EQUIPOTENTIAL LINES

Electric fields

One way to look at the force between charges is to say that the charge alters the space around it by generating an electric field **E**. Any other charge placed in this field then experiences a Coulomb force. We thus regard the **E** field as transmitting the Coulomb force.

To define the electric field **E** more precisely, consider a small positive test charge q at a given location. As long as everything else stays the same, the Coulomb force exerted on the test charge q is proportional to q. Then the force per unit charge, $\mathbf{F}/q$, does not depend on the charge q, and therefore can be regarded meaningfully to be the electric field **E** at that point.

In defining the electric field, we specify that the test charge q be small because in practice the test charge q can indirectly affect the field it is being used to measure. If, for example, we bring a test charge near the Van de Graaff generator dome, the Coulomb forces from the test charge redistribute charge on the conducting dome and thereby slightly change the **E** field that the dome produces. But secondary effects of this sort have less and less effect on the proportionality between **F** and q as we make q smaller.

So many phenomena can be explained in terms of the electric field, but not nearly as well in terms of charges simply exerting forces on each other through empty space, that the electric field is regarded as having a real physical existence, rather than being a mere mathematically-defined quantity. For example when a collection of charges in one region of space move, the effect on a test charge at a distant point is not felt instantaneously, but instead is detected with a time delay that corresponds to the changed pattern of electric field values moving through space at the speed of light.

Closely associated with the concept of electric field is the pictorial representation of the field in terms of lines of force. These are imaginary geometric lines constructed so that the direction of the line, as given by the tangent to the line at each point, is always in the direction of the **E** field at that

point, or equivalently, is in the direction of the force that would act on a small positive test charge placed at that point. The electric field and the concept of lines of electric force can be used to map out what forces act on a charge placed in a particular region of space.

Figures 1a and 1b show a region of space around an electric dipole, with the electric field indicated by lines of force. The charges in Fig. 1a are identical but opposite in sign. In Fig. 1b the charges have the same sign. Above each figure is a picture of a region around charges in which grass seed has been sprinkled on a glass plate. The elongated seeds have aligned themselves with the electric field at each location, thus indicating its direction at each point.

A few simple rules govern the behavior of electric field lines. These rules can be applied to deduce some properties of the field for various geometrical distributions of charge:

> a) Electric field lines are drawn such that a tangent to the line at a particular point in space gives the direction of the electrical force on a small positive test charge placed at the point.

> b) The density of electric field lines indicates the strength of the **E** field in a particular region. The field is stronger where the lines get closer together.

> c) Electric field lines start on positive charges and end on negative charges. Sometimes the lines take a long route around and we can only show a portion of the line within a diagram of the kind below. If net charge in the picture is not zero, some lines will not have a charge on which to end. In that case they head out toward infinity, as shown in Figure 1b.

You might think when several charges are present that the electric field lines from two charges could meet at some location, producing crossed lines of force. But imagine placing a charge where the two lines intersect. Charges are never confused about the direction of the force acting on them, so along which line would the force lie? In such a case, the electric fields add vectorially at each point, producing a single net **E** field that lies along one specific line of force, rather than being at the intersection of two lines of force. Thus, it can be seen that none of the lines cross each other. It can also be shown that two field lines never merge to become one.

Under certain circumstances, the rules defining these field lines can be used to deduce some general properties of charges and their forces. For example, a property easily deduced from these rules is that a region of space enclosed by a spherically symmetric distribution of charge has zero electric everywhere within that region (assuming no additional charges produce electric fields

inside). Imagine first a spherically symmetric thin shell of positive charge all at a certain distance from the center. Field lines from the shell would have to be radially outward equally in all directions. If these outward pointing lines continued radially inward beneath the shell, they would have nowhere to end. Hence, the field lines must have ended at the surface charge, and there must be zero field everywhere inside. Next, suppose the spherically symmetric distribution of charge surrounding the uncharged region is not merely a thin shell. We can nevertheless consider the charge distribution to be divided up into many thin layers each at a different radius. Each layer contributes its own field lines that end at that layer, producing none of the field lines in the region enclosed by that layer. Then any point in the region of interest is inside all of the thin layers, where all the field lines have ended. We can conclude that the E field is zero at any point within a region surrounded by the spherical distribution of charge.
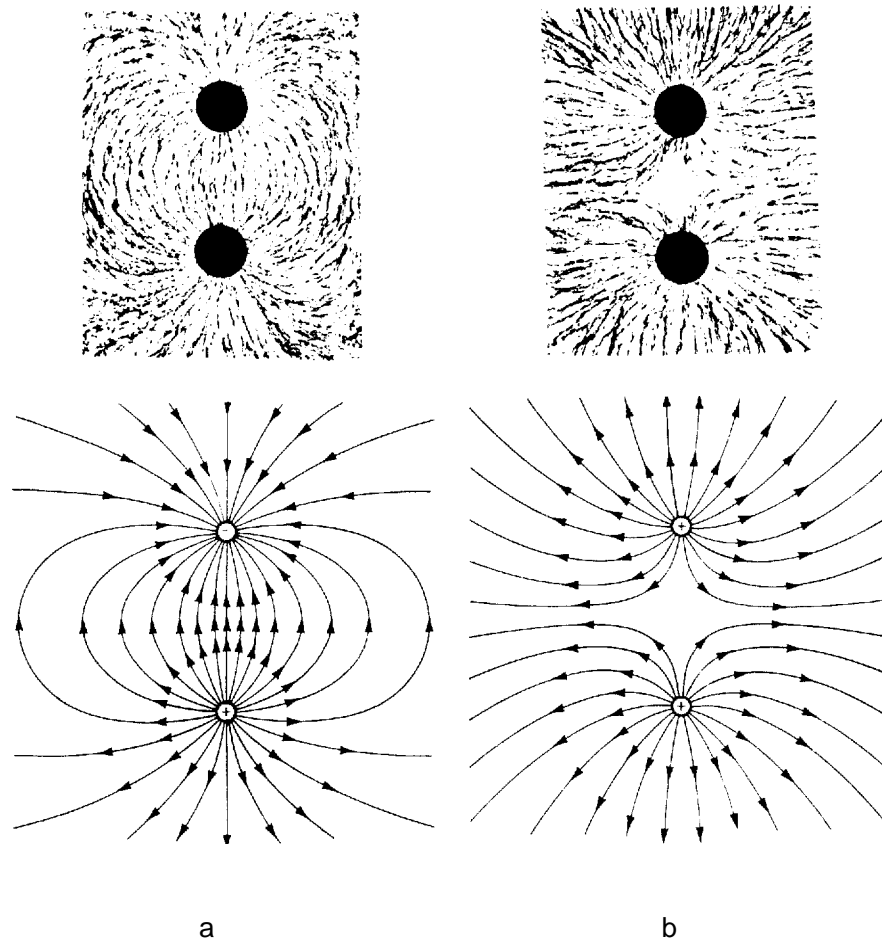

a   b

Figure 1   Above: Grass seeds align themselves with the electric field between two charges. Below: The drawing shows the lines of force associated with the electric field between charges. Figure 1a shows a charge pair with negative charge above and positive charge below. Figure 1b shows two positive charges.

In Figs. 1a and 1b it is seen that the density of field lines is greater near the charges because the lines must converge closer together as they approach a particular charge. It can also be shown that the electric field intensity increases near conducting surfaces that are curved to protrude outward, so that they have a positive curvature. Curvature is defined as the inverse of the radius. A flat surface has zero curvature. A needle point has a very small radius and a large positive curvature. The larger the curvature of a conducting surface, the greater the field intensity is near the surface.

The lightning rod is a pointed conductor. An electrified thundercloud above it attracts charge of the opposite sign to the near end of the rod, but repels charge of the same sign to the far end. The far end is grounded, allowing its charge to escape. The rod thereby becomes charged by electric induction. The cloud can similarly induce a charge of sign opposite to is own in the ground beneath it. The strongest field results near the point of the lightning rod, and is intense enough to transfer a net charge onto the airborne molecules, thus ionizing them. This produces a glow discharge, in which net electric charge is carried up on the ionized molecules in the air to neutralize part of the charge at the bottom of the cloud before it can produce a lightning bolt.

The electric field representation is not the only way to map how a charge affects the space around it. An equivalent scheme involves the notion of electric potential. The difference in electric potential between two points A and B is defined as the work per unit charge required to move a small positive test charge from point A to point B against the electric force. For electrostatic forces, it can be shown that this work depends only on the locations of the points A and B and not on the path followed between them in doing the work. Therefore, choosing a convenient point in the region and arbitrarily assigning its electric potential to have some convenient value specifies the electric potential at every other point in the region as the work per unit test charge done to move a test charge between the points. It is usual to choose either some convenient conductor or else the ground as the reference, and to assign it a potential of zero.

This bears some similarity to how the gravitational potential energy was defined. We could have considered the "electric potential energy" of a test charge in analogy with the gravitational potential energy by considering the work, not the work per unit charge, done in moving a test charge between two points. But just as the gravitational potential energy itself cannot be used to characterize the gravitational field because it depends on the test mass used, the electric potential energy similarly depends on the test charge used. But the force and therefore the work to move the test charge from one location to another is proportional to its charge. Thus the work per unit charge, or electric potential difference, is independent of the test charge used as long as the field does not vary in time, so that the electric potential characterizes the electric field itself throughout

the region of space without regard to the magnitude of the test charge used to probe it.

It is convenient to connect points of equal potential with lines in two dimensional problems; or surfaces in the case of three dimensions. These surfaces are referred to as equipotential surfaces, or equipotentials.

 If a small test charge is moved so that its direction of motion is always perpendicular to the electric field at each location, then the electric force and the direction of motion at each point are perpendicular. No work is done against the electric force, and the potential at each point traversed is therefore the same. Hence a path traced out by moving in a direction perpendicular to the electric field at each point is an equipotential. Conversely, if the test charge is moved along an equipotential, there is no change in potential and therefore no work done on the charge by the electric field. For non-zero electric field this can happen only if the charge is being moved perpendicular to the field at each point on such a path. Therefore, electric field lines and equipotentials always cross at right angles.

Figure 2 shows a region of space around a group of charges. The electric lines of force are indicated with solid lines and arrows. The electric field can also be indicated by equipotential lines, shown as dashed lines in the figure. The mapping of a region of space with equipotential lines or, in the case of 3-D space, with equipotential surfaces, provides the same degree of information as by mapping out the electric field itself throughout the region.
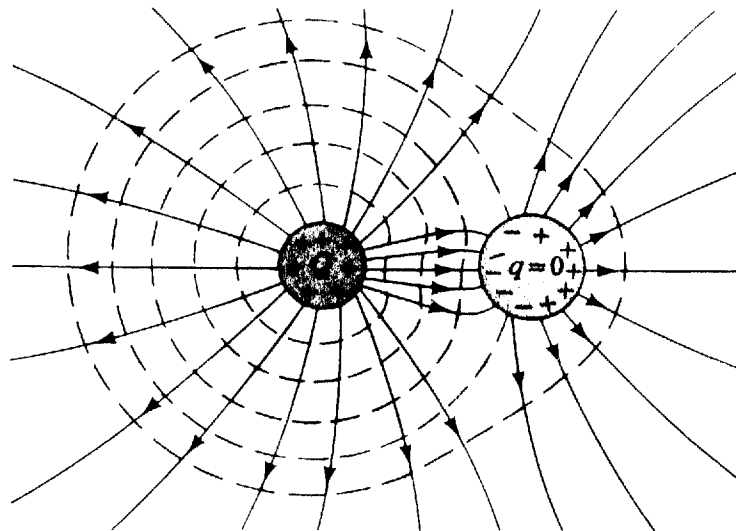

Figure 2　　　Electric field around a group of charges. Lines of force are shown as solid lines. Equipotential lines are shown as dashed lines.

Mapping equipotentials
between oppositely charged conductors

The equipotential apparatus is shown in Fig. 3. The power supply is a source of potential difference which is measured in Newton-meters per Coulomb. When it is connected to the two conductors, a small amount of charge is deposited on each conductor, producing an electric field and maintaining a potential difference, identical to that of the power supply, between the two conductors. The black paper beneath the conductors is weakly conducting to allow a small current to flow. The digital voltmeter measures the potential difference between the point on the paper where the probe is held and the conductor connected to the other lead of the voltmeter.

Choose the conductor geometry for which you will be mapping the field. Start with a circular conductor on the north terminal post (furthest away from you) and a horizontal bar on the south terminal (nearest you). Mount these conductor pieces on the brass bolts which protrude from the black-coated paper. Secure the conductors with the brass nuts. Tighten down the nuts to ensure good electrical contact between the conductors and the paper. The north banana jack (away from
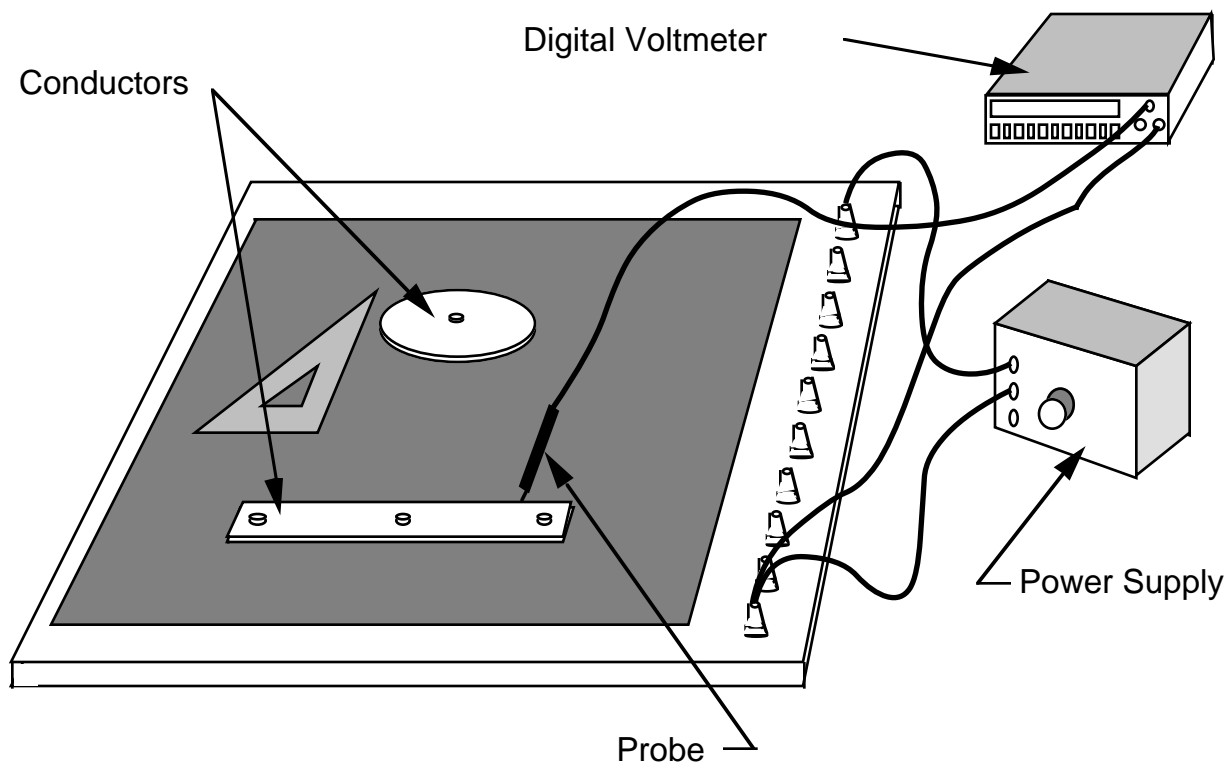
Figure 3   Equipotential Apparatus

you) is red and the south jack is black. The positive terminal of the power supply is connected to the red banana jack and the negative terminal to the black banana jack, thereby connecting the power supply to the conductors.

The digital voltmeter measures the potential difference between the two input electrodes. The black ground lead of the digital voltmeter (its negative terminal) should be connected to the black negative banana jack on the board. You will use the red (positive) lead of the digital voltmeter as a probe.

Setting the potential difference

Adjust the power supply to maintain the desired voltage between the two conductors by following these steps. Connect the red lead of the digital voltmeter to the red banana jack on the board. (The black lead was connected to the black banana jack in the previous step.) Adjust the power supply voltage to 6.00 volts, as read on the digital voltmeter. You may have to change the range setting of the voltmeter to get the proper reading. When this adjustment is completed, disconnect the red lead of the digital voltmeter. You are now ready to take data.

Mapping equipotential lines

Each equipotential line or surface is specified by the same single value of the voltage that all its points have with respect to the conductor. The goal is to locate points at each desired potential in order to trace out the corresponding equipotential line.

Suppose, for example, you want to find an equipotential at 5.00 volts. Place the red probe on the surface of the black paper and move it around until the digital voltmeter reads 5 volts. This point is then at a potential of 5 volts above that of the bar conductor.

To simplify mapping the region, the bar is inscribed with marks at every two centimeters, and we have provided graph paper containing a scale drawing of the apparatus in which we include a scale marked with two centimeter rulings. Use a straight edge as a guide for the probe. Line the straight edge with one of the scribe marks on the bar and slide the probe along the straight edge, maintaining good electrical contact, to find the point where the potential reads 5 volts. Record how far up the straight edge this point is, and mark the point on the scale drawing. Label the point as a 5 volt point. Now, move the straight edge to another scribe mark, similarly find the 5 volt potential point, and mark that point on the scale drawing. Repeat this process until you have a good idea of the shape of the 5 volt equipotential in the region. Draw a line connecting all the 5 volt points to show the 5 volt potential surface.

Now, go after the 4 volt potential using the same technique. Then, do the same for the 3 volt , 2 volt and 1 volt potential surfaces. For each case, draw a smooth curve connecting the points at the given equal potential. The curve is intended to fall along the equipotential between, as well as at, the specific points marked off, so the points should not be connected by straight line segments.

| Finding electric field lines | The result discussed earlier that the electric field is everywhere at right angles to the equipotential surfaces

and the fact that electric fields start on positive charges and end on negative charges can now be used to draw the field lines in the region where you have traced the equipotentials. On your drawing, place your pen at a point representing the bar conductor surface and draw a line perpendicular to the bar going toward the nearest equipotential line. As your line approaches the equipotential, be sure that it curves to meet the line at a right angle. Proceed similarly to the next equipotential, and so on until your line ends on the drawing of the round conductor. Keep in mind that each conductor itself is an equipotential. Return to the bar in the graph and construct a new line starting at an appropriate distance (say 2 cm) from the first line. Continue constructing electric field lines to map the entire area on your graph

| Conclusions | The electric field lines and equipotential surfaces that you have mapped should exhibit features consistent with the properties of the Coulomb

interaction. You should examine your plot and verify that it shows these features. Here are some suggested questions you might be able to address based on examining and analyzing your data.

Do the electric field lines approach conducting surfaces everywhere at right angles?

Electric fields are more intense near surfaces with larger curvature. Does the electric field intensity increase near the circular electrode as compared to the flat bar electrode, and if so, how?

Is the density of electrical potential lines related to the intensity of the electrical force?

Are there electric fields and equipotentials above the plane of the paper whose equipotential lines were mapped? If so, could we use the probe to map them? How else could they be mapped?

QUESTIONS

- The following list of questions is intended to help you prepare for this laboratory session. If you have read and understood this write-up, you should be able to answer most of these questions. Some of these questions may be asked in the quiz preceding the lab.

- What are three properties of electric lines of force?

- Why do electric lines of force never cross?

- How do the electric lines of force represent an increasing field intensity?

- How can you prove from properties of electric field lines that a spherical distribution of charge surrounding an uncharged region produces no electric field anywhere within the region?

- In what way are equipotential lines oriented with respect to the electric field lines?

- Are the electric field representation and the equipotential line representation equivalent in terms of how much information they contain about the electric field?

- What are the units of potential difference?

- What are units of electric field?

- Why must the **E** field be perpendicular to the surface of an ideal conductor?