Physics Laboratory Works 1, part II

COMPENSATION AND BRIDGE MEASUREMENTS

In physical measurements the observed physical quantities are often changed to electrical quantities. The most often used quantities are resistivity and voltage. Current is also a common output quantity of sensors, but it is usually defined by measuring the voltage over a known resistor.

There are some principal problems in voltage and resistance measurements that one should be aware of and keep in mind during the measurements. Often the problem is that the measurement affects the object under study. This usually means that there is an internal current in the object caused by the measurement coupling.

Nowadays there usually exists a commercial equipment for resistance and voltage measurements which is applicable to the problem under investigation. However, acquisition of this kind of a device could be very expensive or the device could not be available when needed. Therefore it is good to know some basic methods to carry out these measurements.

In this work we use the Wheatstone's bridge coupling for measuring resistor. In Wheatstone's coupling the measured resistor is compared to a known resistor. The electromotive force (EMF) of a voltage source is defined with a compensation method, where the voltage is compared to the accurately known voltage of a standard element. Finally, the internal resistance of the voltage source is defined.

1 Resistor measurement with Wheatstone's bridge

The Wheatstone's bridge coupling is presented in Figure 1, where R_x is the measured resistor, R is the known resistor, AB is a homogeneous resistance wire (e.g. potentiometer) and R_e is a bias resistor, which limits the current flow in the circuit. The bridge is in balance when the roll of the potentiometer (point C) is adjusted in such a way that $U_{\mathrm{DC}} = 0$. Then the points C and D are in the same potential and there is no current flow in the interval CD and

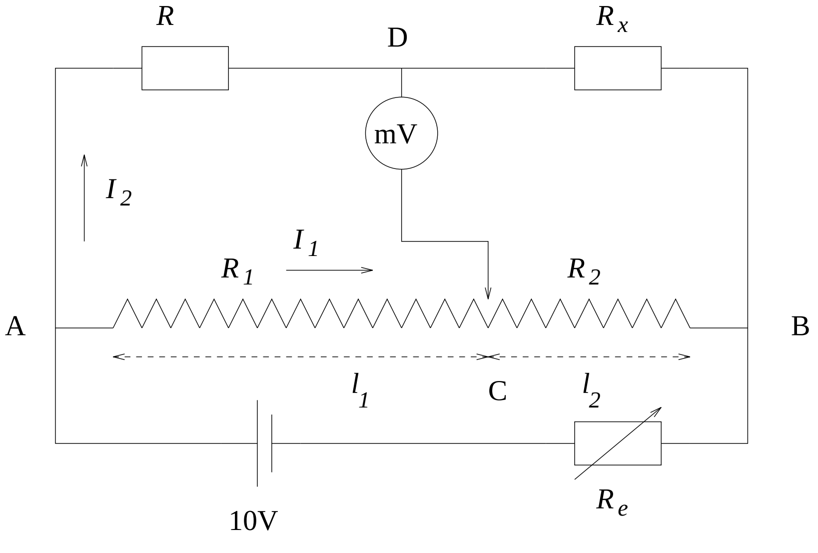

Figure 1: Wheatstone's bridge coupling (see text).

$$I_1 R_1 = I_2 R \tag{1}$$
$$I_1 R_2 = I_2 R_x . \tag{2}$$

By dividing equation (1) by (2) we obtain for R_x

$$R_x = \frac{R_2}{R_1} R. \tag{3}$$

Since the resistance wire of the potentiometer used in this work is homogeneous, the ratio R_2/R_1 is equal to the corresponding ratio of wire lengths l_2/l_1. Therefore

$$R_x = \frac{l_2}{l_1} R. \tag{4}$$

In the potentiometer the wire lengths are given as proportional units between 0 – 1000.

2 Measuring the electromotive force with compensation method

2.1 Properties of a voltage source

The electromotive force (EMF) E of a voltage source is the potential difference between its poles when there is no current flow through the source, that is the potential difference in an open circuit. Every real voltage source has an internal resistance R_i which causes an internal voltage loss when current I drifts through the source. The situation is as in Figure 2.

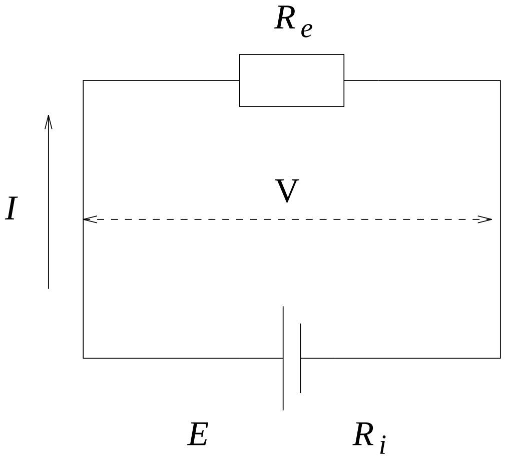

Figure 2: Voltage source in a closed circuit (see text).

If the outside resistor in the circuit is R_e, we get

$$E = IR_i + IR_e = IR_i + V. \tag{5}$$

Consequently,

$$V = E - IR_i \tag{6}$$

where V is the terminal voltage, or the potential difference between the poles of the voltage source in a closed circuit.

From equation (6) it can be seen that $V = E$ when $I = 0$. Since $I = E/(R_i + R_e)$, $I = 0$ when $R_e = \infty$ (open circuit). Thus, EMF can be measured only with such a method where there is no current flow through the voltage source. This kind of example is the compensation method used in this work. If we instead connect a voltmeter between the poles of the voltage source, there will be current flow through the source, and we obtain the terminal voltage V according to equation (6) which is smaller than E. Equation (6) also shows that while R_i increases the terminal voltage approaches zero.

2.2 Measurement of the EMF of a voltage source

Figure 3 presents a compensation coupling with which the electromotive force of a voltage source can be measured. The potentiometer of the coupling is of the same kind as in the previous resistance bridge coupling. E_x is the studied voltage source and E_n is a standard element. The standard element is a voltage source for which the output voltage is known accurately.

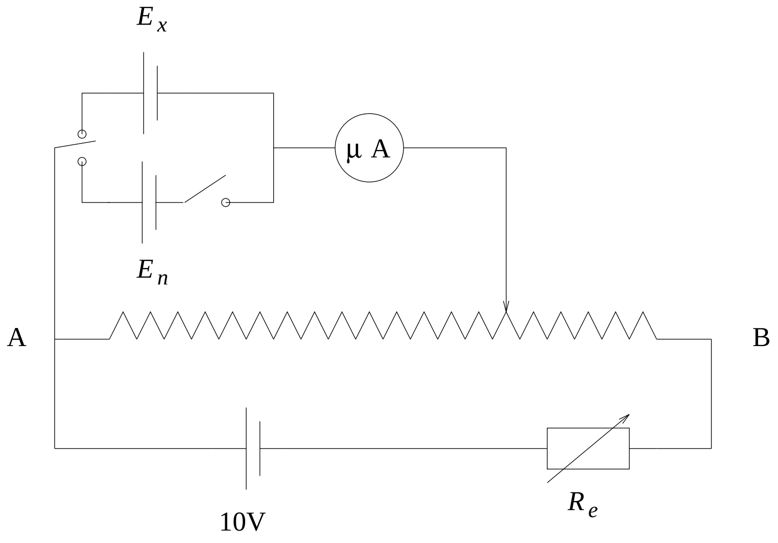

Figure 3: Compensation coupling for measuring the electromotive force (see text).

When E_n and E_x are switched off a current flow I appears in the main circuit. Let us switch E_x on to the circuit and find the point C_x of the potentiometer for which there is no current flow through the μA-meter. If R_x is the resistance of the interval AC_x, then

$$IR_x = E_x. \tag{7}$$

Correspondingly, by switching the standard element on to the circuit and by finding the point C_n for which there is no current flow through the μA-meter. Since the current in the main circuit is the same in both cases we get

$$IR_n = E_n. \tag{8}$$

From two previous equations we now get

$$E_x = \frac{R_x}{R_n} E_n = \frac{l_x}{l_n} E_n. \tag{9}$$

2.3 Quantification of the internal resistance of a voltage source

In order to measure the internal resistance we add a load circuit to the voltage source E_x (Figure 4). In this case a current flow I passes through the voltage source. As previously, we get the EMF of the voltage source with a compensation method and correspondingly the terminal voltage V_x [in equation (9) E_x can be replaced with V_x] when the load circuit is switched on. The internal resistance of the voltage source is then

$$R_i = \frac{1}{I}(E_x - V_x). \tag{10}$$

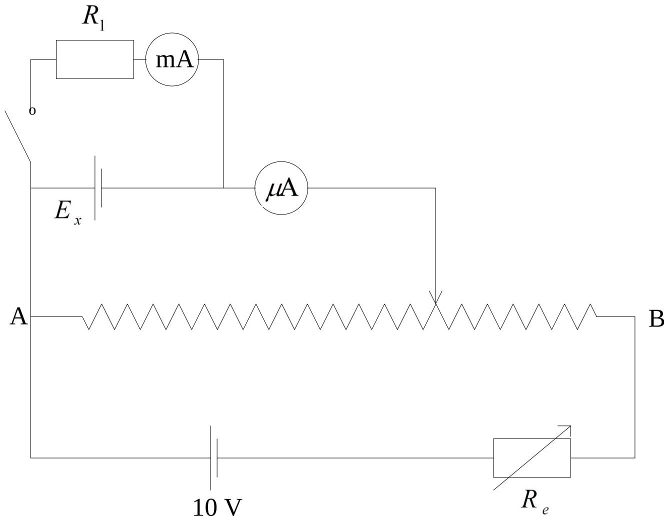

Figure 4: Coupling for defining the internal resistance of a voltage source (see text).

3 Execution of the work

In the first part of the work, the internal resistance of a multimeter is measured with a Wheatstone's bridge for three measuring ranges defined by the tutor using the coupling presented in Figure 1. The obtained results are compared to the manufacturers values, which can be read from the bottom of the meter. In order to obtain accurate results the reference resistor should be selected so that the equilibrium of the bridge is near the midpoint of the resistance wire. A digital multimeter is used as a mV-meter.

In the second part, the electromotive force of a battery is measured with the compensation coupling in Figure 3. For comparison the terminal voltage of the battery is measured with a multimeter. As the standard element E_n an adjustable voltage source (the output is set to 1 V with a multimeter) is used. In the calculations this voltage and other multimeter readings can be considered accurate. The measured voltage source should not be loaded too much. Before the measurements, resistor R_e is adjusted so that the compensation point (no current flow through the μA-meter) is near B when E_x is switched on to the circuit. This way the whole length of the wire can be utilized. The measurement will be repeated four times by changing the R_e slightly.

In the last part, the internal resistance of the battery is measured with the coupling in Figure 4 for different loads of E_x. The load current is altered between $0 - 100$ mA with R_1 (ten observations) and R_i is measured for each load of E_x using equation (10). In order to get the terminal voltage V_x from equation (9) one of the previously used values of R_e must be applied so that the corresponding l_n value can be used in the calculation. The best results are obtained by loading the studied voltage source only temporarily. In other words, the switch is closed only while making observations. In the report R_i is presented as a function of I.

Error estimations are required in the first two parts of the work. The accuracy of lengths l_1 and l_2 is observed experimentally by finding the change in the potentiometer slider that causes a change in the current meter. Also the errors of the used resistors must be catered for.

Equipments

Two adjustable voltage sources, 100 Ω potentiometer, 0.1 Ω step decade resistor, 1 kΩ decade bias resistor, Keithley 160 B multimeter, two digital multimeters, 1.5 V battery, switch.

Related bibliography

H. Young and R. Freedman. *Sears and Zemansky's University Physics with modern physics* (10th edition). Addison Wesley Longman, Inc., pages 799-864, 2000.