Trigonometry Formulae

- Degree/Radian Relationship: $180° = \pi$ radians
- Length of Arc: $s = r\theta$ (θ in radians)
- Area of Sector: $A = \frac{1}{2}r^2\theta$ (θ in radians)
- Angular Velocity: $\omega = \frac{\theta}{t}$ (θ in radians) Linear Velocity: $v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$
- Let (x, y) be a point other than the origin on the terminal side of an angle θ in standard position.

We have the following definitions, where $r = \sqrt{x^2 + y^2}$:

$$\cos(\theta) = \frac{x}{r} \quad \sec(\theta) = \frac{r}{x} \quad \sin(\theta) = \frac{y}{r} \quad \csc(\theta) = \frac{r}{y} \quad \tan(\theta) = \frac{y}{x} \quad \cot(\theta) = \frac{x}{y}$$

We can write all trigonometric functions in terms of cosine and sine as follows:

$$\sec(\theta) = \frac{1}{\cos(\theta)} \quad \csc(\theta) = \frac{1}{\sin(\theta)} \quad \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

- Even: Cosine is an even function, so $\cos(-x) = \cos(x)$
- Odd: Sine and Tangent are odd functions, so $\sin(-x) = -\sin(x)$, $\tan(-x) = -\tan(x)$.
- Cofunction Identities:

$$\cos(90° - \theta) = \sin(\theta),\ \sin(90° - \theta) = \cos(\theta),\ \tan(90° - \theta) = \cot(\theta), \cot(90° - \theta) = \tan(\theta),$$
$$\sec(90° - \theta) = \csc(\theta),\ \csc(90° - \theta) = \sec(\theta)$$

- Sum and Difference Identities:

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

- Double-Angle Identities:

$\cos(2A) = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\sin(2A) = 2 \sin A \cos A$

$\tan(2A) = \dfrac{2 \tan A}{1 - \tan^2 A}$

- Half-Angle Identities:

$\sin(\frac{A}{2}) = \pm\sqrt{\dfrac{1 - \cos A}{2}}$

$\cos(\frac{A}{2}) = \pm\sqrt{\dfrac{1 + \cos A}{2}}$

$\tan(\frac{A}{2}) = \pm\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$

• Product-to-Sum Identities:

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

• Sum-to-Product Identities:

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

• Law of Sines: In any plane triangle ABC, with sides a, b, and c, we have:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

• Law of Cosines: In any plane triangle ABC, with sides a, b, and c, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

• Law of Tangents: In any plane triangle ABC, with sides a, b, and c, we have:

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$$
$$\frac{b+c}{b-c} = \frac{\tan\left(\frac{B+C}{2}\right)}{\tan\left(\frac{B-C}{2}\right)}$$
$$\frac{c+a}{c-a} = \frac{\tan\left(\frac{C+A}{2}\right)}{\tan\left(\frac{C-A}{2}\right)}$$

• In any plane triangle ABC, with sides a, b, and c, the area $\mathcal{A}$ is given by any of the following:

(a) $\mathcal{A} = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$

(b) (Heron's formula) $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$