Application of Wheatstone Bridge Circuit in Water Level Measurement System

Arsali
University of Sriwijaya
Inderalaya, South Sumatra, Indonesia.

Abstract

A conceptual model of simple water level meter based on the tide gauge's principle of the stilling well float gauge type has been designed by applying the Wheatstone Bridge (WB) circuit as a part of measuring device. By using this model it can be found that the change of water level is linearly correlated with the change of electric current reading at the galvanometer of the WB. Furthermore, the accuracy of the meter reading can be improved by adjusting the variable resistor while keeping the balance condition of the WB. The implementation of this model in turn will improve the ability of water level monitoring system in various observation fields.

Keywords: *stilling well float tide gauge, Wheatstone bridge, balance and unbalance conditions, sensitivity and accuracy of instrument.*

Introduction

Measurement of ground water or surface water levels such as river, lake, or ocean is very critical in order to get information in term of water availability on earth (Williams, 1973). In broader context this activity has to be done regularly with the objective to comprehend the change of water resource condition due to changes of water and land use, season, and even local climate as well as global scope.

There are numerous tools and methods to measure the water level, which depend on the situation, condition, and requirement. Basically, the measurement principle is carried out by determining the height difference (vertical distance) between surface water and certain point of reference. In case of change of the water level, the measurement frequency will highly depend on the frequency of the change mentioned above, whereas the number of observation points will require proper coordination and synchronization of the measurement system in the field.

Besides fulfilling the scientific objective which heavily focus on accuracy aspect (Sayer & Mansingh., 2000), the practical objective of water level measurement needs to consider aspects of economic and operational simplicity in order to be effective and sustainable. The last two aspects are very predominate policy in development and improvement of science and technology in many developing countries. The water level measurement system which mentioned here is designed by considering the last two aspects while still keeping the first aspect.

Wheatstone Bridge as Measuring Instrument

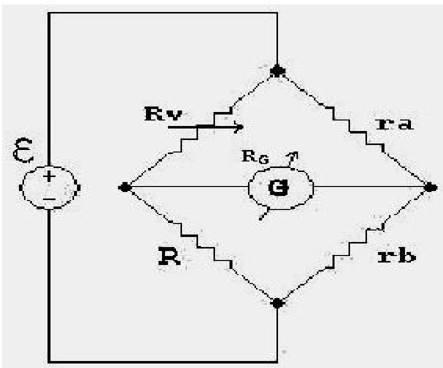

Figure 1. Wheatstone Bridge circuit

The principle operation of the Wheatstone Bridge (WB) can de described by referring to Figure 1. Two main components which function as meter indicator of measurements are Galvanometer *G* and variable resistor R_v ; the other two resistors which function as sensor components are resistors r_a and r_b. The values of R_V, r_a, and r_b can be changed according to condition, tool, field and also measurement objectives, but not Galvanometer R_G and circuit resistance *R*.

In general, the measured current at Galvanometer which is based on the voltage divider principle can be expressed as (Theraja, 1978):

$$I_G = \left[\frac{r_a}{r_a + r_b} - \frac{R_v}{R_v + R} \right] \frac{\varepsilon}{R_G} \qquad (1a)$$

or

$$I_G = -\left[\frac{r_b}{r_a + r_b} - \frac{R}{R_v + R} \right] \frac{\varepsilon}{R_G} \qquad (1b)$$

In numerous cases of measurement, such as the one described in this article, the serial resistors of r_a+r_b have a constant value, so that basically I_G is merely the function of two independent variables, i.e. r_a (or r_b) and R_v.

There are two specific conditions which fulfill the general condition previously described in equations (1a) or (1b). The first is known as unbalance condition of the WB which is characterized by $I_G \neq 0$. The second is known as balance condition of the WB which is characterized by $I_G = 0$ and will be fulfilled if r_a (or r_b) and R_v have met the following relationship :

$$\frac{r_a}{r_a + r_b} = \frac{R_v}{R_v + R} \qquad (2a)$$

or

$$\frac{r_b}{r_a + r_b} = \frac{R}{R_v + R} \qquad (2b)$$

Application of Wheatstone Bridge Circuit on Stilling Well Float Tide Gauge

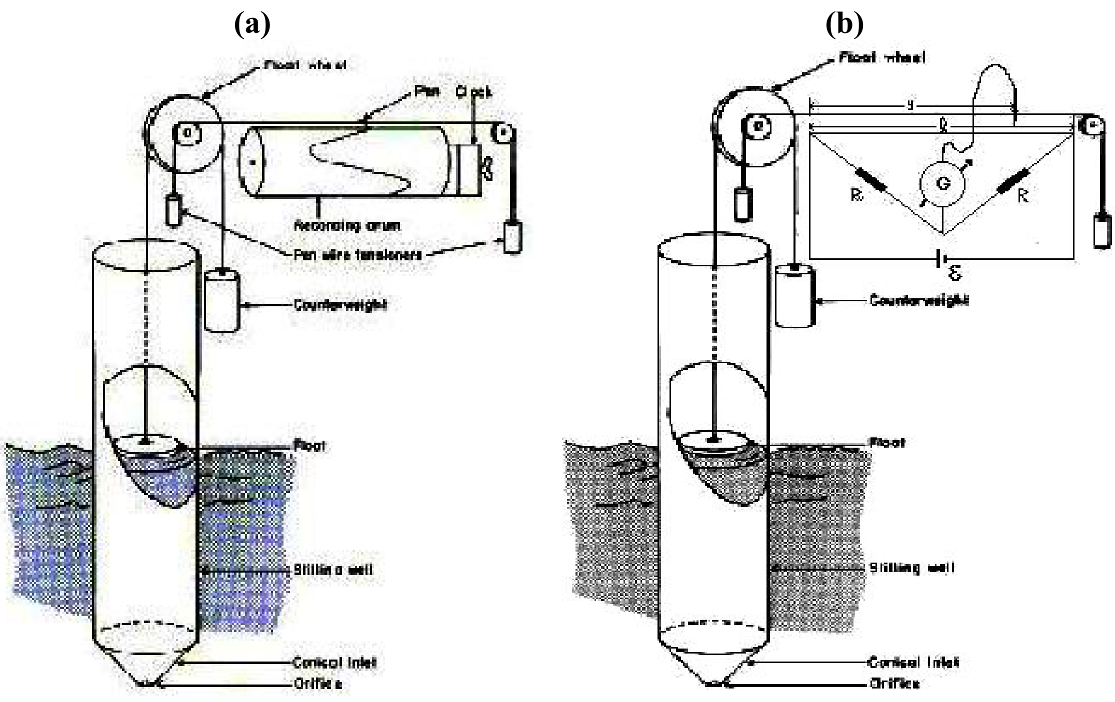

Figure 2. (a) Basic scheme of Stilling Well Float Tide Gauge. (b) Application of WB circuit on Stilling Well Float Tide Gauge (Adapted from IOC, 1985).

The basic model of *Simple Stilling Well Float Tide Gauge* is illustrated in Figure 2a (IOC, 1985). By merely substituting the recording drum, clock, and pen of the system with an appropriate WB circuit will result in the modified system such as illustrated in Figure 2.b. The apparent difference between the previous system (see Figure 1) and this modified WB system is that serial resistance of r_a+r_b is represented by metal wire which has total length of ℓ and a constant resistance value.

By using the WB design such as illustrated in Figure 2b, equations (1a) and (1b) can be rewritten as :

$$I_G = \left[\frac{l-y}{l} - \frac{R_v}{R_v + R}\right]\frac{\varepsilon}{R_G} \tag{3a}$$

or

$$I_G = -\left[\frac{y}{l} - \frac{R}{R_v + R}\right]\frac{\varepsilon}{R_G} \tag{3b}$$

and balance condition requirement such as described by equations (2a) and (2b) can be rewritten as :

$$\frac{y}{l} = \frac{R}{R_v + R} \tag{4}$$

In specifying WB as measuring instrument, physical dimensions for every part of system are specified in such manner so that *y* could represents the water surface position, with values in the range of 0 (the lowest) to ℓ (the highest). Meanwhile, R_v as illustrated in Figure 1, is variable resistance that function as adjusting parameter in order to produce balance condition of WB. R_V should be set up from values of 0 to ∞ in order to get values of *y*/ℓ in the range of 0 to 1.

Measurement: Analysis of Sensitivity and Accuracy

There are two approach methods used for obtaining information of surface water level *y*. The first method is carried out by current measurement on galvanometer I_G directly on the unbalance condition of WB. The *y* value subsequently could be calculated using equations (3a) or (3b). This calculation was conducted by using any R_V value which is previously determined. The second method is conducted by adjusting R_V value after I_G measurement until the balance condition of the WB is obtained (which is shown by I_G = 0). This R_V value is subsequently used to calculate *y* through equation (4). The flow chart of the two methods is illustrated in Figure 3.

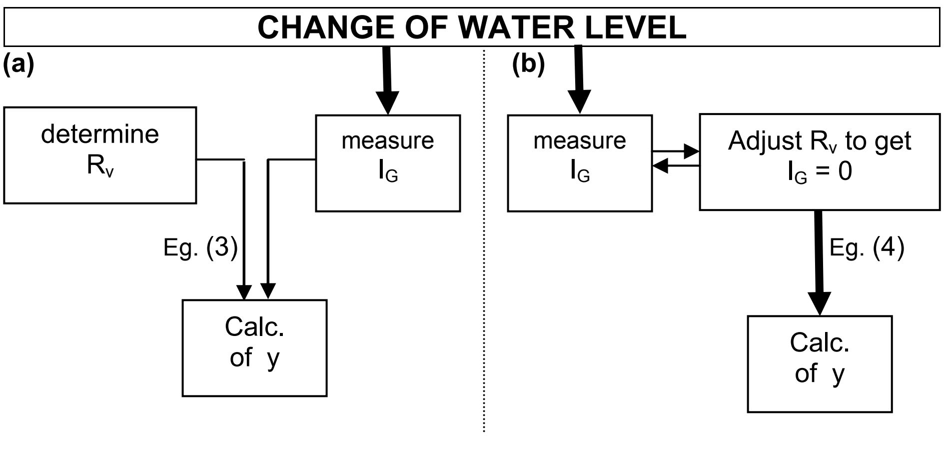

Figure 3. Working diagram of water level measurement (a). By Measuring I_G (unbalanced condition). (b). By adjusting the R_v (balance condition).

The advantage of the first method is in term of speed. This process can be done quickly by direct measurement of I_G on unbalance condition and calculating the y value using equations (3a) or (3b) which need relatively simple effort. The disadvantage of this method is that current I_G which is depend on R_V value could have broad range of meter reading, which results in reduced accuracy of galvanometer reading.

In the second method, information of I_G value was not needed, except that I_G should have zero value. Measurement of I_G close to zero value result in small range of galvanometer reading as well as improving the sensitivity of the meter. The disadvantage of this method is the requirement of added process and broad domain of R_y which create more complex operational condition and more time to complete the process.

Measurement of electric current through the Galvanometer

From equations (3a) or (3b) it can be proved that the maximum and the minimum values of I_G is $\pm\varepsilon/R_G$, depending on the values of y and R_v, as graphically shown in Figure 4. We have known that procedurally y and R_v do not need to change simultaneously, so that depiction I_G to each independent variable can be conducted separately.

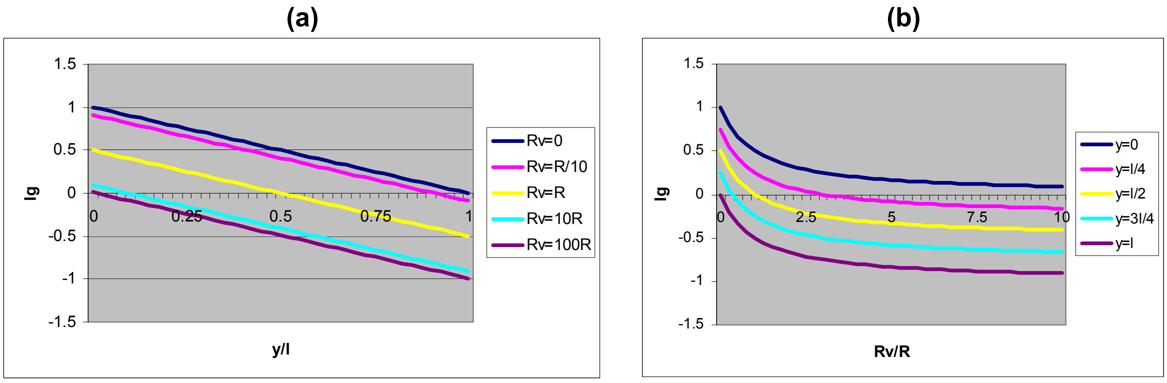

Figure 4. Graph of I_G based on equations (3a) or (3b). (a) I_G versus y. (b) I_G versus R_v.

Figure 4a shows the linearity of I_G as function of y for many values of R_v, while Figure 4b shows the reversal connection of I_G to R_v for many values of y, in accordance with the equations (3a) or (3b).

Sensitivity and Accuracy of the WB Circuit

The galvanometer as a part of the WB circuit has its sensitivity that is depending on its internal resistance R_G and other components of such WB such as the operating voltage ε, the length of the wire metal l, and the other two resistors, R_v and R. Based on equations (3a) or (3b), the sensitivity of the galvanometer to measure I_G according to the change of y can be formulated as

$$\left(\frac{\partial I_G}{\partial y}\right)_{R_v} = -\frac{\varepsilon}{R_G \ell} \quad (5)$$

in case of change of R_v, the sensitivity of the measurement of I_G can be formulated as

$$\left(\frac{\partial I_G}{\partial R_v}\right)_{y} = -\frac{\varepsilon}{R_G}\frac{R}{(R_v + R)^2} \quad (6)$$

Figure 5a shows the graph of $(\partial I_G/\partial y)_{Rv}$ expressing the sensitivity of measurement of the current I_G through the galvanometer that is caused by the change of water level y. The graph expresses that constant sensitivity of I_G as $-\varepsilon/R_G\ell$, independent of y. Another, the sensitivity of I_G in connection with the change of the variable of resistance $(\partial I_G/\partial R_v)_y$ yielding degradation pattern by the inverse power of two, as shown in Figure 5b.

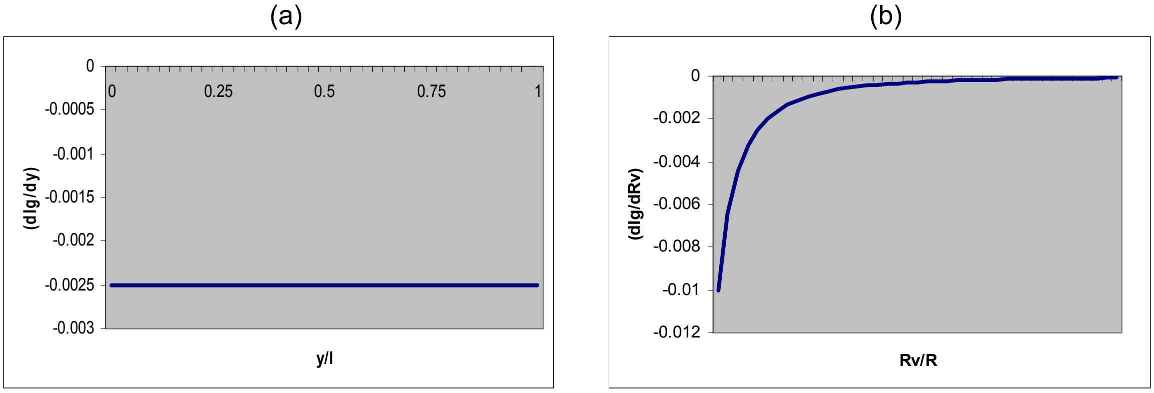

Figure 5. Sensitivity of I_G measurement in the galvanometer as change of y (a) and R_v (b).

Figure 4a showed that maximum and minimum levels of I_G could be minimized by adjusting R_V value which in turn will improve I_G current reading accuracy. For example, suppose that there is an error of current measurement on galvanometer with magnitude of ΔI_G. Based on the equation (5), the error produced in determining y value is as follow

$$\Delta y = -\frac{R_G \ell}{\varepsilon} \Delta I_G \tag{7}$$

Meanwhile, as mentioned previously, the maximum value of I_G at unbalance condition is

$$I_{Gm}^{UB} = \pm \frac{\varepsilon}{R_G} \tag{8}$$

If N number of smallest reading scales of galvanometer is available, then the value of maximum reading error due to the reading can be written as

$$\Delta I_{Gm}^{UB} = \pm \frac{\varepsilon}{R_G} \frac{1}{N} \tag{9}$$

Therefore, the maximum error of y at unbalance condition can be described using the following relation

$$\Delta y_m^{UB} = -\frac{R_G \ell}{\varepsilon} \Delta I_{Gm}^{UB} = \left(-\frac{R_G \ell}{\varepsilon} \right) \left(\pm \frac{\varepsilon}{R_G} \frac{1}{N} \right)$$

$$\Delta y_m^{UB} = \mp \frac{\ell}{N} \tag{10}$$

Because I_G is always set up to zero value, its maximum reading value is equal to the error of I_G at balance condition as follow

$$I_{G\,m}^{B} = \Delta I_G = \pm \frac{\varepsilon}{R_G \ell} \Delta y \tag{10}$$

In similar manner, the maximum reading error in this case can be written as

$$\Delta I_{Gm}^{B} = \pm \frac{\varepsilon}{R_G \ell} \frac{\Delta y}{N} \tag{11}$$

Therefore, the maximum error value of y at balance condition can be determined using the following relation

$$\Delta y_m^B = -\frac{R_G \ell}{\varepsilon} \Delta I_{Gm}^{B} = \left(-\frac{R_G \ell}{\varepsilon} \right) \left(\pm \frac{\varepsilon}{R_G \ell} \frac{\Delta y}{N} \right)$$

$$\Delta y_m^B = \mp \frac{\Delta y}{N} \tag{12}$$

In case that the error of y at balance condition expressed by equation (12) is equal to the maximum error at balance condition expressed by equation (10), then the following relation is apply

$$\Delta y_m^B = \mp \frac{\Delta y_m^{UB}}{N} = \pm \frac{\ell}{N^2} \tag{13}$$

It can be shown that water level determination at balance condition is capable to improve its accuracy N times than that of unbalance condition of the WB. Therefore, measurement process at balance condition is very critical in order to improve accuracy by using WB circuit.

In attempt to have the WB back to its balance condition every time after finishing I_G measurement by adjusting R_V value, not only we capable to recalculate the more accurate of y value which have previously discussed on the second method, but also to improve the values of reading measurement accuracy which has already mentioned in the first method. Therefore, it is suggested to use water level determination method that represent the combination of the two methods above, as illustrated in Figure 6.

In this combination method (see Figure 6), initial information is from variable resistance R_V which is arbitrary chosen and water surface level y position (with unknown value) subsequently receive by WB circuit resulting in current value at galvanometer I_G. Using equations (3a) or (3b) as well as values of R_V and I_G mentioned above, the initial information of water surface level y_1 will be achieved. This y_1 value which is incorporated into equation (4) will produce the closest approximation value of R_V at balance condition of WB, whereas the exact value of R_V at balance condition will be confirmed by adjusting variable resistance value of R_V in order to achieve the exact balance condition which is shown by value of I_G = 0. The R_V value at exact balance condition of WB in one side by using equation (4) will produce second information of water surface level y_2 as the weighted value of y_1

mentioned above, whereas on the other side in conjunction with I_G value from the new measurement will provide y_1 value on the next stage by using equations (3a) or (3b).

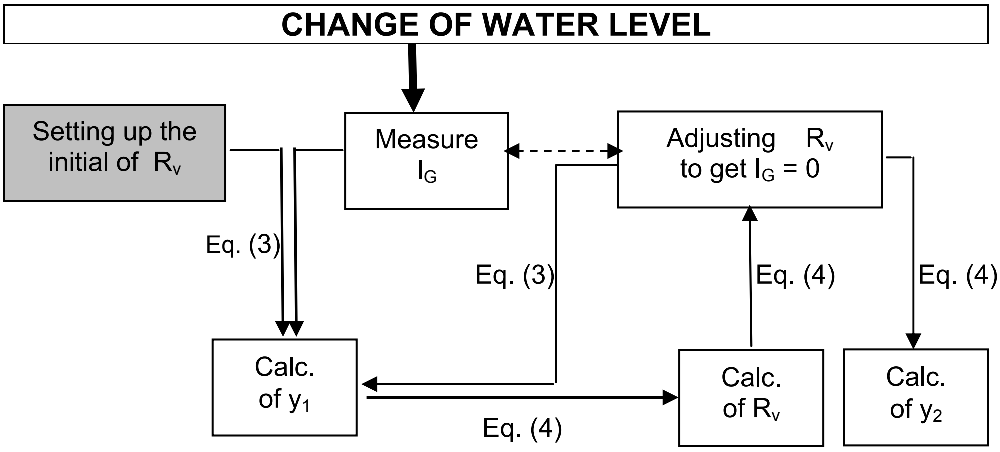

Figure 6. Working diagram of the method of water level determination y is the combination of the measurement of the electric current through galvanometer I_G in the unbalance condition and adjusting the variable resistor R_v in balance condition. Except at the initial process (see the grey box) the value of R_v is always determined in balance condition, before applying to equations (3a) or (3b). Read the text for more information.

Implementation of Tide Gauge in the water level monitoring system

Indonesia as part of maritime continent with more than 17000 islands, 80000 km coastal long, and various river and lake, needs instrument of water level measurement with the amount, quality, and adequate management system to realize a tradition of observation and measurement with accurately, quickly, and continually data (Nontji,1987).

As we have been discussed at great length, conceptually WB is able to measure with the good accurateness in the water level measurement system based on the Stilling Well Float Tide Gauge, whereas technically this instrument is expected can be completed by the electronic control system that is capable to transforms, records, and communicates data of result of measurement that is possibly doing of data management from various station which is distributed in various location. The advantage of WB as measurement instrument which relative simply, is also expected can become the selected technology that is cheap and easy to operated, so that can be more guaranteed permanence process of monitoring in the future.

Conclusions

It is concluded that the WB circuit is feasible to implement in real water level measurement system based on the three simple reasons: good accurateness, simple operation and maintenance, and economic consideration.

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