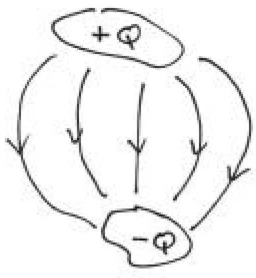
L3 Dielectrics

Lecture outline:

- Capacitance.
- Dielectric materials.
- Polarization of materials.
- Gauss s law in presence of dielectrics.

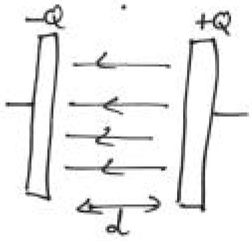
L3.1 Dielectrics

Capacitance: consider 2 conductors separated by an insulator.
Potential difference between the conductors is proportional to Q: V=Q/C, where C is the capacitance.
(1 Farad =1 Coulomb/1 Volt)



L3.2 Dielectrics

Parallel-plate capacitor:
(Area A)



$$E = \frac{Q}{\varepsilon_0 A}, \quad V = Ed$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 AE}{Ed} = \frac{\varepsilon_0 A}{d}$$

L3.3 Dielectrics

Potential energy of a capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy per unit volume (energy density):

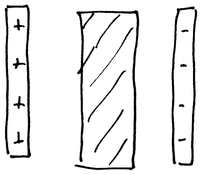
$$u = \frac{U}{Ad} = \frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\frac{(\varepsilon_0 A/d)(Ed)^2}{Ad}$$

So $u = \frac{1}{2}\varepsilon_0 E^2$ and energy resides in the electric field.

L3.4 Dielectric constant

Dielectrics are insulating materials – not a conductor.

Introduce a block of dielectric material between capacitor plates:



C increases: $C_{diel} = K\, C_0$,

K = dielectric constant, or relative permittivity.

L3.5 Dielectric constant

TABLE 27-2

SOME PROPERTIES OF DIELECTRICS[a]

MATERIAL	DIELECTRIC CONSTANT κ	DIELECTRIC STRENGTH (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transfomer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

L3.6 Dielectric constant

If V is kept constant, $Q = K C_0 V$, so charge on plates increases.

If Q is kept constant, $V = Q/(K C_0)$, so voltage between plates decreases.

<u>Example</u>: $A = 10^{-2}$ m^2, d=1mm,

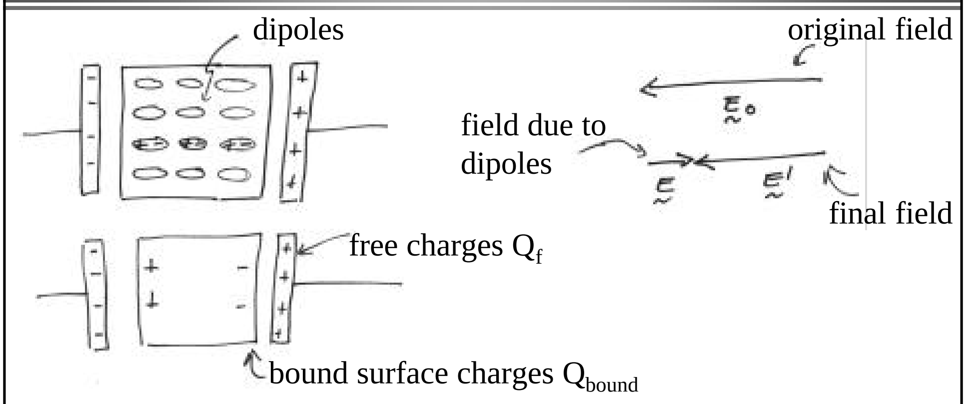
then $C = \varepsilon_0 A/d = 8.85 \times 10^{-12} \times 10^{-2}/10^{-3} = 88.5$ pF.

With polystyrene, $C = 2.6 \times 88.5 = 230$ pF.

If V =100V, $Q = 2.6 \times 88.5 \times 100 = 2.3 \times 10^{-8}$ C,

Surface charge density $\sigma = Q/A = 2.3 \times 10^{-6}$ Cm^{-2}

L3.7 Polarization of materials



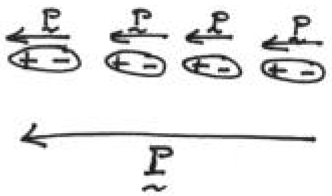
The dielectric has become polarized. The field inside is reduced.

L3.8 Polarization of materials

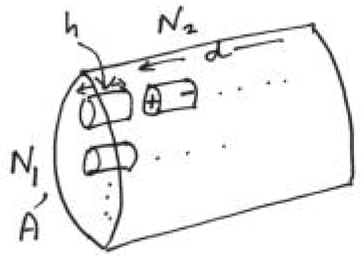
Polarization vector **P**.
The sum of all the elementary dipoles, in unit volume:

P = n **p** = dipole moment per unit volume,

where n = N (number of dipoles)/V (volume of dielectric).



L3.9 Polarization of materials

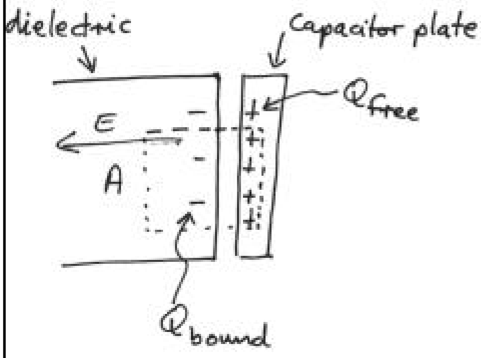


Number of dipoles = N
= N_1N_2
Moment = p = q h.
So P = n p = (N_1N_2/Ad)qh.
Use N_2h = d,
N_1q =Q_{bound}
=charge at surface.

So P = Q_{bound} /A = σ_{bound} = surface charge density.
(units: Cm^{-2})

L3.10 Gauss s law

Gauss s law in the presence of dielectrics:



$$EA = \frac{Q_{free} \quad Q_{bound}}{\varepsilon_0} = \frac{Q_{total}}{\varepsilon_0}$$

so

$$E = \frac{\sigma_{total}}{\varepsilon_0}, \quad \text{where} \quad \sigma_{total} = \frac{Q_{total}}{A}$$

But

$$Q_{free} = KC_0V, \qquad Q_{total} = C_0V$$

so $$Q_{free} = KQ_{total}$$

L3.11 Gauss s law

This means that $$E = \frac{Q_{free}}{K\varepsilon_0 A} = \frac{\sigma_{free}}{K\varepsilon_0}$$

and E has decreased.

General rule: to get E in a dielectric, replace ε_0 by $K\varepsilon_0$.

Eg, for a point charge, $$E = \frac{1}{4\pi K\varepsilon_0}\frac{q}{r^2}.$$

Gauss's law in a dielectric is now $$\oint K\mathbf{E}\cdot d\mathbf{a} = \frac{q_{\text{enclosed}}(\text{free})}{\varepsilon_0}$$