchronous machine that has a number of uniformly distributed slots. Coils are to be laid in these slots and connected in such a way that the current in each phase winding would produce a magnetic field in the air gap around the stator periphery as closely as possible the ideal sinusoidal distribution. Fig.(b) is a picture of a coil.



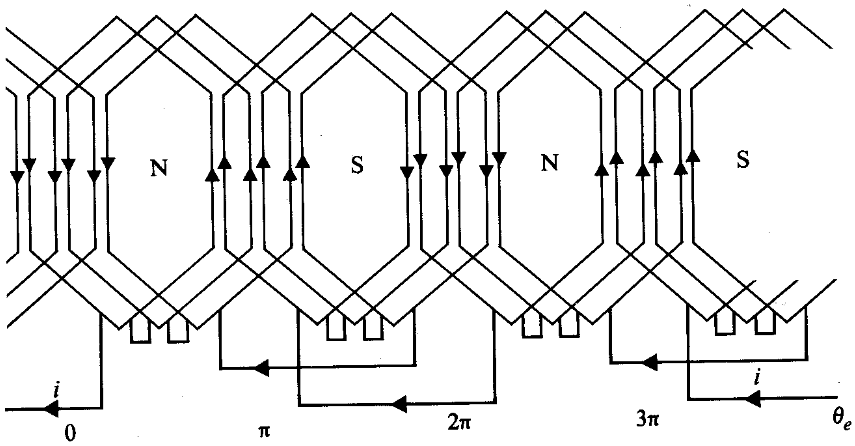


Multiturn coil for an ac winding before and after pulling. *(The Harland Engineering Company of Canada Ltd.)*

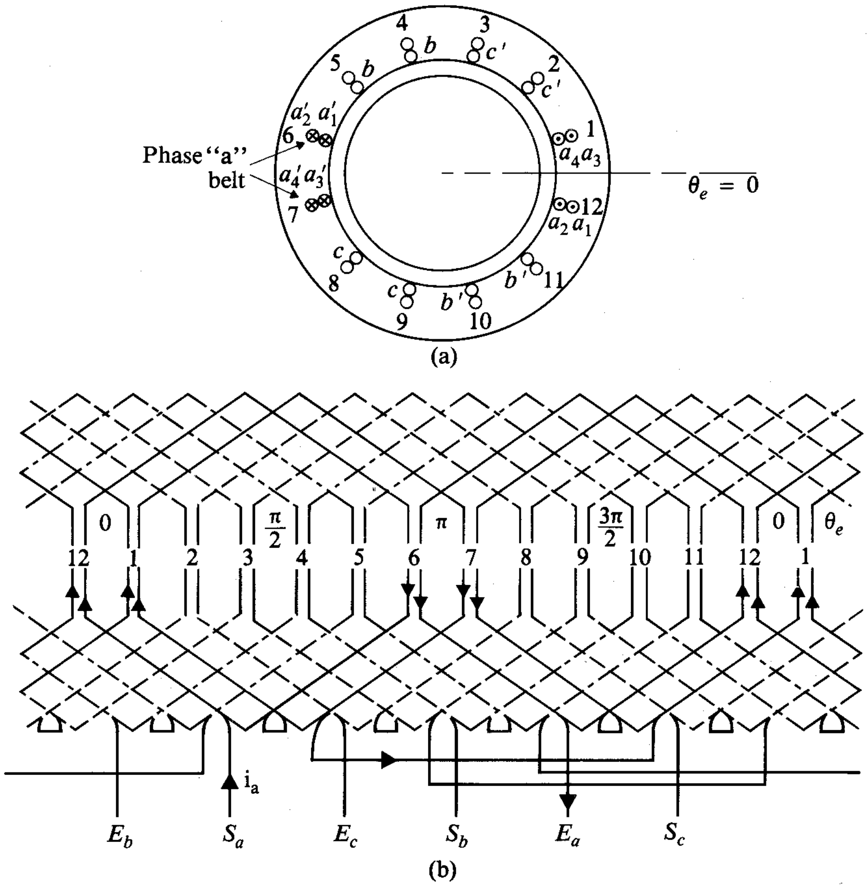
(a) (b)

Pictures of (a) stator lamination and (b) coil of a synchronous machine

As illustrated below, these coils are connected to form a three phase winding. Each phase is able to produce a specified number of magnetic poles (in the diagram below, four magnetic poles are generated by a phase winding). The windings of the three phase are arranged uniformly around the stator periphery and are labeled in the sequence that phase *a* is 120° (electrical) ahead of phase *b* and 240° (electrical) ahead of phase *c*. It is noted that in the diagrams above, two coil sides are laid in each slot. This type of winding is known as the double layer winding. In the case that there is only one coil side in each slot, the winding is known as the single layer winding.



Single-phase, whole-coil distributed winding.



Two-pole, three-phase, double-layer full-pitch winding (S = start of phase winding; E = end).

Rotating Magnetic Fields

Magnetic Field of a Distributed Phase Winding

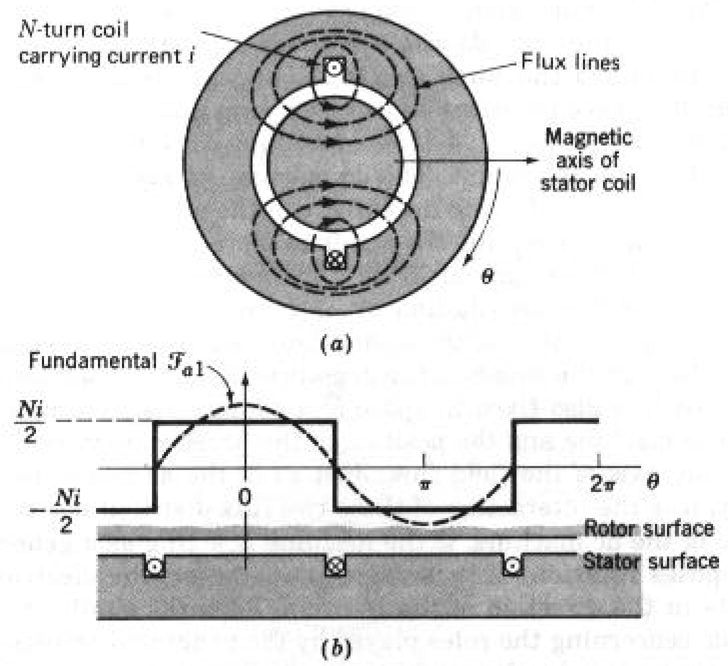
The magnetic field distribution of a distributed phase winding can be obtained by adding the fields generated by all the coils of the winding. The diagram below plots the profiles of *mmf* and field strength of a single coil in a uniform air gap. If the permeability of the iron is assumed to be infinite, by Ampere's law, the *mmf* across each air gap would be $Ni_a/2$, where N is the number of turns of the coil and i_a the current in the coil. The *mmf* distribution along the air gap is a square wave. Because of the uniform air gap, the spatial distribution of magnetic field strength is the same as that of *mmf*.

It can be shown analytically that the fundamental component is the major component when the square wave *mmf* is expanded into a Fourier Series, and it can written as

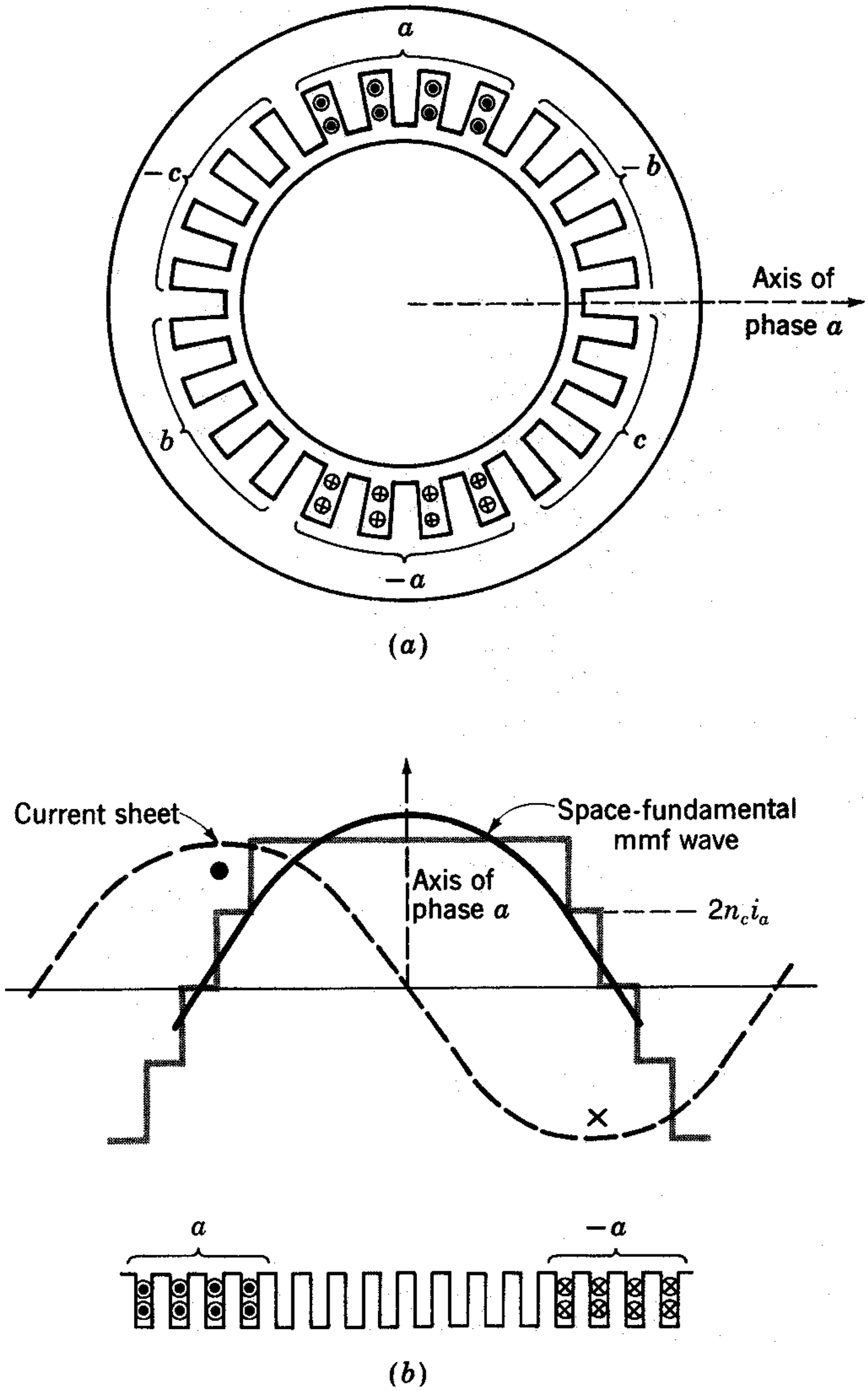
$$F_{a1} = \frac{4}{p} \frac{Ni_a}{2} \cos q$$

where q is the angular displacement from the magnetic axis of the coil.

When the field distributions of a number of distributed coils are combined, the resultant field distribution is close to a sine wave, as shown in the diagram in the next page. The fundamental component of the resultant *mmf* can be obtained by adding the fundamental components of these individual coils, and it can expressed as



The mmf of a concentrated full-pitch coil.



The mmf of one phase of a distributed two-pole three-phase winding with full-pitch coils.

$$F_{a1} = \frac{4}{p} \frac{k_p N_{ph}}{P} i_a \cos q$$

where N_{ph} is the total number of turns of the phase winding, which is formed by these coils, k_p is known as the distribution factor of the winding, which is defined by

$$k_p = \frac{Fundamental\ mmf\ of\ a\ distributed\ winding}{Fundamental\ mmf\ of\ a\ concentrated\ winding}$$

and P is the number of poles.

In some windings, short pitched coils (the distance between two sides of coil is smaller than that between two adjacent magnetic poles) are used to eliminate a certain harmonic, and the fundamental component of the resultant *mmf* is then expressed as

$$F_{a1} = \frac{4}{p} \frac{k_w N_{ph}}{P} i_a \cos q$$

where $k_w = k_d k_p$ is the winding factor, k_d is known as the pitching factor, which is defined by

$$k_d = \frac{Fundamental\ mmf\ of\ a\ short\ pitch\ winding}{Fundamental\ mmf\ of\ a\ full\ pitch\ winding}$$

and $k_w N_{ph}$ is known as the effective number of turns of the phase winding.

Let $i_a = I_m \cos wt$, and we have

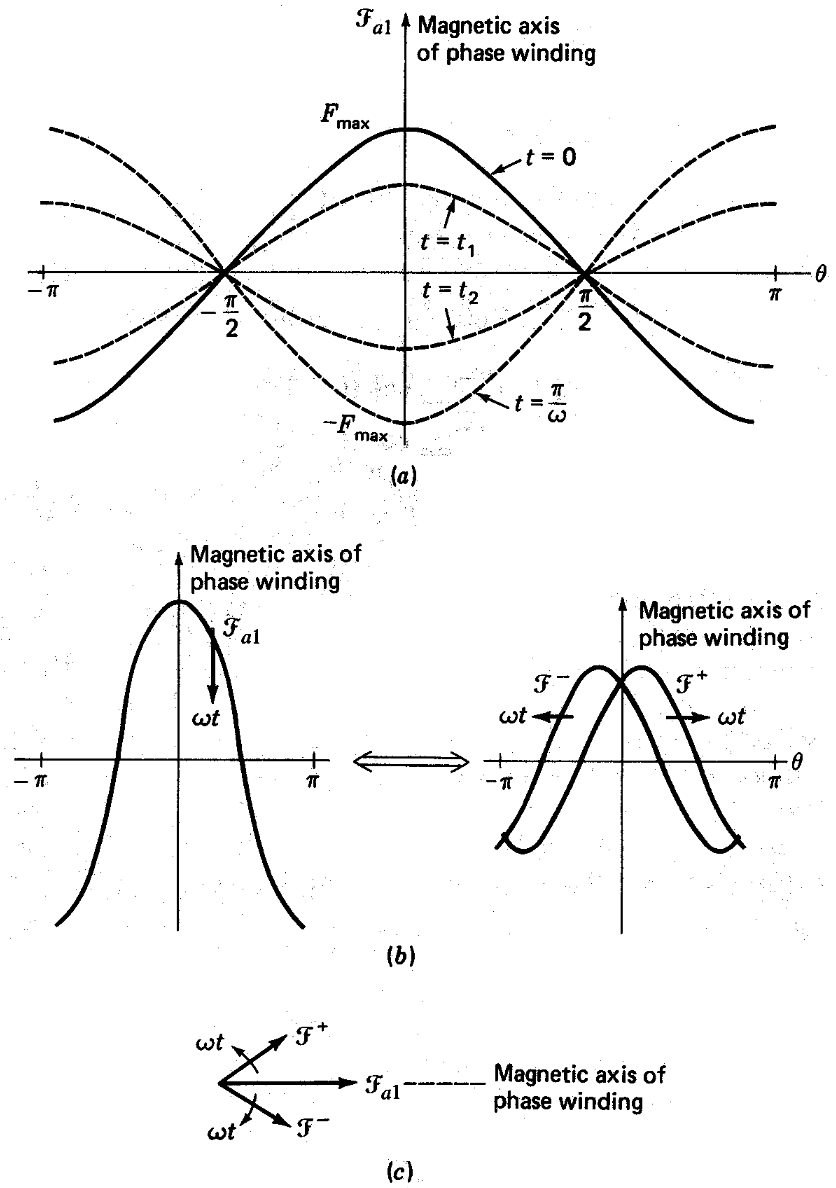
$$\begin{aligned} F_{a1} &= \frac{4}{p} \frac{k_w N_{ph}}{P} I_m \cos wt \cos q \\ &= F_m \cos wt \cos q \end{aligned}$$

where $$F_m = \frac{4}{p} \frac{k_w N_{ph}}{P} I_m$$

The *mmf* of a distributed phase winding is a function of both space and time. When plotted at different time instants as shown below, we can see that it is a pulsating sine wave. We call this type of *mmf* as a ***pulsating mmf***.

Because $\cos a \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$, the above expression of the *mmf* fundamental component can be further written as

$$\begin{aligned} F_{a1} &= \frac{F_m}{2} \cos(q - wt) + \frac{F_m}{2} \cos(q + wt) \\ &= F_+ + F_- \end{aligned}$$



Single-phase-winding fundamental mmf: (*a*) mmf distribution of a single-phase winding at various times; (*b*) total mmf $\mathcal{F}_{a1}$ decomposed into two traveling waves $\mathcal{F}^+$ and $\mathcal{F}^-$; (*c*) phasor decomposition of $\mathcal{F}_{a1}$.

It can be shown that the first term in the above equation stands for a rotating *mmf* in the +*q* direction and the second a rotating *mmf* in the –*q* direction. That is a pulsating *mmf* can be resolved into two rotating *mmf*'s with the same magnitudes and opposite rotating directions, as shown above. For a machine with uniform air gap, the above analysis is also applicable to the magnetic field strength and flux density in the air gap.

Magnetic Field of Three Phase Windings

Once we get the expression of *mmf* for a single phase winding, it is not difficult to write the expressions of *mmf's* for three single phase windings placed 120^o (electrical) apart and excited by balanced three phase currents:

$$F_{a1} = F_m \cos wt \cos q = \frac{F_m}{2}\cos(q - wt) + \frac{F_m}{2}\cos(q + wt)$$

$$F_{b1} = F_m \cos\left(wt - 120^o\right)\cos\left(q - 120^o\right)$$

$$= \frac{F_m}{2}\cos(q - wt) + \frac{F_m}{2}\cos\left(q + wt - 240^o\right)$$

$$F_{c1} = F_m \cos\left(wt - 240^o\right)\cos\left(q - 240^o\right)$$

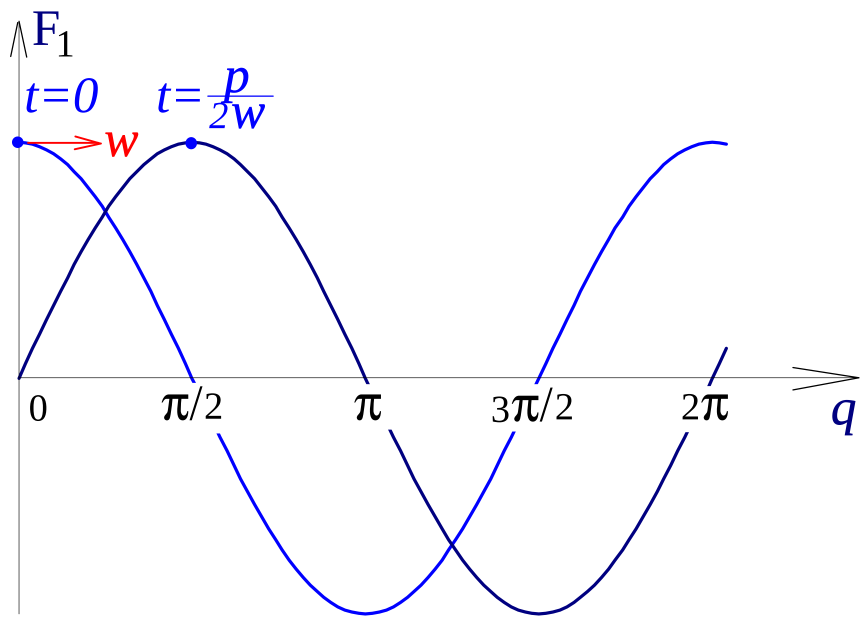
and

$$= \frac{F_m}{2}\cos(q - wt) + \frac{F_m}{2}\cos\left(q + wt - 480^o\right)$$

Therefore, the resultant *mmf* generated by a three phase winding is

$$F_1 = F_{a1} + F_{b1} + F_{c1} = \frac{3F_m}{2}\cos(q - wt)$$

Note that $\cos(q + wt) + \cos\left(q + wt - 240^o\right) + \cos\left(q + wt - 480^o\right) = 0$



Rotating *mmf* in $+q$ direction

The above diagram plots the resultant *mmf* F_1 at two specific time instants: $t=0$ and $t=p/2w$. It can be readily observed that ***F_1 is a rotating mmf in the +q direction (a®b®c) with a constant magnitude $3F_m/2$***. The speed of this rotating *mmf* can be calculated as

$$w_f = \frac{dq}{dt} = \frac{p/2}{p/2w} = w \qquad \text{rad/s (electrical)}$$

When expressed in mechanical radians per second and revolutions per minute, the speed of the rotating mmf can be expressed as

$$w_f = \frac{w}{P/2} \qquad \text{rad/s (mechanical)}$$

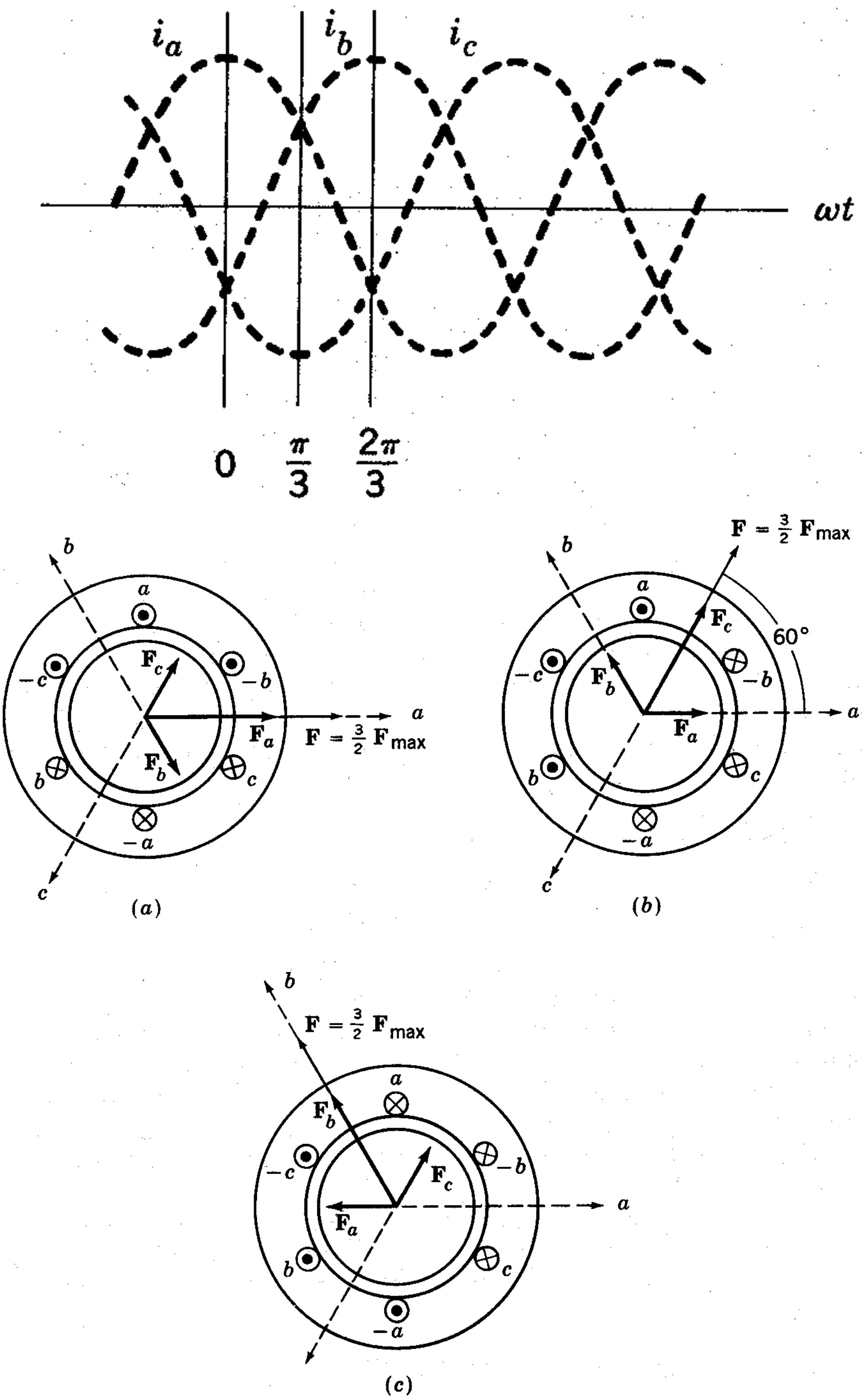
and

$$n_f = \frac{60 w_f}{2p} = \frac{120 f}{P} \qquad \text{rev/min}$$

respectively. Again, for a machine with uniform air gap, the above analysis for *mmf* is also valid for the magnetic field strength and the flux density in the air gap. Therefore, ***the speed of a rotating magnetic field is proportional to the frequency of the three phase excitation currents, which generate the field.***

Comparing with the relationship between the rotor speed and the frequency of the induced *emf* in a three phase winding derived earlier, we can find that the rotor speed equals the rotating field speed for a given frequency. In other words, the rotor and the rotating field are rotating at a same speed. We call this speed ***synchronous speed***, and use specific symbols **w_{syn} *(mechanical rad/s) and* n_{syn} *(rev/min)*** to indicate it.

The above analytical derivation can also be done graphically by using adding the *mmf* vectors of three phases, as illustrated in the diagrams below. When $wt=0$, phase *a* current is maximum and the *mmf* vector with a magnitude F_m of phase *a* is on the magnetic axis of phase a, while the *mmf's* of phases *b* and *c* are both of magnitude $F_m/2$ and in the opposite directions of their magnetic axes since the currents of these two phases are both $-I_m/2$. Therefore, the resultant *mmf* $F_1=3F_m/2$ is on the magnetic axis of phase *a*. When $wt=p/3$, $i_c=-I_m$ and $i_a=i_b=I_m/2$. The resultant *mmf* $F_1=3F_m/2$ is on the axis of phase *c* but in the opposite direction. Similarly, when $wt=2p/3$, $i_b=I_m$ and $i_a=i_c=-I_m/2$. Hence the resultant *mmf* $F_1=3F_m/2$ is in the positive direction of the magnetic axis of phase *b*. In general, the resultant *mmf* is of a constant magnitude $3F_m/2$ and will be in the positive direction of the magnetic axis of a phase winding when the current in that phase winding reaches positive maximum. The speed of the rotating *mmf* equals the angular frequency in electrical rad/s.



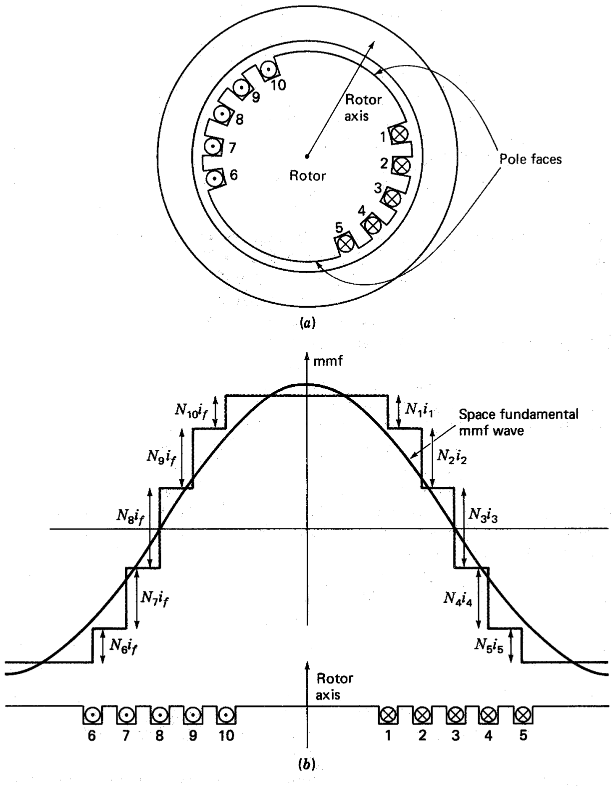
The production of a rotating magnetic field by means of three-phase currents.

In the case of a synchronous generator, three balanced *emf's* of frequency $f=Pn/120$ Hz are induced in the three phase windings when the rotor is driven by a prime mover rotating at a speed n. If the three phase stator circuit is closed by a balanced three phase electrical load, balanced three phase currents of frequency f will flow in the stator circuit, and these currents will generate a rotating magnetic field of a speed $n_f = 120f/P = n$.

When the stator winding of a three phase synchronous motor is supplied by a balanced three phase power supply of frequency f, the balanced three phase currents in the winding will generate a rotating magnetic field of speed $n_f = 120f/P$. This rotating magnetic field will drag the magnetized rotor, which is essential a magnet, to rotate at the same speed $n=n_f$. On the other hand, this rotating rotor will also generate balanced three phase *emf's* of frequency f in the stator winding, which would balance with the applied terminal voltage.

Rotor Magnetic Field

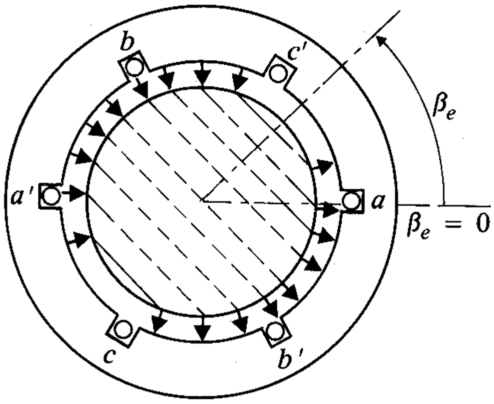
Using the method of superposition on the *mmf*'s of the coils which form the rotor winding, we can derive that the distributions of the *mmf* and hence the flux density in the air gap are close to sine waves for a round rotor synchronous machine with uniform air gap, as illustrated below.



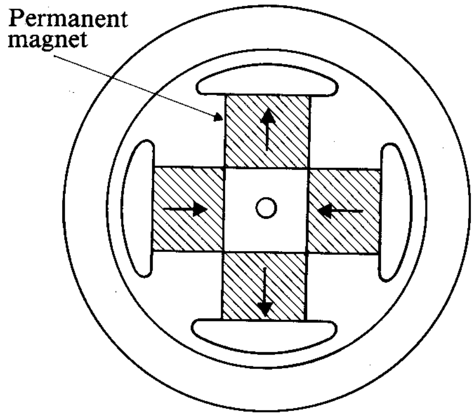
The mmf of a distributed winding on the rotor of a round-rotor generator.

In the case of a salient pole rotor, the rotor poles are shaped so that the resultant *mmf* and flux density would distribute sinusoidally in the air gap, and thus the induced *emf* in the stator windings linking this flux will also be sinusoidal.

The field excitation of a synchronous machine may be provided by means of permanent magnets, which eliminate the need for a DC source for excitation. This can not only save energy for magnetic excitation but also dramatically simplify the machine structures, which is especially favorable for small synchronous machines, since this offers more flexibility on machine topologies. The diagram below illustrates the cross sections of two permanent magnet synchronous machines.



Cylindrical-rotor, permanent magnet machine



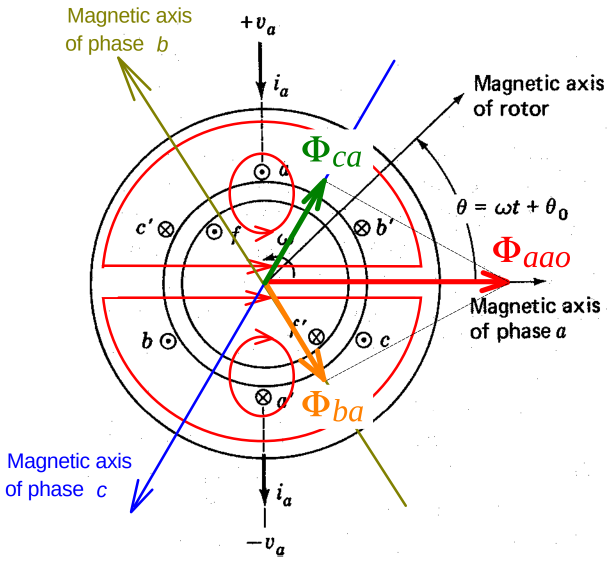
Salient-pole, permanent magnet machine

Per Phase Equivalent Electrical Circuit Model

The diagram below illustrates schematically the cross section of a three phase, two pole cylindrical rotor synchronous machine. Coils *aa*', *bb*', and *cc*' represent the distributed stator windings producing sinusoidal *mmf* and flux density waves rotating in the air gap. The reference directions for the currents are shown by dots and crosses. The field winding *ff*' on the rotor also represents a distributed winding which produces sinusoidal *mmf* and flux density waves centered on its magnetic axis and rotating with the rotor.

The electrical circuit equations for the three stator phase windings can be written by the Kirchhoff's voltage law as

$$v_a = R_a i_a + \frac{d l_a}{dt}$$



Schematic diagram of a three phase
cylindrical rotor synchronous machine

$$v_b = R_b i_b + \frac{d l_b}{dt}$$

$$v_c = R_c i_c + \frac{d l_c}{dt}$$

where v_a, v_b, and v_c are the voltages across the windings, R_a, R_b, and R_c are the winding resistances, and l_a, l_b, and l_c are the total flux linkages of the windings of phases a, b, and c, respectively. For a symmetric three phase stator winding, we have

$$R_a = R_b = R_c$$

The flux linkages of phase windings a, b, and c can be expressed in terms of the self and mutual inductances as the following

$$\begin{aligned} l_a &= l_{aa} + l_{ab} + l_{ac} + l_{af} = L_{aa} i_a + L_{ab} i_b + L_{ac} i_c + L_{af} i_f \\ l_b &= l_{ba} + l_{bb} + l_{bc} + l_{bf} = L_{ba} i_a + L_{bb} i_b + L_{bc} i_c + L_{bf} i_f \\ l_c &= l_{ca} + l_{cb} + l_{cc} + l_{cf} = L_{ca} i_a + L_{cb} i_b + L_{cc} i_c + L_{cf} i_f \end{aligned}$$

where

$$\begin{aligned} L_{aa} &= L_{bb} = L_{cc} = L_{aao} + L_{al} \\ L_{ab} &= L_{ba} = L_{ac} = L_{ca} = -L_{aao}/2 \\ L_{af} &= L_{afm} \cos q \\ L_{bf} &= L_{afm} \cos\left(q - 120^o\right) \end{aligned}$$

$$L_{cf} = L_{afm} \cos\left(q - 240^o\right)$$

for a balanced three phase machine, $L_{aao}=F_{aao}/i_a$, $L_{al}=F_{al}/i_a$, F_{aao} is the flux that links all three phase windings, F_{al} the flux that links only phase *a* winding and $q=wt+q_o$.

When the stator windings are excited by balanced three phase currents, we have

$$i_a + i_b + i_c = 0$$

The total flux linkage of phase *a* winding can be further written as

$$\begin{aligned} l_a &= \left(L_{aao} + L_{al}\right)i_a - L_{aao}i_b / 2 - L_{aao}i_c / 2 + L_{afm}i_f \cos\left(wt + q_o\right) \\ &= \left(L_{aao} + L_{al}\right)i_a - L_{aao}\left(i_b + i_c\right)/2 + L_{afm}i_f \cos\left(wt + q_o\right) \\ &= \left(L_{aao} + L_{al}\right)i_a + L_{aao}i_a / 2 + L_{afm}i_f \cos\left(wt + q_o\right) \\ &= \left(3L_{aao}/2 + L_{al}\right)i_a + L_{afm}i_f \cos\left(wt + q_o\right) \\ &= L_s i_a + L_{afm}i_f \cos\left(wt + q_o\right) \end{aligned}$$

Similarly, we can write

$$l_b = L_s i_b + L_{afm}i_f \cos\left(wt + q_o - 120^o\right)$$

and $$l_c = L_s i_c + L_{afm}i_f \cos\left(wt + q_o - 240^o\right)$$

where $L_s=3L_{aao}/2+L_{al}$ is known as the ***synchronous inductance***.

In this way, the three phase windings are mathematically de-coupled, and hence for a balanced three phase synchronous machine, we just need to solve the circuit equation of one phase. Substituting the above expression of flux linkage into the circuit equation of phase *a*, we obtain

$$v_a = R_a i_a + L_s \frac{di_a}{dt} + \frac{d l_{af}}{dt}$$

In steady state, the above equation can be expressed in terms of voltage and current phasors as

$$\mathbf{V}_a = \mathbf{E}_a + \left(R_a + jwL_s\right)\mathbf{I}_a = \mathbf{E}_a + \left(R_a + jX_s\right)\mathbf{I}_a$$

where $X_s = wL_s$ is known as the ***synchronous reactance***, and

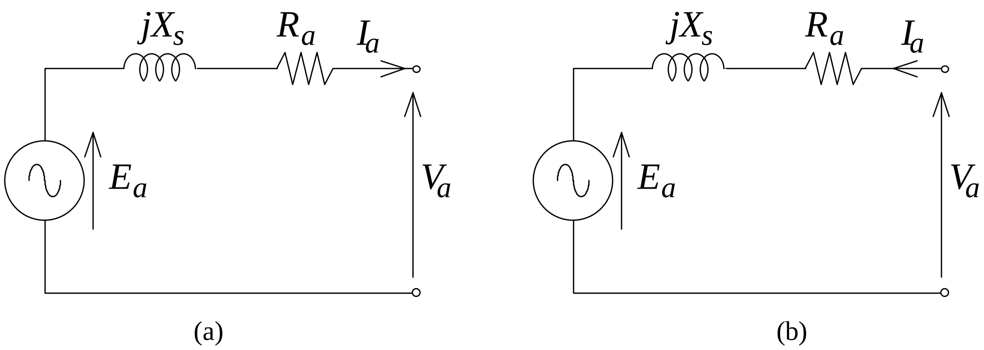
$$\mathbf{E}_a = j\frac{wL_{afm}\mathbf{I}_f}{\sqrt{2}} = j\frac{2p}{\sqrt{2}} fk_w N_{ph}\mathbf{\Phi}_f = j4.44\, fk_w N_{ph}\mathbf{\Phi}_f$$

is the induced *emf* phasor, noting that $L_{afm}\mathbf{I}_f = l_{afm} = k_w N_{ph}\mathbf{\Phi}_f$, I_f is the DC current in the rotor winding, and $\mathbf{\Phi}_f$ the rotor magnetic flux in the air gap.

It should be noticed that the above circuit equation was derived under the assumption that the phase current flows into the positive terminal, i.e. the reference direction of the phase current was chosen assuming the machine is a motor. In the case of a generator, where the phase current is assumed to flow out of the positive terminal, the circuit equation becomes

$$\mathbf{V}_a = \mathbf{E}_a - \left(R_a + jX_s\right)\mathbf{I}_a$$

The following circuit diagrams illustrate the per phase equivalent circuits of a round rotor synchronous machine in the motor and generator mode respectively.



Synchronous machine per phase equivalent circuits
in (a) generator, and (b) motor reference directions

Experimental Determination of Circuit Parameters

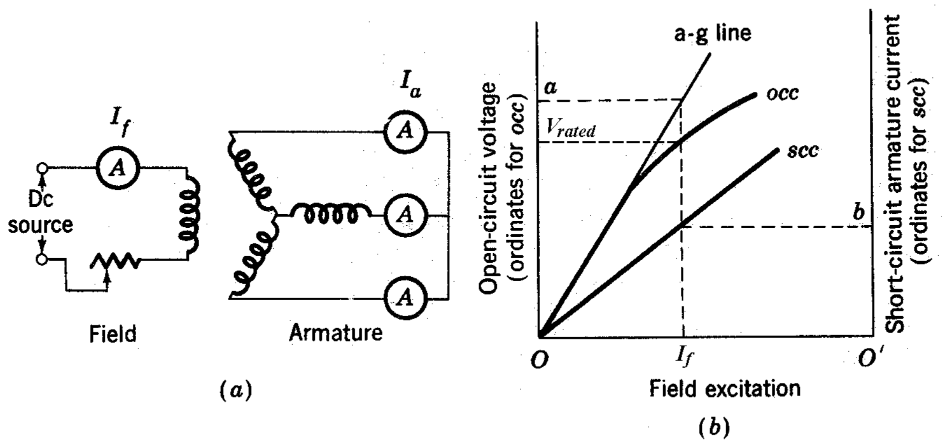
In the per phase equivalent circuit model illustrated above, there are three parameters need to be determined: winding resistance R_a, synchronous reactance X_s, and induced *emf* in the phase winding E_a. The phase winding resistance R_a can be determined by measuring DC resistance of the winding using volt-ampere method, while the synchronous reactance and the induced *emf* can be determined by the open circuit and short circuit tests.

Open Circuit Test

Drive the synchronous machine at the synchronous speed using a prime mover when the stator windings are open circuited. Vary the rotor winding current, and measure stator winding terminal voltage. The relationship between the stator winding terminal voltage and the rotor field current obtained by the open circuit test is known as the ***open circuit characteristic*** of the synchronous machine.

Short Circuit Test

Reduce the field current to a minimum, using the field rheostat, and then open the field supply circuit breaker. Short the stator terminals of the machine together through three ammeters; Close the field circuit breaker; and raise the field current to the value noted in the open circuit test at which the open circuit terminal voltage equals the rated voltage, while maintain the synchronous speed. Record the three stator currents. (This test should be carried out quickly since the stator currents may be greater than the rated value).



(a) Connections for short-circuit test; (b) open- and short-circuit characteristics.

Under the assumptions that the synchronous reactance X_s and the induced *emf* E_a have the same values in both the open and short circuit tests, and that $X_s >> R_a$, we have

$$X_s = \frac{Open\ circuit\ per\ phase\ voltage}{Short\ circuit\ per\ phase\ current}$$

For some machines, the short circuit current is too high if the machine is driven at the synchronous speed. In this case, short circuit test can be performed at a reduced speed say half synchronous speed $n_{syn}/2$ or $f_{rated}/2$. Since E_a µ f, the induced *emf* in the short circuit test is halved. Thus

$$X_s\Big|_{f_{rated}/2} = \frac{\frac{1}{2}V_{oc}\Big|_{f_{rated}}}{I_{sc}\Big|_{f_{rated}/2}}$$

Therefore,

$$X_s\Big|_{f_{rated}} = 2\times X_s\Big|_{f_{rated}/2} = \frac{V_{oc}\Big|_{f_{rated}}}{I_{sc}\Big|_{f_{rated}/2}}$$

Synchronous Machine Operated as a Generator

Electromagnetic Power and Torque

When a synchronous machine is operated as a generator, a prime mover is required to drive the generator. In steady state, the mechanical torque of the prime mover should balance with the electromagnetic torque produced by the generator and the mechanical loss torque due to friction and windage, or

$$T_{pm} = T + T_{loss}$$

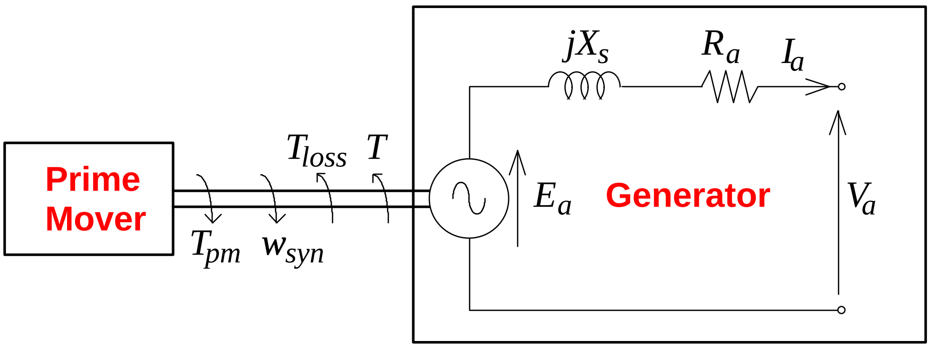
Multiplying the synchronous speed to both sides of the torque equation, we have the power balance equation as

$$P_{pm} = P_{em} + P_{loss}$$

where P_{pm}=$T_{pm}w_{syn}$ is the mechanical power supplied by the prime mover, P_{em}=Tw_{syn} the electromagnetic power of the generator, and P_{loss}=$T_{loss}w_{syn}$ the mechanical power loss of the system. The electromagnetic power is the power being converted into the electrical power in the three phase stator windings. That is

$$P_{em} = Tw_{syn} = 3E_a I_a \cos j_{E_a I_a}$$

where $\varphi_{E_aI_a}$ is the angle between phasors $\mathbf{E}_a$ and $\mathbf{I}_a$.

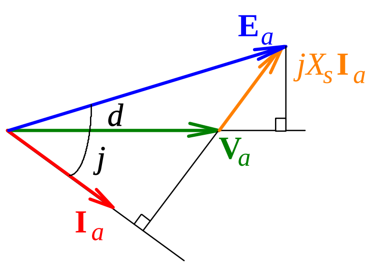


A synchronous machine operated as generator

For larger synchronous generators, the winding resistance is generally much smaller than the synchronous reactance, and thus the per phase circuit equation can be approximately written as

$$\mathbf{V}_a = \mathbf{E}_a - jX_s\mathbf{I}_a$$

The corresponding phasor diagram is shown on the



Generator phasor diagram

right hand side. From the phasor diagram, we can readily obtain

$$E_a \sin d = X_s I_a \cos j$$

When the phase winding resistance is ignored, the output electrical power equals the electromagnetic power, or

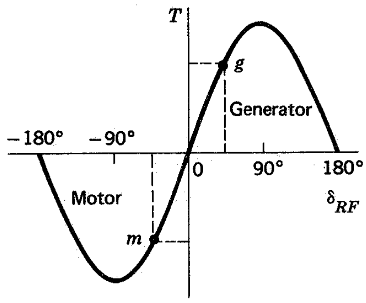
$$P_{em} = P_{out} = 3V_a I_a \cos j$$

Therefore,

$$P_{em} = \frac{3E_a V_a}{X_s} \sin d$$

and

$$T = \frac{P_{em}}{w_{syn}} = \frac{3E_a V_a}{w_{syn} X_s} \sin d$$



Electromagnetic torque *vs.* load angle

where *d* is the angle between the phasors of the voltage and the *emf*, known as the ***load angle***. When the stator winding resistance is ignored, *d* can also be regarded as the angle between the rotor and stator rotating magnetic fields. The electromagnetic torque of a synchronous machine is proportional to the sine function of the load angle, as plotted in the diagram above, where the curve in the third quadrant is for the situation when the machine is operated as a motor, where the electromagnetic torque is negative because the armature current direction is reversed.

Voltage Regulation

The terminal voltage at constant field current varies with the armature current, or load current; that is, the generator has regulation that becomes more marked as the load circuit becomes more inductive and the operating power factor falls. This regulation is defined as

$$VR = \frac{V_{a(NL)} - V_{a(rated)}}{V_{a(rated)}}$$

where $V_{a(NL)}$ is the ***magnitude*** of the no load terminal voltage, and $V_{a(rated)}$ the magnitude of the rated terminal voltage. When a generator is supplying a full load, the required terminal must be the rated voltage. The normalized difference between the magnitudes of the no load voltage and the full load voltage by the rated voltage is defined as the ***voltage regulation***.

This value may be readily determined from the phasor diagram for full load operation. If the regulation is excessive, automatic control of field current may be employed to maintain a nearly constant terminal voltage as load varies.

Synchronous Machine Operated as a Motor

Electromagnetic Power and Torque

When a synchronous machine is operated as a motor to drive a mechanical load, in steady state, the mechanical torque of the motor should balance the load torque and the mechanical loss torque due to friction and windage, that is

$$T = T_{load} + T_{loss}$$

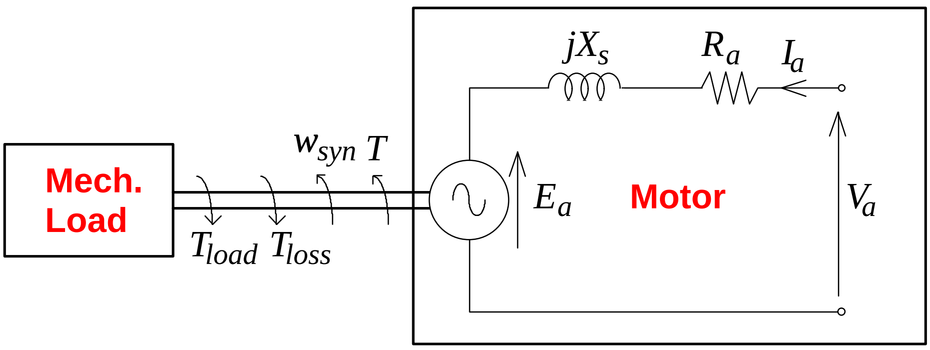
Multiplying the synchronous speed to both sides of the torque equation, we have the power balance equation as

$$P_{em} = P_{load} + P_{loss}$$

where $P_{em}=T\omega_{syn}$ the electromagnetic power of the motor, $P_{load}=T_{load}\omega_{syn}$ is the mechanical power delivered to the mechanical load, and $P_{loss}=T_{loss}\omega_{syn}$ the mechanical power loss of the system. Similar to the case of a generator, the electromagnetic power is the amount of power being converted from the electrical into the mechanical power. That is

$$P_{em} = 3E_a I_a \cos\varphi_{E_a I_a} = T\omega_{syn}$$

where φ_{EaIa} is the angle between phasors $\mathbf{E}_a$ and $\mathbf{I}_a$.

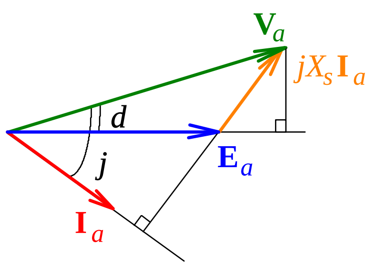


A synchronous machine operated as motor

When the stator winding resistance is ignored, the per phase circuit equation can be approximately written as

$$\mathbf{V}_a = \mathbf{E}_a + jX_s\mathbf{I}_a$$

The corresponding phasor diagram is shown on the right hand side. From the phasor diagram, we can readily obtain



Motor phasor diagram

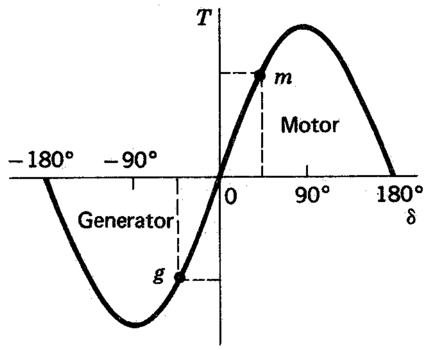
$$V_a \sin\delta = X_s I_a \cos\varphi_{E_a I_a} \qquad \text{where } \varphi_{E_a I_a} = \varphi - \delta$$

Therefore,

$$P_{em} = \frac{3E_a V_a}{X_s}\sin\delta$$

and

$$T = \frac{P_{em}}{\omega_{syn}} = \frac{3E_a V_a}{\omega_{syn} X_s}\sin\delta$$



Electromagnetic torque
vs. load angle

where δ is the ***load angle***. When the stator winding resistance is ignored, δ can also be regarded as the angle between the rotor and stator rotating magnetic fields. In motor mode, the stator field is ahead of the rotor. The electromagnetic torque of a synchronous machine is proportional to the sine function of the load angle, as plotted in the diagram above, where the curve in the third quadrant is for the situation when the machine is operated as a generator, where the electromagnetic torque is negative because the armature current direction is reversed.

Synchronous Motor Power Factor

Assume that a synchronous motor is driving a constant torque load. The active power converted by the machine is constant, no matter what the value of the field current is, since the motor speed is a constant. Thus,

$$T = \frac{3V_a E_a}{\omega_{syn} X_s}\sin\delta = constant$$

or

$$E_a \sin\delta = constant$$

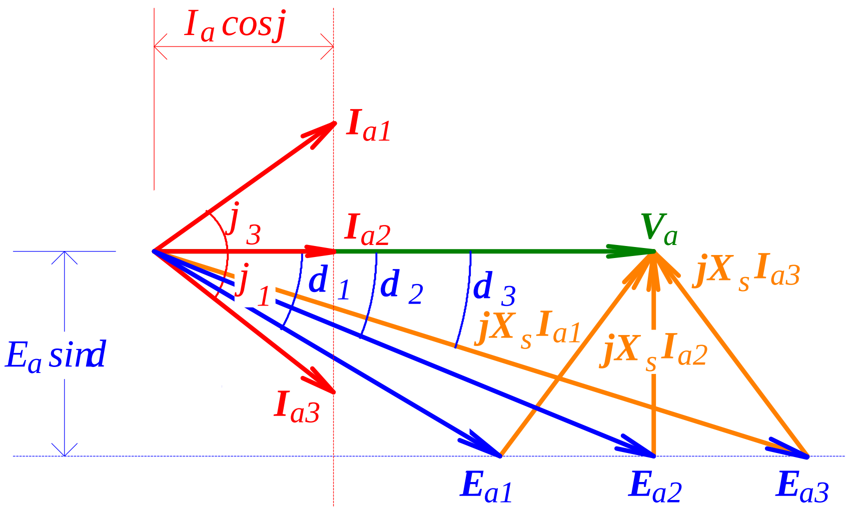
and

$$P_{em} = 3V_a I_a \cos\varphi = constant$$

or

$$I_a \cos\varphi = constant$$

Using the phasor diagram below, we analyze the variation of the power factor angle of a synchronous motor when the rotor field excitation is varied. For a small rotor field current the induced *emf* in the stator winding is also small, as shown by the phasor $\mathbf{E}_{a1}$. This yields a lagging power factor angle $\varphi_1 > 0$. As the excitation current increases, the lagging power factor angle is reduced. At a certain rotor current, the induced *emf* phasor $\mathbf{E}_{a2}$ is perpendicular to the terminal voltage phasor, and hence the stator current phasor is aligned with the terminal voltage, that is a zero power factor angle $\varphi_2 = 0$. When the rotor current further increases, the stator current leads the terminal voltage, or a leading power factor angle $\varphi_3 < 0$. In the phasor diagram, the above two conditions on E_a and I_a mean that they will only be able to vary along the horizontal and the vertical dotted lines, respectively, as shown below.

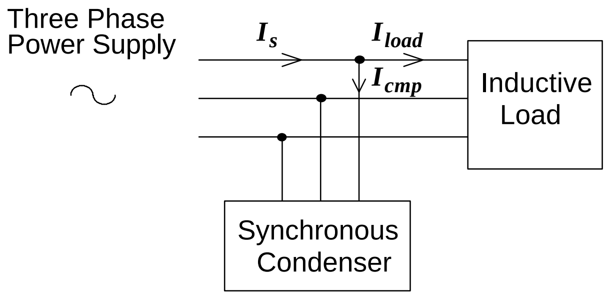


Phasor diagram of a synchronous motor in under excitation,
unit power factor, and over excitation mode

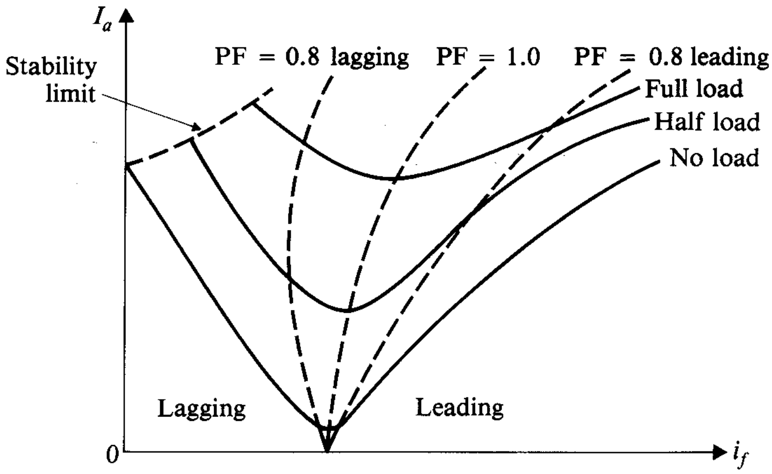
For conversion of a certain amount of active electrical power into mechanical power, a certain amount of magnetic flux is required. In the case of a lagging power factor, the rotor field current is so small that some reactive power is required from the stator power supply, and hence the stator current lags the terminal voltage. This state is known as ***under excitation***. When the rotor field current is just enough to produce the required magnetic flux, a unit power factor is obtained. If the rotor field current is more than required the spurious reactive power is to be exported to the power lines of the power supply. This state is known as ***over excitation***.

In practice, because of this feature, synchronous motors are often run at no active load as ***synchronous condensers*** for the purpose of power factor correction. The diagram underneath the phasor diagram illustrates schematically the power factor compensation for an inductive load, which is common for factories using large induction motor drives, using a synchronous condenser. By controlling the rotor excitation current such that the synchronous condenser draws a line current of leading phase angle, whose imaginary component cancels that of the load current, the total line current would have a minimum imaginary component. Therefore, the overall power factor of the inductive load and the synchronous condenser would be close to one and the magnitude of the overall line current would be the minimum.

It can also be seen that only when the power factor is unit or the stator current is aligned with the terminal voltage, the magnitude of the stator current is minimum. By plotting the magnitude of the stator current against the rotor excitation current, a family of "V" curves can be obtained. It is shown that a larger rotor field current is required for a larger active load to operate at unit power factor.



Power factor compensation for an inductive load using a synchronous condenser

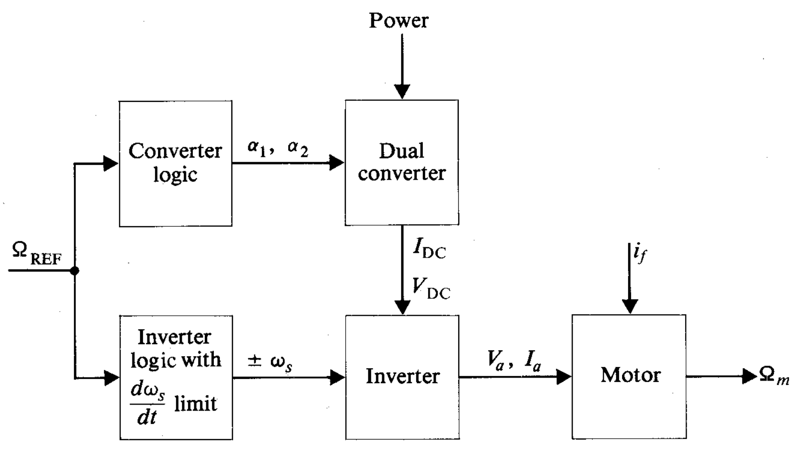


Synchronous motor "V" curves

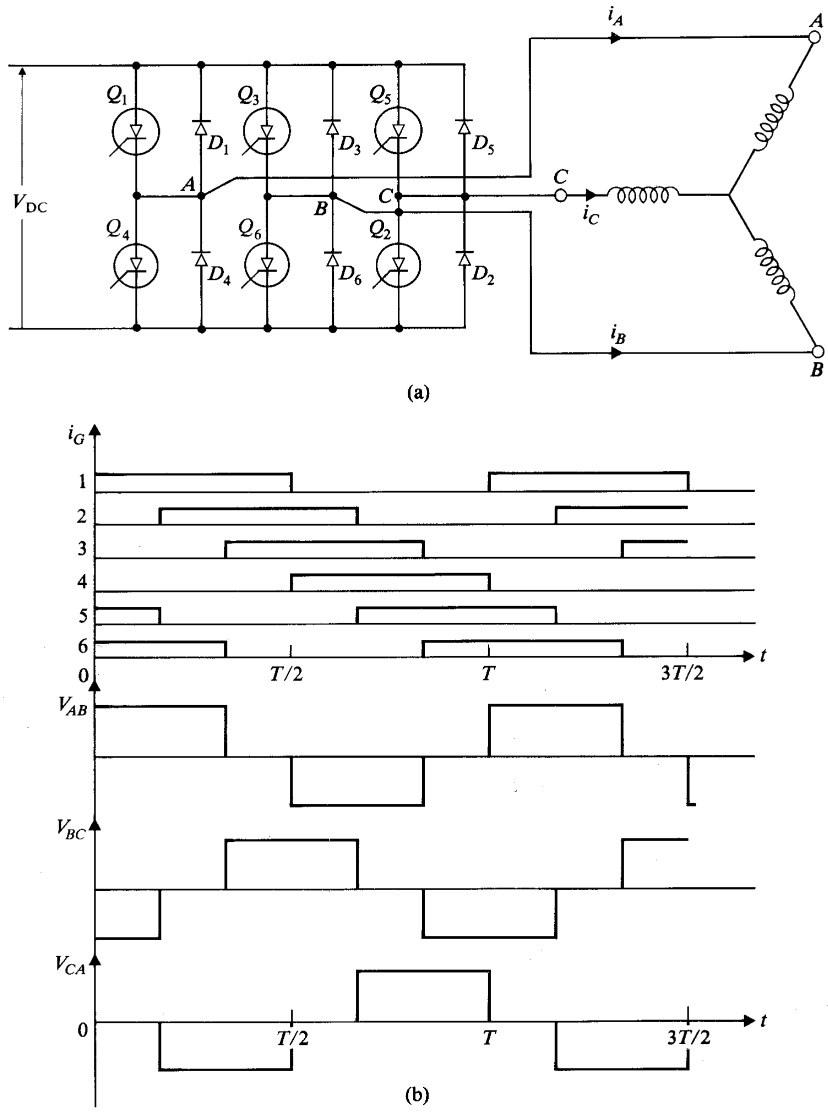
Synchronous Motor Drives

A synchronous motor cannot start in synchronous mode since the inertia and the mechanical load prevent the rotor to catch up with the rotating magnetic field at the synchronous speed. A common practice is to embed a few copper or aluminum bars short circuited by end rings in the rotor and to start the motor as an induction motor (the principle of induction motors is discussed in another chapter). When the rotor speed is close to the synchronous speed, the rotor is energized with a DC power supply and it will catch up or synchronize with the rotating magnetic field. This, however, is not a problem for power electronic inverter controlled synchronous motors because the inverter can ramp up the excitation frequency.

Since the rotor speed is proportional to the stator excitation frequency, the speed of a synchronous motor can only be controlled by varying the stator frequency. A common speed control strategy is the ***variable voltage variable frequency*** (***VVVF***) speed control, in which the ratio between the stator voltage and frequency is kept a constant. Below is the block diagram of an open loop synchronous motor drive. For rotor speeds below the rated speed, VVVF strategy is employed, and the maximum torque that the motor can produce is a constant. When the rotor speed required is higher than the rated speed, the stator voltage is capped to the rated voltage while the frequency is increased. The maximum torque is then reduced as the speed increases. As illustrated by the torque-speed curves in the diagram below, the motor drive is suitable for a constant torque load when the speed is below the rated speed, and would be suitable for a constant power load when the speed is higher than the rated speed.



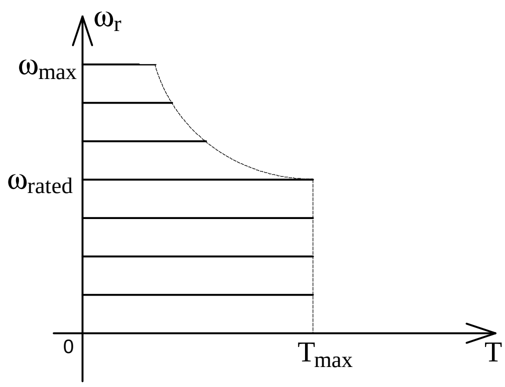
Steady-state model of an open-loop synchronous motor drive.



(a)

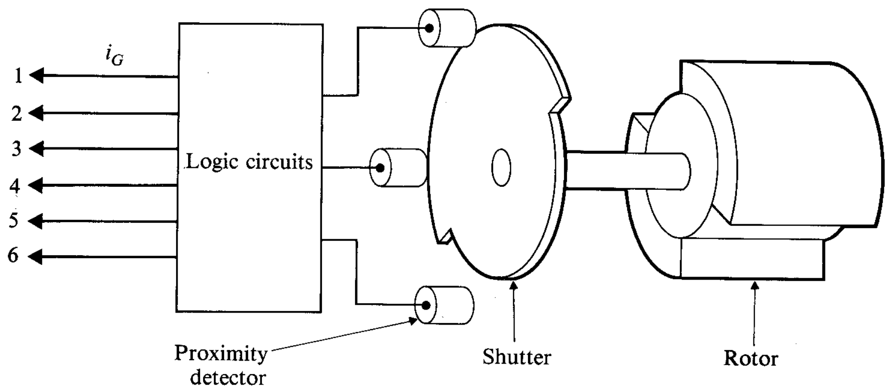
(b)

Inverter power circuit and waveforms

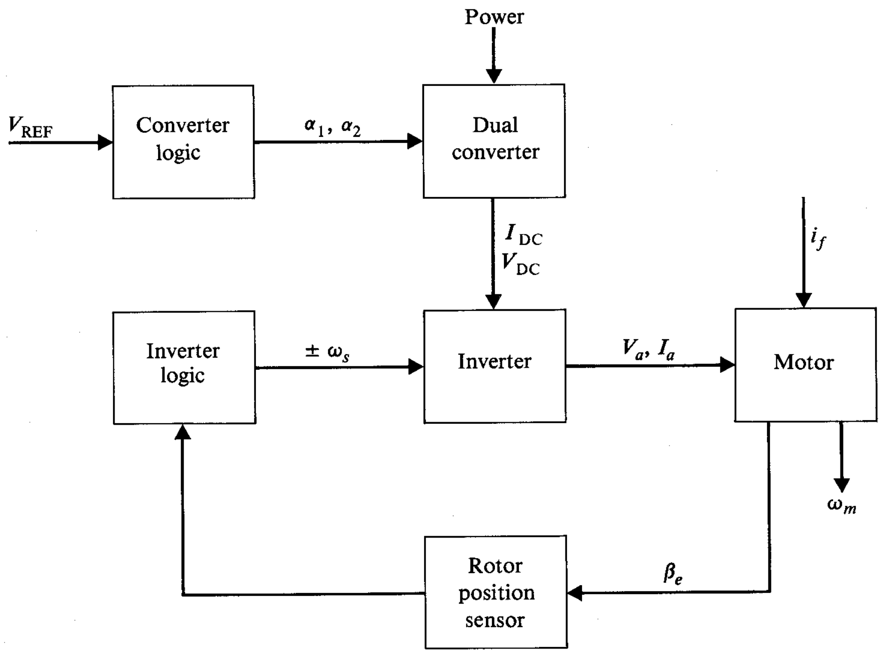


Torque speed curves of a synchronous motor with VVVF control

In the closed loop control, the stator excitation can be controlled according to the rotor position such that stator magnetic field is perpendicular to the rotor field and hence the electromagnetic torque the motor produces is always maximum under any load conditions. The torque speed curve of the motor in this case is essentially same as that of a DC motor. This type of motor drive is known as the brushless DC motors, which will discussed in another chapter. The diagrams below illustrate an optic position sensor and the block diagram of the closed loop synchronous motor drive.



Rotor position sensor



Steady-state model of a closed-loop synchronous-motor drive

Exercises

1. A 6 pole round rotor 3 phase star connected synchronous machine has the following test results:

Open circuit test:	4000 V line to line at 1000 rev/min
	50 A rotor current
Short circuit test:	300 A at 500 rev/min
	50 A rotor current

   Neglect the stator resistance and core losses, and assuming a linear open circuit characteristic, calculate:
   (a) the machine synchronous reactance at 50 Hz,
   (b) the rotor current required for the machine to operate as a motor at 0.8 power factor leading from a supply of 3.3 kV line to line with an output power of 1000 kW,
   (c) the rotor current required for the machine to operate as a generator on an infinite bus of 3.3 kV line to line when delievering 1500 kVA at 0.8 power factor lagging,
   (d) the load angle for (b) and (c), and
   Sketch the phasor diagram for (b) and (c).

   Answer: 7.7 W, 69.54 A, 76 A, 24.8^o, 27.4^o

2. For a 3 phase star connected 2500 kVA 6600 V synchronous generator operating at full load, calculate
   (a) the percent voltage regulation at a power factor of 0.8 lagging,
   (b) the percent voltage regulation at a power factor of 0.8 leading.
   The synchronous reactance and the armature resistance are 10.4 Ω and 0.071 Ω respectively.

   Answer: 44%, -20%

3. Determine the rotor speed in rev/min of the following 3 phase synchronous machines:
   (a) f = 60 Hz, number of poles = 6,
   (b) f = 50 Hz, number of poles = 12, and
   (c) f = 400 Hz, number of poles = 4.

   Answer: 1200 rev/min, 500 rev/min, 12000 rev/min

4. A Y connected 3 phase 50 Hz 8 pole synchronous alternator has a induced voltage of 4400 V between the lines when the rotor field current is 10 A. If this alternator is to generate 60 Hz voltage, compute the new synchronous speed and induced voltage for the same rotor current of 10 A.

   Answer: E_{new}*=5280 V,* $n_{syn\text{-}new}$*=900 rev/min*

5. A 3 phase Y connected 6 pole alternator is rated at 10 kVA 220 V at 60 Hz. Synchronous reactance X_s=3 Ω. The no load line to neutral terminal voltage at 1000 rev/min follows the magnetization curve shown below. Determine
   (a) the rated speed in rev/min,
   (b) the field current required for full load operation at 0.8 power factor lagging.

E (V)	11	38	70	102	131	156	178	193	206	215	221	224
I_f (A)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2

Answer: n_{syn}=1200 rev/min, I_f=1.0 A

6. The alternator of question 3 s rated at 10 kVA 220 V 26.2 A at 1200 rev/min. Determine the torque angle and the field current for unity power factor operation as a motor at rated load.

   Answer: torque angle=32^o, I_f = 0.75 A

7. A 3 phase induction furnace draws 7.5 kVA at 0.6 power factor lagging. A 10 kVA synchronous motor is available. If the overall power factor of the combination is to be unity, determine the mechanical load which can be carried by the motor.

   Answer: 8 kW

8. The following data are taken from the open circuit and short circuit characteristics of a 45 kVA 3 phase Y connected 220 V (line to line) 6 pole 60 Hz synchronous machine:

from the open circuit characteristic:	Line to line voltage = 220 V		
	Field current = 2.84 A		
from the short circuit characteristic:	Armature current (A)	118	152
	Field current (A)	2.20	2.84
from the air gap line:	Field current = 2.20 A		
	Line to line voltage = 202 V		

   Calculate the unsaturated value of the synchronous reactance, and its saturated value at the rated voltage. Express the synchronous reactance in ohms per phase and also in per unit on the machine rating as a base.

   Answer: 0.987 W per phase, 0.92 per unit, 0.836 W per phase, 0.775 per unit

9. From the phasor diagram of a synchronous machine with constant synchronous reactance X_s operating a constant terminal voltage V_t and constant excitation voltage E_f, show that the locus of the tip of the armature current phasor is a circle. On a phasor diagram with terminal voltage chosen as the reference phasor indicate the position of the center of this circle and its radius. Express the coordinates of the center and the radius of the circle in terms of V_t, E_f, and X_s.