ELECTROSTATIC FORCES

Introduction

Electric Charge

It was known since the time of the ancient Greeks that amber rubbed with fur would become "electrified" and attract small objects, an effect also easily seen by rubbing a piece of plastic with wool. The word "electricity" in fact comes from the Greek name for amber. During Benjamin Franklin's time, such astonishing electrical phenomena had been observed that widely-attended public electrical displays had become popular. A public presentation of this kind so strongly impressed Franklin that he bought the lecturer's equipment, and began investigating electrical phenomena on his own, in parallel with other efforts already underway in Europe. Others had already found that there were two kinds of electrical charge, and that charges of the same kind repel while charges of opposite kind attract each other. Franklin designated the two kinds of charge as “positive” and “negative.” But while qualitative understanding was developing rapidly, a quantitative understanding of the forces between electrically charged objects was still lacking.

It was Charles Augustin Coulomb, a French scientist, who first quantitatively measured the electrical attraction and repulsion between charged objects and established that the force was proportional to the product of the charges and inversely proportional to the square of the distance between them. In mks units, the electrostatic force F that two charges q_1 and q_2 a distance r apart exert on each other is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}. \qquad (1)$$

The force acts in a direction along the straight line connecting the two charges, and the force is repulsive when q_1 and q_2 are both positive or both negative, corresponding to a positive value of $q_1 q_2$. The force is attractive when the charges have opposite sign so that $q_1 q_2$ is negative. The quantity ε_0, called the permittivity constant, is equal to 8.854×10^{-12} coulomb^2/(newton-meter^2), and assures that the force will be in newtons when the charge is expressed in coulombs.

The gravitational force similarly exhibits an inverse square dependence on distance between two point masses. Gravitational forces differ however by being always attractive, never repulsive, and by being inherently weaker, with the electrostatic repulsion between two protons being 10^{36} times greater than their gravitational attraction.

This might seem puzzling. Gravitational forces involving massive objects can be literally strong enough to move the Earth, constantly acting to keep it in a nearly circular orbit around the Sun, and certainly we experience gravitational forces on ourselves very directly. But the electrostatic forces between pairs of objects in this laboratory are barely strong enough to lift small bits of lint or paper. Even considering the large mass of the earth, this might seem inconsistent with the statement about electrostatic forces being 10^{36} times stronger than gravitational forces.

The weakness of electrostatic forces between different everyday objects reflects the fact that matter consists of almost exactly equal numbers of positively charged protons and negatively charged electrons thoroughly intermingled with one another, mainly in the form of atoms whose electrons move around positively charged nuclei consisting of protons and neutrons. The electron and proton have equal but opposite charge ($q = 1.602 \times 10^{-19}$ C, in mks units), and the neutron has zero charge. The forces of electrostatic attraction and repulsion acting between particles within this intimate mixture of electrons and protons are indeed substantial. But so close to perfect is the balance between the number of electrons and protons in ordinary matter, and so close to zero is the net charge, that two separate objects near each other hardly exert any electrostatic force at all. Yet if you were standing at arm's length from someone and each of you had one percent more electrons than protons, the force of electrostatic repulsion would be sufficient to lift a weight equal to that of the entire earth.

At the time when atoms first came to be regarded as composed of electrons moving around much heavier positively charged nuclei, a major question was why the tremendous electric forces between the protons and the electrons do not cause them to fuse together. According to the Heisenberg Uncertainty Principle, the electrons must gain a larger and larger momentum, and a larger kinetic energy, the more we try to confine them into a smaller and smaller volume around the protons. It is this behavior, required by the laws of quantum mechanics, that keeps atoms from collapsing under the influence of electrical forces.

Similarly, tremendous electrical repulsive forces act between the positively charged protons confined within the atomic nucleus, and yet these forces do not usually succeed in pushing the nucleus apart. This is because of the additional strong short-range attractive nuclear forces that hold the protons and neutrons together despite the electrical repulsion.

Thus electrical forces and quantum mechanical effects acting together determine the precise properties of a material. With such enormous forces acting in balance within this intimate mixture, it is not hard to understand that matter, tending to keep its positive and negative charges in the finest balance, can have great stiffness and strength.

Also as consequence of the electric and quantum mechanical effects that determine the properties of matter, when atoms combine to form solids it often happens that one or more electrons normally bound to each atom are able to wander around more-or-less freely in the material. These are the *conduction* electrons in metals. When the electrons are not free to move through the material, externally applied voltages cannot produce electric currents in the material, and the material is an insulator, or dielectric (amber, Lucite, wood). A third kind of solid in which some of the bound charges can be freed, thus allowing current to flow in a controlled way, is called a semiconductor. A consequence of Coulomb's law is that if a metallic object has a net charge, the excess electrons will repel each other and be attracted to any region with net positive charge, so that the net charge tends to redistribute itself over the object. As a result, excess charge on an isolated spherical metal conductor will distribute itself uniformly on the outside of the conducting sphere. For the same reason, if a wire connected to a pipe buried in the ground (for example, to the plumbing in the building) touches the electrically charged conducting sphere, its the excess charge will flow into the ground under the influence of the Coulomb force between charges.

Practical applications of Coulomb's law involve unbalanced (net) charge distributed over an extended region, such as an approximately spherical conductor in the present experiment, and not actually charge concentrated at a point. The net charge on each object arises from individual discrete electrons and protons, but the small size of the electrons and protons and their large number make the distribution on each object appear smooth and continuous. Coulomb's law, Eq. (1), then applies by regarding each charged object to be divided into small sub-regions, and by using Eq. (1) to calculate the force that each such sub-region of the first object exerts on each small sub-region of the second object. We could then evaluate an appropriate vector sum to find the net force and torque of one object on the other. In terms of calculus, this means regarding Eq. (1) to involve an integral rather than an ordinary sum.

Applying a mathematical procedure equivalent to that described in the previous paragraph shows that excess charge distributed uniformly over the surface of a sphere exerts a force on a small test charge a distance away as if all the charge on the sphere were concentrated at its center. For this special case Eq. (1) ends up applying in the form given provided that the distance r to the excess charge on the metal sphere is taken as the distance to the center of the sphere.

Conservation of Charge

Amber rubbed with fur acquires a net negative charge because some of the negatively charged electrons are pulled from the fur onto the amber, leaving the fur positively charged. Since electrifying objects by friction involves merely moving the charges from one place to another, the total charge stays the same.

This principle is far more fundamental and general however. New particles can be produced in high-energy reactions such as those at the Fermi National Accelerator. Charged particles are not merely moved from one place to another. Yet in each reaction the number of newly created positively-charged particles always equals the number of new particles that are negatively charged. Since the net charge in all known physical processes stays the same, charge is said to be *conserved*.

The Experiment

The Van de Graaff Electrostatic Generator

The measurements to be done will test the distance dependence in Coulomb's law. The equipment used to produce net charge needed consists of the electrostatic generator illustrated in Fig. 1. We also use a small pith ball hanging by a nylon line. The pith ball is a small insulating sphere with a conducting outer surface that allows excess charge to distribute itself evenly. A known charge q is first placed on the ball, and then the generator is used to place a charge Q on the spherical generator dome. The electrostatic force acts to move the ball away from the generator dome, but is balanced by the gravitational force tending to push the pith ball back. The measured displacement of the ball and the angle of the string determines the component of gravitational force in the direction tending to rotate the pith ball back, and therefore measures the electrostatic force acting on the pith ball. We quickly measure the string angle for several values of the distance R to the center of the dome, and immediately repeat the measurement for the first value of R. The R dependence of the measured electrostatic force can then be compared with that predicted by Coulomb's law.

Figure 1 shows the electrostatic generator. The bottom roller is driven by a motor, causing the continuous rubber belt to turn. Friction between the belt and the wool on the bottom roller transfers charge so that the wool becomes positively charged and the belt negatively charged. The belt carries the negative charge to the top roller where the charge is transferred to the top collector. Then, friction between the top roller and the belt leaves the roller negatively charged and the belt positively charged. The region of positive charge on the belt is carried back to the bottom roller. A contact placed close to the bottom roller is grounded (meaning that it is attached to a nearby pipe that ultimately is connected to the earth). The flow of charge to or from the ground cancels of the excess charge on the part of the belt near the contact. By this mechanism, the belt and roller system act as a pump, doing work on the negative electric charge against the repulsive force of the charge already on the dome, and depositing it on the dome with a high potential energy per unit charge, corresponding to a high voltage.

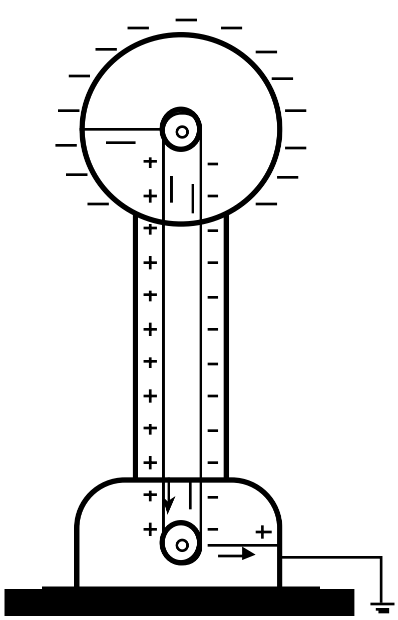

Figure 1
Van de Graaff generator

The charges generated in this experiment are not dangerous, but you might experience some disconcerting shocks by not following the instructions precisely as given. You may also pick up charge that can subject you to a minor electrical shock, about as strong as a carpet shock, when you touch a grounded object.

Experimental Procedure

The measurements to be made and later analyzed using the set-up in Fig. 2 require three basic steps:

a) Aligning the pith ball with respect to the electrostatic generator's conducting dome.
b) Placing a charge q on the ball and determining its value.
c) Measuring the displacement between the charge Q on the generator and q on the ball.

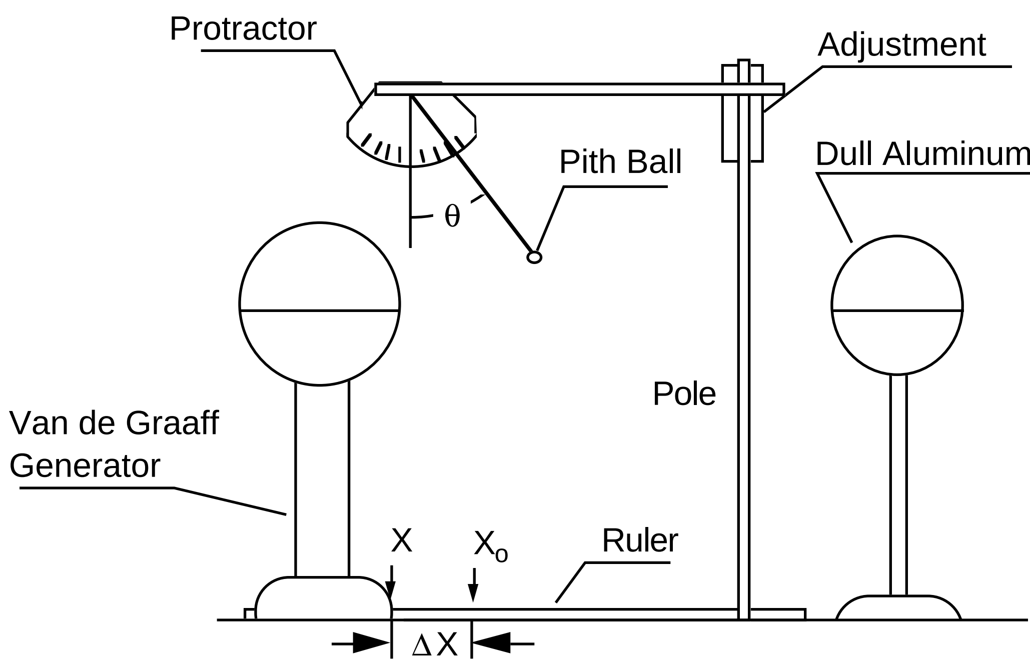

Figure 2
Apparatus for Coulomb's Law Experiment

Alignment of the Equipment

As shown in Fig. 3, the electrostatic generator is arranged so it can be displaced along the ruler fastened to your laboratory bench. In addition, each set-up includes a dull aluminum sphere grounded to a water pipe and mounted on an insulating rod of Lucite; the aluminum sphere is for use in grounding the conducting generator dome or the pith ball when necessary to remove the excess charge.

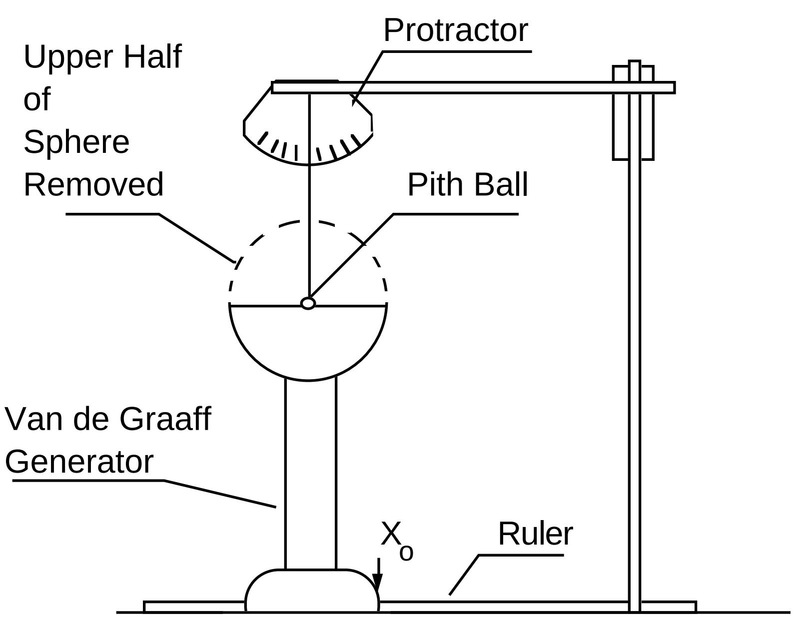

Figure 3
Alignment of Apparatus

First you need to determine the position of the voltage generator so that the pith ball, hanging vertically, is at its center. To do this, ground the generator sphere by touching it with the dull aluminum sphere. Remove the top half of the dome, place the small plastic ruler across a diameter for guidance in locating the center, and shift the generator until the midpoint between the two pith balls lies directly above the center of the sphere. If necessary, adjust the height of the horizontal bar supporting the pith balls so their midpoint coincides with the center of the generator sphere (Fig. 3). Record the position of the right side of the generator along the table. This measured location along the ruler is called X_0 in the equations we will be using later.

Now move the generator along the ruler, in the direction away from the vertical pole, until the pith ball clears the dome, and replace the top of the dome. If the experiment were done with the Van de Graaff generator too close to the vertical pole, the charge induced in the conducting pole would in turn exert Coulomb forces to redistribute the charge in the dome, making it further from spherically symmetric.

Charging the Pith Ball

The measurements described next must be done quickly because the charge on the pith balls dissipates in time. The rate of loss depends on the humidity in the air. Therefore, be sure you know exactly what to do, and once you start, continue taking measurements until you finish, saving the calculations for later.

First, ground both pith balls by touching them with the dull aluminum sphere. Then wrap the fur around the pointed end of the rubber rod and briskly rub the rod with the fur to produce a net negative charge on the end of the rod. Then bring the charged pointed end close to the pith balls. A positive charge will first be induced on the sides of the pith balls closest to the rod, as shown in Fig. 4, causing them to be attracted to the rod. After the rod and pith balls make contact, a negative charge (consisting of electrons) will pass from the rod to the balls. When enough negative charge has been transferred, the balls will fly away from the rod and will repel each other. If the balls do not fly away from the rod within 10 seconds, they are too dry and must be moistened by breathing gently on them. The separation between the two charged balls should be between 2 and 6 cm. Do not touch the pith balls or they may be partially discharged. Place the rod on the table and measure the distance r between the centers of the balls by holding the plastic calipers just below them. The measured value of r is needed in order to calculate the charge q on the pith balls.

Lift one of the charged balls by its line and drape it over the insulated plastic peg mounted on the meter stick without touching or discharging the other ball. Now slide the electrostatic generator

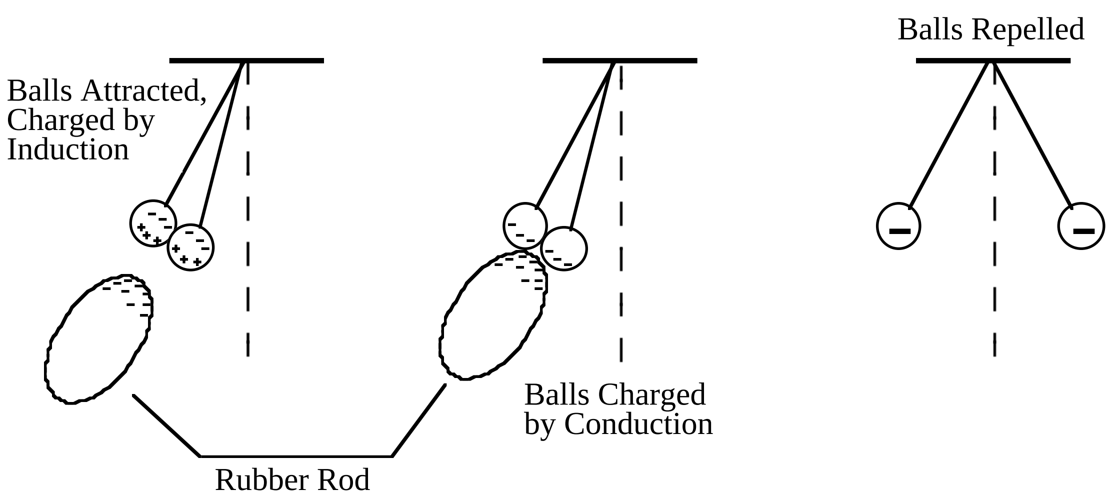

Figure 4
Charging the Pith Balls

along the table until the charged ball is about 5 cm from the generator sphere. Use the power cord switch to run the generator for several short bursts until the ball is deflected between 10° to 15° as measured by the protractor at the line support point. Record the deflection angle θ and the horizontal position X of the generator. Now the shift the generator sideways so as to decrease the deflection angle and again record X and θ. Make six separate measurements to obtain values of θ for $\Delta X = X - X_0$ between 10 cm and 60 cm (see Fig. 5). The data obtained will be used to determine the total charge Q on the generator sphere and to verify the inverse square dependence on distance in Coulomb's law.

Immediately after the last measurement, return the generator to the position of the first measurement and record the value of θ. How closely these two values of θ agree provides information about how much the dissipation of the charge affected your results.

Data Analysis

You probably will have noticed by this point that the Van de Graaff generator can produce impressive electrical discharges. Therefore, to minimize chances of accidental damage to the computer equipment, leave the computer at your workstation turned off until you have taken all your data and have grounded the electrostatic generator dome by leaving the dull aluminum sphere resting in contact with it.

Calculating the Charge on the Pith Ball

Figure 5 shows the vector diagram for the three forces acting on each ball in equilibrium. They are: the tension **T** in the nylon line, the weight $\mathbf{F_W}$ and the Coulomb force $\mathbf{F_C}$. The mass *m* is written on each ball in units of milligrams and the length of the nylon line must be measured.

a) Show that if each pith ball has mass *m* and the two hang from strings of length *L* separated by a distance *r* because of their charge, the charge q on each is

$$q = \left(\frac{2\pi\varepsilon_0 mgr^3}{\sqrt{L^2 - \frac{r^2}{4}}} \right)^{1/2} . \qquad (2)$$

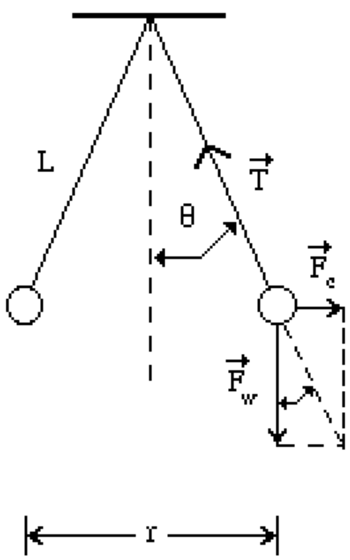

Figure 5
Two Charged Pith Balls

b) Calculate the charge *q* (in coulombs) on the pith ball.

c) Calculate the number of electrons that make up this charge. In your conclusions, discuss the magnitude of this number.

Verifying the Inverse Square Law

With the help of Fig. 6 it can be shown by trigonometry, with $\Delta X = X - X_0$ being that the distance from alignment that the generator was moved, that the distance R between the center of the generator sphere and the center of the ball is given by

$$R^2 = (|\Delta X| + L\sin\theta)^2 + (L - L\cos\theta)^2 \qquad (3)$$

and that the angle α between the line joining the two centers and the horizontal is

$$\alpha = \tan^{-1}[(L - L\cos\theta)/(|\Delta X| + L\sin\theta)].$$

By resolving the forces perpendicular to the thread supporting the ball, it can be shown that the electrical force F acting on the ball is given in terms of the observed θ by

$$F = \frac{mg\sin\theta}{\cos(\theta - \alpha)}.$$

If we make the assumption that α is small, this force can be written as

$$F = mg\tan\theta. \qquad (4)$$

Equation (4) can be used to calculate the force F for each measured value of θ and therefore at each R in Eq. (3). We seek to compare the observed dependence of F on R with that in Coulomb's law

$$F = \frac{qQ}{4\pi\varepsilon_0 R^2}. \qquad (5)$$

It is possible to use the program GAX to have the computer make these calculations, plot the results, and then fit the data to a power law equation to obtain values for the R power and the constant $qQ/4\pi\varepsilon_o$.

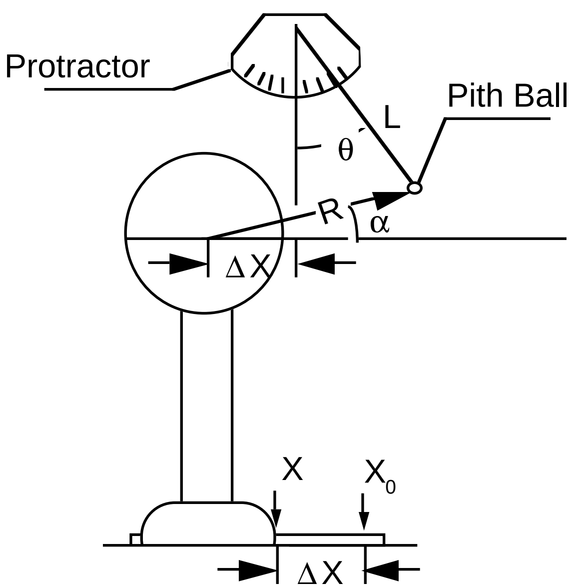

Figure 6
Geometric Considerations

First enter the raw data in the first two columns x and y in the program GAX. You might want to re-label these columns as X and Theta. From the drop down DATA menu select 'new column' and from that select the 'calculated' option.

Define the new column as R and use equation (3) to calculate R using X and Theta columns by entering the following into the space designated 'new column formula':

sqrt(("X"-0.24+0.56*(sin("Theta")))^2+(0.056*(1 - cos("Theta")))^2)

where in this calculation 0.24 is X_o and 0.56 is L. These values will be different for your set up. You should have these measurements ready when you enter the data. Note: the quantities in quotes refer to previously defined columns and can be inserted into the formula by clicking on 'columns' and selecting the column of interest. Likewise function like sqrt, sin, cos and tan can be inserted into the formula by clicking on the corresponding function button.

In a similar way choose to calculate a fourth column for F using equation (4). In this case you will need to type in the following for the 'new column formula' for F:

(45*10^-6)*9,8*tan("Theta")

where 45 is the amount in milligrams of the mass of one of the pith balls (they should be closely matched). The amount in milligrams for each pith ball is written on each pith ball.

Once you have numbers in these two new columns, go to the plot and click on the vertical axis label. A box will open allowing you to change the quantity plotted on the vertical axis. Choose F, Do the same to change the horizontal axis to read R. You should see an inverse relation on the plot where as R gets larger F becomes smaller. You would like to verify that this is an inverse quadratic relation.

Use the mouse to drag the arrow horizontally across the data so that it is highlighted both in the graph and in the Data Table window. From the drop down 'Analyze' menu, select the 'Automatic Curve Fit' option. When the box opens, select the 'Power' formula from the set of 'Stock functions…" The variable B should tell you to what power the data depends on R. Ideally it should be -2. Sometimes the fit might need a little help. One possibility would be to insert a - sign in front of the K in the formula.

- Using the value K in your analysis, evaluate the power dependence of the force F to the radius R. Is it an inverse square relationship?

- Using the value A in your analysis, calculate the charge the Van de Graaff generator dome. Also indicate the sign of the charge.

- What charge did you determine was on the pith balls?

- How did the first θ value differ when re-measured at the end, and what does this tell you about any experimental error caused by charge leaking off the pith balls and the generator?

- Do your data satisfy Coulomb's law to within reasonable experimental error? Explain the possible sources of any disagreement.

- How do the values of the charges you measured compare with your expectations? How hard would it be to place a coulomb or two of charge on the dome of the sphere?

QUESTIONS

- The following list of questions is intended to help you prepare for this laboratory session. If you have read and understood this write-up, you should be able to answer most of these questions. Some of these questions may be asked in the quiz preceding the lab.
- State Coulomb's Law.
- How many types of electric charge exist?
- In a nucleus there are several protons, all of which have positive charge. Why does the electrostatic repulsion fail to push the nucleus apart?
- What does it mean to say that charge is conserved?
- An electron with a charge of -1.602×10^{-19} C can combine with a positron having charge $+1.602 \times 10^{-19}$ C to yield only uncharged products. Is charge conserved in this process?
- How does the Van de Graaff generator operate?
- Is humidity in the room a concern in this experiment? Why or why not?
- As time passes, the pith balls lose their excess charge. Where does it go?
- Why is it possible to use the formula for the force between two point charges, Eq. (1), for the force between the charged pith ball and the dome of the Van de Graaff generator when the electrified dome is not even approximately a point charge?
- After the Van de Graaff generator has been running and is turned off with its dome still charged, how would the charge distribution in the grounded aluminum sphere be affected by bringing it near the dome without making contact? Explain this effect in terms of the electrostatic forces acting and the properties of the metallic sphere.
- Why does the experiment require using two pith balls rather than one?
- Prove Eq. (2) of this lab write-up.